CHAPTER 1

1. (a) From Fig. 11a, the atom at the center of the cube is surround by four equidistant nearest neighbors that lie at the corners of a tetrahedron. Therefore the distance between nearest neighbors in silicon $(a = 5.43\ \text{Å})$ is

$$1/2[(a/2)^2 + (\sqrt{2}a/2)^2]^{1/2} = 3a/4 = 2.35\ \text{Å}\ .$$

   (b) For the (100) plane, there are two atoms (one central atom and 4 corner atoms each contributing $1/4$ of an atom for a total of two atoms as shown in Fig. 4a) for an area of a^2, therefore we have

$$2/a^2 = 2/(5.43\times10^{-8})^2 = 6.78\times10^{14}\,\text{atoms/cm}^2$$

   Similarly we have for (110) plane (Fig. 4a and Fig. 6)

$$(2+2\times1/2+4\times1/4)/\sqrt{2a}^{\,2} = 9.6\times10^{15}\,\text{atoms/cm}^2\,,$$

   and for (111) plane (Fig. 4a and Fig. 6)

$$(3\times1/2+3\times1/6)/1/2(\sqrt{2}a)\left(\sqrt{\left(\frac{3}{2}\right)}a\right) = \frac{2}{\left(\frac{\sqrt{3}}{2}\right)a^2} = 7.83\times10^{14}\,\text{atoms/cm}^2.$$

2. (a) For the simple cubic, a unit cell contains $1/8$ of a sphere at each of the eight corners for a total of one sphere.

   $\therefore$ Maximum fraction of cell filled

   $=$ no. of sphere $\times$ volume of each sphere/unit cell volume

   $= 1\times4\pi(a/2)^3/a^3 = 52\ \%$

   (b) For a face-centered cubic, a unit cell contains $1/8$ of a sphere at each of the eight corners for a total of one sphere. The fcc also contains half a sphere at each of the six faces for a total of three spheres. The nearest neighbor distance is $1/2(a\sqrt{2})$. Therefore the radius of each sphere is $1/4(a\sqrt{2})$.

   $\therefore$ Maximum fraction of cell filled

   $= (1+3)\{4\pi[(a/2)/4]^3/3\}/a^3 = 74\ \%.$

   (c) For a diamond lattice, a unit cell contains $1/8$ of a sphere at each of the eight corners for a total of one sphere, $1/2$ of a sphere at each of the six faces for a total of three spheres, and 4 spheres inside the cell. The diagonal distance between $(1/2, 0, 0)$ and $(1/4, 1/4, 1/4)$ shown in Fig. 9a is

$$\mathrm{D} = \frac{1}{2}\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{4}\sqrt{3}$$

The radius of the sphere is $\mathrm{D}/2 = \frac{a}{8}\sqrt{3}$

$\therefore$ Maximum fraction of cell filled

$$= (1+3+4)\left[\frac{4\pi}{3}\left(\frac{a}{8}\sqrt{3}\right)\right]^3 / a^3 = \pi\sqrt{3}/16 = 34\%.$$

This is a relatively low percentage compared to other lattice structures.

3. Taking the reciprocals of these intercepts we get $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. The smallest three integers having the same ratio are 6, 4, and 3. The plane is referred to as (643) plane.

4. The heights at X, Y, and Z point are $\frac{3}{4}, \frac{1}{4}$, and $\frac{3}{4}$.

5. Length of diagonal $= \left[a^2 + a^2 + a^2\right]^{½} = r + 2r + r$

$[3a^2]^{½} = 4r.$

Thus: $a = 4r/[3]^{½}$

6. (a) The lattice constant for GaAs is $5.65\ \mathrm{\AA}$, and the atomic weights of Ga and As are 69.72 and 74.92 g/mole, respectively. There are four gallium atoms and four arsenic atoms per unit cell, therefore

$$4/a^3 = 4/(5.65\times10^{-8})^3 = 2.22\times10^{22} \text{ Ga or As atoms/cm}^2,$$

$$\text{Density} = (\text{no. of atoms/cm}^3 \times \text{atomic weight})/\text{Avogadro constant}$$
$$= 2.22\times10^{22}(69.72+74.92)/6.02\times10^{23} = 5.33 \text{ g/cm}^3.$$

(b) If GaAs is doped with Sn and Sn atoms displace Ga atoms, donors are formed, because Sn has four valence electrons while Ga has only three. The resulting semiconductor is n-type.

7. $E_g(T) = 1.17 - \frac{4.73\times10^{-4}T^2}{(T+636)}$ for Si

$\therefore E_g(100\ \mathrm{K}) = 1.163$ eV, and $E_g(600\ \mathrm{K}) = 1.032$ eV

$$E_g(T) = 1.519 - \frac{5.405\times10^{-4}T^2}{(T+204)} \text{ for GaAs}$$

$\therefore E_g(100\text{ K}) = 1.501$ eV, and $E_g(600\text{ K}) = 1.277$ eV.

8. Effective mass $m^* = (d^2E/dp^2)^{-1}$. Using given relation between E and p then ultimately leads to: $m^* = m_0(1+2\alpha E)^3$.

9. The density of holes in the valence band is given by integrating the product $N(E)[1-F(E)]dE$ from top of the valence band (E_V taken to be $E=0$) to the bottom of the valence band E_{bottom}:

$$p = \int_0^{E_{bottom}} N(E)[1-F(E)]dE \quad (1)$$

where $1-\text{F(E)} = 1-\{1/[1+e^{(E-E_F)/kT}]\}=[1+e^{(E-E_F)/kT}]^{-1}$

If $E_F - E \gg kT$ then

$$1-F(E) \sim \exp[-(E_F-E)/kT] \qquad (2)$$

Then from Appendix H and , Eqs. 1 and 2 we obtain

$$p = 4\pi[2m_p/h^2]^{3/2}\int_0^{E_{bottom}} E^{1/2}\exp[-(E_F-E)/kT]dE \qquad (3)$$

Let $x \equiv E/kT$, and let $E_{bottom} = -\infty$, Eq. 3 becomes

$$p = 4\pi(2m_p/h^2)^{3/2}(k\text{T})^{3/2}\exp[-(E_F/kT)]\int_0^{-\infty} x^{1/2}\text{e}^x\text{d}x$$

where the integral on the right is of the standard form and equals $\sqrt{\pi}/2$.

$$\therefore p = 2[2\pi m_p kT/h^2]^{3/2}\exp[-E_F/kT]$$

By referring to the top of the valence band as E_V instead of $E=0$ we have,

$$p = 2(2\pi m_p kT/h^2)^{3/2}\exp[-(E_F-E_V)/kT]$$

or $\quad p = N_V\exp[-(E_F-E_V)/kT]$

where $\quad N_V = 2(2\pi m_p kT/h^2)^3$.

10. From Eq. 18

$$N_V = 2(2\pi m_p kT/h^2)^{3/2}$$

The effective mass of holes in Si is

$$\begin{aligned} mp &= (N_V/2)^{2/3}(h^2/2\pi kT) \\ &= \left(\frac{2.66\times10^{19}\times10^{6}\,\mathrm{m}^{-3}}{3}\right)^{2/3}\frac{(6.625\times10^{-34})^2}{2\pi(1.38\times10^{-23})(300)} \\ &= \ 9.4\times10^{-31}\,\mathrm{kg} = 1.03\ m_0. \end{aligned}$$

Similarly, we have for GaAs

$$m_p = 3.9\times10^{-31}\,\mathrm{kg} = 0.43\ m_0.$$

11. Using Eq. 19

$$\begin{aligned} E_i &= (E_C+E_V)/2 + (kT/2)\ln(N_V/N_C) \\ &= (E_C+E_V)/2 + (3kT/4)\ln[(m_p/m_n)(6)^{2/3}] \end{aligned} \qquad (1)$$

At 77 K

$$\begin{aligned} E_i &= (1.16/2) + (3\times1.38\times10^{-23}T)/(4\times1.6\times10^{-19})\ln(1.0/0.62) \\ &= 0.58 + 3.29\times10^{-5}T = 0.58 + 2.54\times10^{-3} = 0.583\ \mathrm{eV}. \end{aligned}$$

At 300 K

$$E_i = (1.12/2) + (3.29\times10^{-5})(300) = 0.56 + 0.009 = 0.569\ \mathrm{eV}.$$

At 373 K

$$E_i = (1.09/2) + (3.29\times10^{-5})(373) = 0.545 + 0.012 = 0.557\ \mathrm{eV}.$$

Because the second term on the right-hand side of the Eq.1 is much smaller compared to the first term, over the above temperature range, it is reasonable to assume that E_i is in the center of the forbidden gap.

12.

$$\begin{aligned} KE &= \frac{\int_{E_C}^{E_{top}} (E-E_C)\sqrt{E-E_C}\,e^{-(E-E_F)/kT}\,\mathrm{d}E}{\int_{E_C}^{E_{top}} \sqrt{E-E_C}\,e^{-(E-E_F)/kT}\,\mathrm{d}E}\Bigg|_{x\equiv(E-E_C)} \\ &= kT\frac{\int_0^\infty x^{3/2}\mathrm{e}^{-x}\mathrm{d}x}{\int_0^\infty x^{1/2}\mathrm{e}^{-x}\mathrm{d}x} = kT\frac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{3}{2}\right)} = kT\frac{1.5\times0.5\times\sqrt{\pi}}{0.5\sqrt{\pi}} \\ &= \frac{3}{2}kT. \end{aligned}$$

13. (a) $p = mv = 9.109\times10^{-31}\times10^{5} = 9.109\times10^{-29}\,\text{kg}-\text{m/s}$

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}}{9.109\times10^{-26}} = 7.27\times10^{-9}\,\text{m} = 72.7\ \text{Å}$$

(b) $\lambda_n = \dfrac{m_0}{m_p}\lambda = \dfrac{1}{0.063}\times72.7 = 1154\ \text{Å}$.

14. $p \approx N_A \approx 10^{17}\,cm^{-3}$. Now at 300 °K:$n_i = 9.65\times10^{9}$.

But from Eq. 19: $n_i(200)/n_i(300) = [\exp(-E_g/(2k\,200))]/[\exp(-E_g/(2k\,300))]$

For silicon: . $E_g = 1.12$ eV.

$\rightarrow n_i(77) = 9.65\times10^{9}[\exp(-1.12/(2\,k\ 200))]/[\exp(-1.12/(2\,k\ 300))] = 1.9473\times10^{5}\,\text{cm}^{-3}$.

From Eq. 17, $n \approx n_i^2/N_A \approx (1.9473\times10^{5})^2/10^{17} \approx 3.7923\times10^{-7}\,\text{cm}^{-3}$.

15. From $Ec - E_F = kT\ln[N_C/(N_D - N_A)]$

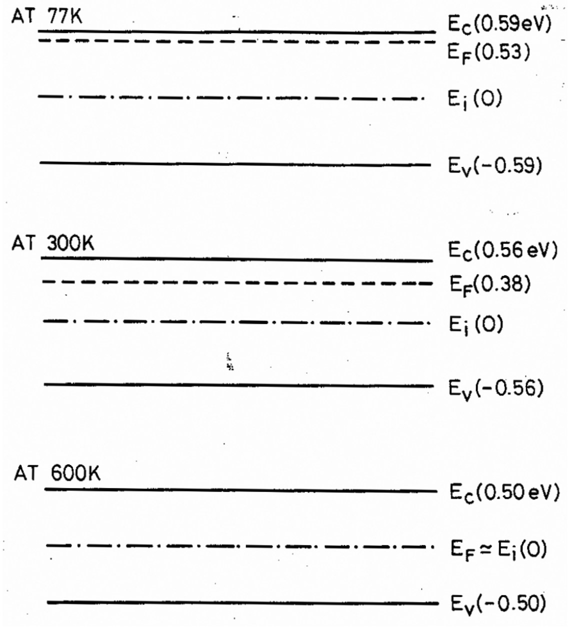
which can be rewritten as $N_D - N_A = N_C\exp[-(E_C - E_F)/kT]$

Then $N_D - N_A = 2.86\times10^{19}\exp(-0.20/0.0259) = 1.26\times10^{16}\,\text{cm}^{-3}$

or $N_D = 1.26\times10^{16} + N_A = 2.26\times10^{16}\,\text{cm}^{-3}$

A compensated semiconductor can be fabricated to provide a specific Fermi energy level.

16. From Fig. 28a we can draw the following energy-band diagrams:



17. Choose the valance band-edge as the zero reference. So then: $E_c = 1.12$ eV and $E_i \sim 0.56$ eV. The given Fermi level E_F is: 0.92 eV. The sample is clearly n-type.

Using Eq. 13 with $N_c = 2.86\times10^{19}\,\text{cm}^{-3} : n = N_c \exp[(E_F - E_c)/(kT)]$

$\rightarrow n = 2.86\times10^{19} \exp[(0.92-1.12)/(0.0259)] = 1.2668\times10^{16}\ \text{cm}^{-3}$.

Hence: $N_D \sim n = 1.2668\times10^{16}\ \text{cm}^{-3}$.

Thus: $p = n_i^2/n \sim (9.65\times10^9)^2/1.2668\times10^{16} = 7350.45\ \text{cm}^{-3}$.

18. From Eq. 27: $p + N_D = n + N_A$.

$p + 8\times10^{16} = n + 10^{16}$

$p + 7\times10^{16} = n.$

Using: $pn = n_i^2$ leads to: $p(p + 7\times10^{16}) = (9.65\times10^9)^2$. Solving leads to: $p = 1330.32\ \text{cm}^{-3}$.

Since: $p = n_i \exp[(E_i - E_F)/(kT)]$

$1330.32 = 9.65\times10^9 \exp[(E_i - E_F)/(kT)].$

Thus: $(E_i - E_F)/(kT) = -15.797$

$E_F = E_i + 0.40914 = 1.12/2 + 0.40914 = 0.96914$ eV.

19. (1) $N_D = 2N_A (2) 2N_A$

20. Assuming complete ionization, the Fermi level measured from the intrinsic Fermi level is 0.35 eV for 10^{15}cm^{-3}, 0.45 eV for 10^{17}cm^{-3}, and 0.54 eV for 10^{19}cm^{-3}.

The number of electrons that are ionized is given by

$$n \cong N_D[1 - F(E_D)] = N_D/[1 + e^{-(E_D - E_F)/kT}]$$

Using the Fermi levels given above, we obtain the number of ionized donors as

$n = 10^{15}\text{cm}^{-3}$ for $N_D = 10^{15}\text{cm}^{-3}$

$n = 0.93\times10^{17}\text{cm}^{-3}$ for $N_D = 10^{17}\text{cm}^{-3}$

$n = 0.27\times10^{19}\text{cm}^{-3}$ for $N_D = 10^{19}\text{cm}^{-3}$

Therefore, the assumption of complete ionization is valid only for the case of 10^{15}cm^{-3}.

21. $$N_D^+ = \frac{10^{16}}{1+e^{-(E_D-E_F)/kT}} = \frac{10^{16}}{1+e^{-0.135}}$$

$$= \frac{10^{16}}{1+\frac{1}{1.145}} = 5.33\times10^{15}\,\text{cm}^{-3}$$

The neutral donor $= 10^{16} - 5.33\times10^{15}\,\text{cm}^{-3} = 4.67\times10^{15}\,\text{cm}^{-3}$

$\therefore$ The ratio of $\frac{N_D^O}{N_D^+} = \frac{4.76}{5.33} = 0.876$.

CHAPTER 2

1. (a) For intrinsic Si, $\mu_n = 1450, \mu_p = 505,$ and $n = p = n_i = 9.65 \times 10^9$

We have $\rho = \dfrac{1}{qn\mu_n + qp\mu_p} = \dfrac{1}{qn_i(\mu_n + \mu_p)} = 3.31 \times 10^5$ Ω-cm

(b) Similarly for GaAs, $\mu_n = 9200, \mu_p = 320,$ and $n = p = n_i = 2.25 \times 10^6$

We have $\rho = \dfrac{1}{qn\mu_n + qp\mu_p} = \dfrac{1}{qn_i(\mu_n + \mu_p)} = 2.92 \times 10^8$ Ω-cm.

2. The mobility $= q * \tau / m$. Using MKS (SI) units:

$\rightarrow\ 1000 \times 10^{-4} m^2/Vs = [1.6 \times 10^{-19}\tau] / (0.26 \times 9.1 \times 10^{-31} Kg).$

$\rightarrow \tau = 1.4787 \times 10^{-13} s.$

3. (a) $p = 5 \times 10^{15}\,\text{cm}^{-3},\ n = n_i^2/p = (9.65 \times 10^9)^2 / 5 \times 10^{15} = 1.86 \times 10^4\,\text{cm}^{-3}$

$\mu_p = 410\ \text{cm}^2/\text{V-s},\ \mu_n = 1300\ \text{cm}^2/\text{V-s}$

$\rho = \dfrac{1}{q\mu_n n + q\mu_p p} \approx \dfrac{1}{q\mu_p p} = 3\ \Omega\text{-cm}$

(b) $p = N_A - N_D = 2 \times 10^{16} - 1.5 \times 10^{16} = 5 \times 10^{15}\,\text{cm}^{-3},\ n = 1.86 \times 10^4\,\text{cm}^{-3}$

$\mu_p = \mu_p(N_A + N_D) = \mu_p(3.5 \times 10^{16}) = 290\ \text{cm}^2/\text{V-s},$

$\mu_n = \mu_n(N_A + N_D) = 1000\ \text{cm}^2/\text{V-s}$

$\rho = \dfrac{1}{q\mu_n n + q\mu_p p} \approx \dfrac{1}{q\mu_p p} = 4.3\ \Omega\text{-cm}$

(c) $p = N_A(\text{Boron}) - N_D + N_A(\text{Gallium}) = 5 \times 10^{15}\,\text{cm}^{-3},\ n = 1.86 \times 10^4\,\text{cm}^{-3}$

$\mu_p = \mu_p(N_A + N_D + N_A) = \mu_p(2.05 \times 10^{17}) = 150\ \text{cm}^2/\text{V-s},$

$\mu_n = \mu_n(N_A + N_D + N_A) = 520\ \text{cm}^2/\text{V-s}$

$\rho = 8.3\ \Omega\text{-cm}.$

4. Since $\dfrac{1}{\mu} = \dfrac{1}{\mu_1} + \dfrac{1}{\mu_2}$

$\therefore\ \dfrac{1}{\mu} = \dfrac{1}{250} + \dfrac{1}{500}\quad \mu = 167\ \text{cm}^2/\text{V-s}.$

5. (a) $(N_v/N_c) = [m_h/m_e]^{3/2}$

$\rightarrow m_h = m_e(N_v/N_c)^{2/3} = 9.1\times10^{-32}[2.5/3.0]^{2/3} = 8.058\times10^{-32}\,\text{Kg}.$

(b) $n_i^2 = [N_v N_c]\exp[-E_g/(k_B T)] \rightarrow$

$$E_g = (k_B T)Ln\{[N_v N_c]/n_i^2\}$$
$$= [1.38\times10^{-23}\times300/1.6\times10^{-19}]\ln(7.5\times10^{18}) \rightarrow$$
$$E_g = 1.1245\ \text{eV}.$$

(c) Fermi energy ~ $[E_g/2]$ below conduction band.

$\rightarrow E_F = -0.56228$ eV with respect to the conduction band.

(d) Thermal velocity $v_{th} = [3k_B T/m]^{1/2}$.

$\rightarrow v_{th} = [3\times1.38\times10^{-23}\times300/8.058\times10^{-32}]^{1/2} \sim 3.9261\times10^{5}$ m/s.

6 Hall coefficient,

$$R_H = \frac{V_H A}{I B_z W} = \frac{10\times10^{-3}\times1.6\times10^{-3}}{2.5\times10^{-3}\times(30\times10^{-9}\times10^{4})\times0.05} = 426.7\ \text{cm}^3/\text{C}$$

Since the sign of R_H is positive, the carriers are holes. From Eq. 22

$$p = \frac{1}{qR_H} = \frac{1}{1.6\times10^{-19}\times426.7} = 1.46\times10^{16}\ \text{cm}^{-3}$$

Assuming $N_A \approx p$, from Fig. 7 we obtain $\rho = 1.1\ \Omega$-cm

The mobility μ_p is given by Eq. 15b

$$\mu_p = \frac{1}{qp\rho} = \frac{1}{1.6\times10^{-19}\times1.46\times10^{16}\times1.1} = 380\ \text{cm}^2/\text{V-s}.$$

7. At the limit when $d \gg s, CF = \dfrac{\pi}{\ln 2} = 4.53$. Then from Eq. 16

$$\rho = \frac{V}{I}\times W\times CF\,\frac{10\times10^{-3}}{1\times10^{-3}}\times50\times10^{-4}\times4.53 = 0.226\ \Omega\text{-cm}$$

From Fig. 6, $CF = 4.2(d/s = 10)$; using the $a/d = 1$ curve we obtain

$$V = \rho\cdot I/(W\cdot CF) = \frac{0.226\times10^{-3}}{50\times10^{-4}\times4.2} = 10.78\ \text{mV}.$$

8. $\sigma = q(\mu_n n + \mu_p p) = q\mu_p(bn + n_i^2/n)$

From the condition $d\sigma/dn = 0$, we obtain

$$n = n_i/\sqrt{b}$$

Therefore

$$\frac{\rho_m}{\rho_i} = \frac{\dfrac{1}{q\mu_p(bn_i/\sqrt{b}+\sqrt{b}n_i)}}{\dfrac{1}{q\mu_p n_i(b+1)}} = \frac{b+1}{2\sqrt{b}}.$$

9. Since $R \propto \rho$ and $\rho = \dfrac{1}{qn\mu_n + qp\mu_p}$, hence $R \propto \dfrac{1}{n\mu_n + p\mu_p}$

From Einstein relation $D \propto \mu$

$$\mu_n/\mu_p = D_n/D_p = 50$$

$$\frac{R_1}{0.5R_1} = \frac{\dfrac{1}{N_D\mu_n}}{\dfrac{1}{N_D\mu_n + N_A\mu_p}}$$

We have $N_A = 50\,N_D$.

10. Yes, in case of non-uniform internal doping.

11. (a) From Eq. 31, $J_n = 0$ and

$$\mathrm{E}(x) = -\frac{D_n}{\mu_n}\frac{dn/dx}{n} = -\frac{kT}{q}\frac{N_0(-a)e^{-ax}}{N_0e^{-ax}} = +\frac{kT}{q}a$$

(b) $\mathscr{E}(x) = 0.0259(10^4) = 259$ V/cm .

12. At thermal and electric equilibria,

$$J_n = q\mu_n n(x)\mathrm{E} + qD_n\frac{dn(x)}{dx} = 0$$

$$\mathrm{E}(x) = -\frac{D_n}{\mu_n}\frac{1}{n(x)}\frac{dn(x)}{dx} = -\frac{D_n}{\mu_n}\frac{1}{N_0 + (N_L - N_0)(x/L)}\frac{N_L - N_0}{L}$$

$$= -\frac{D_n}{\mu_n}\frac{N_L - N_0}{LN_0 + (N_L - N_0)x}$$

$$V = \int_0^L -\frac{D_n}{\mu}\frac{N_L - N_0}{LN_0 + (N_L - N_0)x} = -\frac{D_n}{\mu_n}\ln\frac{N_L}{N_0}.$$

13. $\Delta n = \Delta p = \tau_p G_L = 10\times10^{-6}\times10^{16} = 10^{11}\ \text{cm}^{-3}$

$$n = n_{no} + \Delta n = N_D + \Delta n = 10^{15} + 10^{11} \approx 10^{15}\ \text{cm}^{-3}$$

$$p = \frac{n_i^2}{N_D} + \Delta p = \frac{(9.65\times10^9)^2}{10^{15}} + 10^{11} \approx 10^{11}\ \text{cm}^{-3}.$$

14. (a) $\tau_p \approx \dfrac{1}{\sigma_p v_{th} N_t} = \dfrac{1}{5\times10^{-15}\times10^{7}\times2\times10^{15}} = 10^{-8}\text{s}$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{9\times10^{-8}} = 3\times10^{-4}\ \text{cm}$$

$$S_{tr} = v_{th}\sigma_s N_{sts} = 10^7\times2\times10^{-16}\times10^{10} = 20\ \text{cm/s}$$

(b) The hole concentration at the surface is given by Eq. 67

$$p_n(0) = p_{no} + \tau_p G_L\left(1 - \frac{\tau_p S_{lr}}{L_p + \tau_p S_{lr}}\right)$$

$$= \frac{(9.65\times10^9)^2}{2\times10^{16}} + 10^{-8}\times10^{17}\left(1 - \frac{10^{-8}\times20}{3\times10^{-4} + 10^{-8}\times20}\right)$$

$$\approx 10^9\ \text{cm}^{-3}.$$

15. Total excess holes $= \int \Delta p(x)\, A\, dx$ where the integration goes from 0 to $2\ \mu m$.

Here:

$$\Delta p(x) = 5\times10^{14}(1 - 0.5\times10^4 x),\ \text{and}\ A = 10^{-4}\text{cm}^2.$$

Thus excess hole number $= 5\times10^6$.

Next, rate of total hole recombination: (excess hole number)/(hole lifetime)

$$= (5\times10^6)/(10^{-4}) = 5\times10^{10}\text{s}^{-1}.$$

16. $\sigma = qn\mu_n + qp\mu_p$

Before illumination

$$n_n = n_{no},\ p_n = p_{no}$$

After illumination

$$n_n = n_{no} + \Delta n = n_{no} + \tau_p G,$$
$$p_n = p_{no} + \Delta p = p_{no} + \tau_p G$$

$$\Delta\sigma = [q\mu_n(n_{no} + \Delta n) + q\mu_p(p_{no} + \Delta p)] - (q\mu_n n_{no} + q\mu_p p_{no})$$
$$= q(\mu_n + \mu_p)\tau_p G.$$

17. (a) $J_{p,\text{diff}} = -qD_p \dfrac{dp}{dx}$

$$= -1.6\times10^{-19}\times12\times\frac{1}{12\times10^{-4}}\times10^{15}\exp(-x/12)$$

$$= 1.6\exp(-x/12)\ \text{A/cm}^2$$

(b) $J_{n,\text{drift}} = J_{total} - J_{p,\text{diff}}$

$$= 4.8 - 1.6\exp(-x/12)\ \text{A/cm}^2$$

(c) $\because J_{n,\text{drift}} = qn\mu_n\text{E}$

$\therefore 4.8 - 1.6\exp(-x/12) = 1.6\times10^{-19}\times10^{16}\times1000\times\text{E}$

$\text{E} = 3 - \exp(-x/12)$ V/cm.

18. Diffusion current density $J_n = qD_n dn/dx = kT\mu_n dn/dx$. In this case:

$J_n = kT\mu_n dn/dx = 1.38\times10^{-23}\times300\times1500(0-10^{16})/(10^{-4}) = 621\ \text{A/cm}^2$.

The field required for same current density:

$E = J_n/(nq\mu_n) = 621/(1.6\times10^{-19}\times10^{16}\times1500)$.

$\rightarrow E = 258.75$ V/cm.

19. In steady state, the recombination rate at the surface and in the bulk is equal

$$\frac{\Delta p_{n,\,\text{bulk}}}{\tau_{p,\,\text{bulk}}} = \frac{\Delta p_{n,\,\text{surface}}}{\tau_{p,\,\text{surface}}}$$

so that the excess minority carrier concentration at the surface

$$\Delta p_{n,\,\text{surface}} = 10^{14}\cdot\frac{10^{-7}}{10^{-6}} = 10^{13}\ \text{cm}^{-3}$$

The generation rate can be determined from the steady-state conditions in the bulk

$$G = \frac{10^{14}}{10^{-6}} = 10^{20}\ \text{cm}^{-3}\text{s}^{-1}$$

From Eq. 62, we can write

$$D_p = \frac{\partial^2\Delta p}{\partial x^2} + G - \frac{\Delta p}{\tau_p} = 0$$

The boundary conditions are $\Delta p(x=\infty) = 10^{14}\text{cm}^{-3}$ and $\Delta p(x=0) = 10^{13}\text{cm}^{-3}$

Hence $\Delta p(x) = 10^{14}(1 - 00.9e^{-1/L_p})$

Where $Lp = \sqrt{10\cdot10^{-6}} = 31.6\ \mu\text{m}$.

20. (a). The potential barrier height

$$\varphi = \varphi - X_{Bm} = 4.2 - 4.0 = 0.2 \text{ volts}.$$

(b). The portion of injection current that reaches the opposite surface by diffusion is given by

$$\alpha_0 = \frac{J_p(W)}{J_p(0)} = \frac{1}{\cosh(W/L_p)}$$

$$L_p \equiv \sqrt{D_p \tau_p} = \sqrt{50 \times 50 \times 10^{-6}} = 5 \times 10^{-2} \text{ cm}$$

$$\therefore \alpha_0 = \frac{1}{\cosh(10^{-2}/5 \times 10^{-2})} = 0.98$$

Therefore, 98% of the injected current can reach the opposite surface.

21. (a) For $\mathrm{E} = 0$ we have

$$\frac{\partial p}{\partial t} = -\frac{p_n - p_{no}}{\tau_p} + D_p \frac{\partial^2 p_n}{\partial x^2} = 0$$

at steady state, the boundary conditions are $p_n(x = 0) = p_n(0)$ and $p_n(x = W) = p_{no}$. Therefore

$$p_n(x) = p_{no} + [p_n(0) - p_{no}] \left[\frac{\sinh\left(\frac{W - x}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)} \right]$$

$$J_p(x = 0) = -qD_p \left. \frac{\partial p_n}{\partial x} \right|_{x=0} = q[p_n(0) - p_{no}] \frac{D_p}{L_p} \coth\left(\frac{W}{L_p}\right)$$

$$J_p(x = W) = -qD_p \left. \frac{\partial p_n}{\partial x} \right|_{x=W} = q[p_n(0) - p_{no}] \frac{D_p}{L_p} \frac{1}{\sinh\left(\frac{W}{L_p}\right)}.$$

22. The potential barrier height

$$\phi_B = \phi_m - \chi = 4.2 - 4.0 = 0.2 \text{ volts}.$$

23. The number of electrons occupying the energy level between E and $E + dE$ is $dn = N(E)F(E)dE$

where $N(E)$ is the density-of-state function, and $F(E)$ is Fermi-Dirac distribution function. Since only electrons with an energy greater than $E_F + q\phi_m$

and having a velocity component normal to the surface can escape the solid, the thermionic current density is

$$J = \int qv_x = \int_{E_p + q\phi_m}^{\infty} \frac{4\pi(2m)^{3/2}}{h^3} v_x E^{1/2} e^{-(E-E_F)/kT} dE$$

where v_x is the component of velocity normal to the surface of the metal. Since the energy-momentum relationship

$$E = \frac{P^2}{2m} = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

Differentiation leads to $dE = \frac{PdP}{m}$

By changing the momentum component to rectangular coordinates,

$$4\pi P^2 dP = dp_x dp_y dp_z$$

$$J = \frac{2q}{mh^3} \int_{p_{x0}}^{\infty} \int_{p_y=-\infty}^{\infty} \int_{p=-\infty}^{\infty} p_x e^{-(p_x^2+p_y^2+p_z^2-2mE_f)/2mkT} dp_x dp_y dp_z$$

Hence $= \frac{2q}{mh^3} \int_{p_{x0}}^{\infty} e^{-(p_x^2-2mE_f)/2mkT} p_x dp_x \int_{-\infty}^{\infty} e^{-(p_y^2/2mkT} dp_y \int_{-\infty}^{\infty} e^{-p_z^2/2mkT} dp_z$

where $p_{x0}^2 = 2m(E_F + q\phi_m)$.

Since $\int_{-\infty}^{\infty} e^{-ax^2} dx = \left(\frac{\pi}{a}\right)^{1/2}$, the last two integrals yield $(2\pi mkT)^{1/2}$.

The first integral is evaluated by setting $\frac{p_x^2 - 2mE_F}{2mkT} = u.$

Therefore we have $du = \frac{p_x dp_x}{mkT}$

The lower limit of the first integral can be written as

$$\frac{2m(E_F + q\phi_m) - 2mE_F}{2mkT} = \frac{q\phi_m}{kT}$$

so that the first integral becomes $mkT = \int_{q\phi_m/kt}^{\infty} e^{-u} du = mKTe^{-q\phi_m/kT}$

Hence $J = \frac{4\pi qmk^2}{h^3} T^2 e^{-q\phi_m/kT} = A^* T^2 \exp\left(\frac{-q\phi_m}{kT}\right).$

24. Equation 79 is the tunneling probability

$$\beta = \sqrt{\frac{2m_n(qV_0 - E)}{\hbar^2}} = \sqrt{\frac{2(9.11\times10^{-31})(20-2)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}} = 2.17\times10^{10}\,\mathrm{m}^{-1}$$

$$T = \left\{1 + \frac{[20\times\sinh(2.17\times10^{0}\times3\times10^{-10})]^2}{4\times2\times(20-2)}\right\}^{-1} = 3.19\times10^{-6}.$$

25. Equation 79 is the tunneling probability

$$\beta = \sqrt{\frac{2m_n(qV_0 - E)}{\hbar^2}} = \sqrt{\frac{2(9.11\times10^{-31})(6-2.2)(1.6\times10^{-19})}{(1.054\times10^{-34})^2}} = 9.99\times10^{9}\,\mathrm{m}^{-1}$$

$$T(10^{-10}) = \left\{1 + \frac{[6\times\sinh(9.99\times10^{9}\times10^{-10})]^2}{4\times2.2\times(6-2.2)}\right\}^{-1} = 0.403$$

$$T(10^{-9}) = \left\{1 + \frac{[6\times\sinh(9.99\times10^{9}\times10^{-9})]^2}{4\times2.2\times(6-2.2)}\right\}^{-1} = 7.8\times10^{-9}.$$

26. Thermal velocity $v_{th} = \sqrt{\frac{2E_{th}}{m_0}} = \sqrt{\frac{2kT}{m_0}}$

$$= \sqrt{\frac{2\times1.38\times10^{-23}\times300}{9.1\times10^{-31}}}$$

$$= 9.5\times10^{4}\,\mathrm{m/s} = 9.5\times10^{6}\ \mathrm{cm/s}$$

For electric field of $100\ \mathrm{V/cm}$, drift velocity

$$v_d = \mu_n\mathrm{E} = 1350\times100 = 1.35\times10^{5}\,\mathrm{cm/s} \lll v_{th}$$

For electric field of $10^4\ \mathrm{V/cm}$.

$$\mu_n\mathrm{E} = 1350\times10^{4} = 1.35\times10^{7}\,\mathrm{cm/s} \approx v_{th}.$$

The value is comparable to the thermal velocity, the linear relationship between drift velocity and the electric field is not valid.

27. From Fig. 22

As $\mathrm{E} = 10^3\ \mathrm{V/s}$

$v_d \approx 1.3\times10^{6}\,\mathrm{cm/s}$ (Si) and $v_d \approx 8.7\times10^{6}\ \mathrm{cm/s}$ (GaAs)

$t \approx 77$ ps (Si) and $t \approx 11.5$ ps (GaAs)

As $E = 5\times10^4\ \mathrm{V/s}$

$v_d \approx 10^{7}\,\mathrm{cm/s}$ (Si) and $v_d \approx 8.2\times10^{6}\ \mathrm{cm/s}$ (GaAs)

$t \approx 10$ ps (Si) and $t \approx 12.2$ ps (GaAs).

CHAPTER 3

1. $V_{bi} = V_T \ln[(N_D N_A)/n_i^2] = 25.8 \ln[2\times10^{15}\times2\times10^{18}/(1.45\times10^{10})^2]$.

Then, $V_{bi} = 0.788$ V.

$W = [2(\varepsilon_{si}/q)(N_A + N_D)/(N_D N_A)V_{bi}]^{1/2}$. Putting in the values:

$W = [2\times11.9\times8.85\times10^{-14}/(1.6\times10^{-19})(2\times10^{18} + 2\times10^{15})/(2\times10^{18}\times2\times10^{15})0.788]^{½}$

So: $W = 7.2\times10^{-5}\,\text{cm}$.

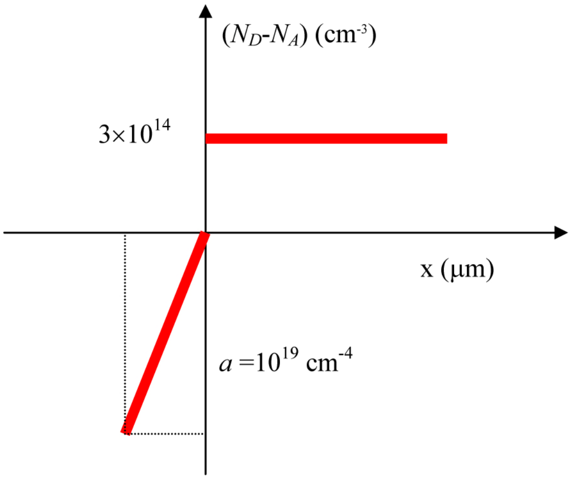
$$x_n = WN_A/[N_A + N_D] = 7.2\times10^{-5} 2\times10^{18}/[2\times10^{18} + 2\times10^{15}]$$
$$= 7.19\times10^{-5}\,\text{cm}$$

$x_p = WN_D/[N_A + N_D] = 7.2\times10^{-5} 2\times10^{15}/[2\times10^{18} + 2\times10^{15}] \sim 10^{-7}\,\text{cm}$.

$E_{\max} = qN_D x_n/\varepsilon_{Si} = 1.6\times10^{-19}\times2\times10^{15}\times7.19\times10^{-5}/(11.9\times8.85\times10^{-14})$.

$E_{\max} = 2.1847\times10^4$ V/cm.

2. The impurity profile is,



The overall space charge neutrality of the semiconductor requires that the total negative space charge per unit area in the p-side must equal the total positive space charge per unit area in the n-side, thus we can obtain the depletion layer width in the n-side region:

$$\frac{0.8\times8\times10^{14}}{2} = W_n\times3\times10^{14}$$

Hence, the n-side depletion layer width is:

$$W_n = 1.067\ \mu\text{m}$$

The total depletion layer width is $1.867\ \mu\text{m}$.

We use the Poisson's equation for calculation of the electric field $E(x)$.

In the n-side region,

$$\frac{d\mathrm{E}}{dx} = \frac{q}{\varepsilon_s} N_D \Rightarrow \mathrm{E}(x_n) = \frac{q}{\varepsilon_s} N_D x + K$$

$$\mathrm{E}(x_n = 1.067 \mu\mathrm{m}) = 0 \Rightarrow K = -\frac{q}{\varepsilon_s} N_D \times 1.067 \times 10^{-4}$$

$$\therefore \mathrm{E}(x_n) = \frac{q}{\varepsilon_s} \times 3 \times 10^{14} (x - 1.067 \times 10^{-4})$$

$$\mathrm{E}_{\max} = \mathrm{E}(x_n = 0) = -4.86 \times 10^3 \text{ V/cm}$$

In the p-side region, the electrical field is:

$$\frac{d\mathrm{E}}{dx} = \frac{q}{\varepsilon_s} N_A \Rightarrow \mathrm{E}(x_p) = \frac{q}{2\varepsilon_s} \times ax^2 + K'$$

$$\mathrm{E}(x_p = -0.8 \mu\mathrm{m}) = 0 \Rightarrow K' = -\frac{q}{2\varepsilon_s} \times a \times (0.8 \times 10^{-4})^2$$

$$\therefore \mathrm{E}(x_p) = \frac{q}{2\varepsilon_s} \times a \times [x^2 - (0.8 \times 10^{-4})^2]$$

$$\mathrm{E}_{\max} = \mathrm{E}(x_p = 0) = -4.86 \times 10^3 \text{ V/cm}$$

The built-in potential is:

$$V_{bi} = \int_{-x_p}^{x_n} \mathrm{E}(x) dx = -\int_{-x_p}^{0} \mathrm{E}(x) dx \bigg|_{p-side} - \int_{0}^{x_n} \mathrm{E}(x) dx \bigg|_{n-side} = 0.52 \text{ V.}$$

3. From $V_{bi} = -\int \mathrm{E}(x) dx$, the potential distribution can be obtained

With zero potential in the neutral p-region as a reference, the potential in the p-side depletion region is

$$V_p(x) = -\int_0^x \mathrm{E}(x) dx = -\int_0^x \frac{q}{2\varepsilon_s} \times a \times [x^2 - (0.8 \times 10^{-4})^2] dx$$

$$= -\frac{qa}{2\varepsilon_s} \left[\frac{1}{3} x^3 - (0.8 \times 10^{-4})^2 x - \frac{2}{3} (0.8 \times 10^{-4})^3 \right]$$

$$= -7.596 \times 10^{11} \times \left[\frac{1}{3} x^3 - (0.8 \times 10^{-4})^2 x - \frac{2}{3} (0.8 \times 10^{-4})^3 \right]$$

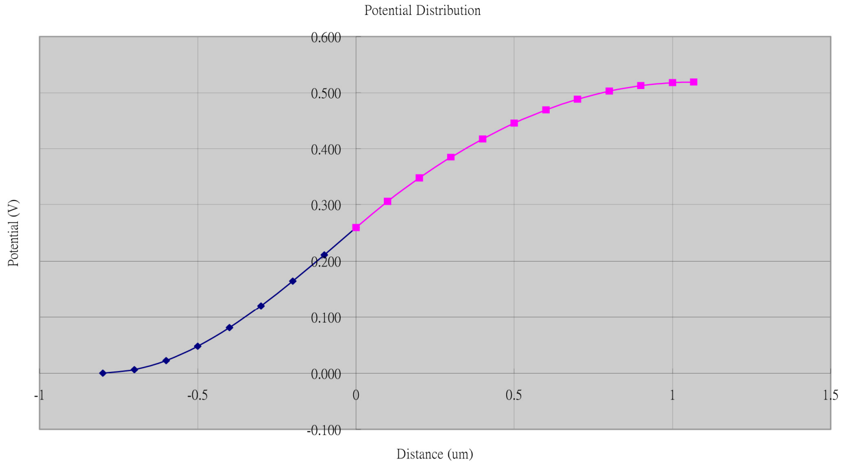
With the condition $V_p(0) = V_n(0)$, the potential in the n-region is

$$V_n(x) = -\frac{q}{\varepsilon_s} \times 3 \times 10^{14} \left(\frac{1}{2} x^2 - 1.067 \times 10^{-4} x + \frac{0.8^3}{9} \times 10^{-7} \right)$$

$$= -4.56 \times 10^7 \times \left(\frac{1}{2} x^2 - 1.067 \times 10^{-4} x - \frac{0.8^3}{9} \times 10^{-7} \right)$$

The potential distribution is

Distance	*p*-region	*n*-region
–0.8	0.000	
–0.7	0.006	
–0.6	0.022	
–0.5	0.048	
–0.4	0.081	
–0.3	0.120	
–0.2	0.164	
–0.1	0.211	
0	0.259	0.259413333
0.1		0.305788533
0.2		0.347603733
0.3		0.384858933
0.4		0.417554133
0.5		0.445689333
0.6		0.469264533
0.7		0.488279733
0.8		0.502734933
0.9		0.512630133
1		0.517965333
1.067		0.518988825



4. The intrinsic carriers density in Si at different temperatures can be obtained by using Fig.22 in Chapter 2 :

Temperature (K)	Intrinsic carrier density (n_i)
250	1.50×10^{8}
300	9.65×10^{9}
350	2.00×10^{11}
400	8.50×10^{12}
450	9.00×10^{13}
500	2.20×10^{14}

The V_{bi} can be obtained by using Eq. 12, and the results are listed in the following table.

T	ni	$Vbi(V)$
250	1.500E+08	0.777
300	9.65E+9	0.717
350	2.00E+11	0.653
400	8.50E+12	0.488
450	9.00E+13	0.366
500	2.20E+14	0.329

Thus, the built-in potential is decreased as the temperature is increased.

The depletion layer width and the maximum field at 300 K are

$$W=\sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}}=\sqrt{\frac{2\times11.9\times8.85\times10^{-14}\times0.717}{1.6\times10^{-19}\times10^{15}}}=0.9715\ \mu\text{m}$$

$$\mathrm{E}_{\max} = \frac{qN_D W}{\varepsilon_s} = \frac{1.6\times10^{-19}\times10^{15}\times9.715\times10^{-5}}{11.9\times8.85\times10^{-14}} = 1.476\times10^{4}\ \text{V/cm.}$$

5. $$\mathrm{E}_{\max} \approx \left[\frac{2qV_R}{\varepsilon_s}\left(\frac{N_A N_D}{N_A+N_D}\right)\right]^{1/2} \Rightarrow 4\times10^{5} = \left[\frac{2\times1.6\times10^{-19}\times30}{11.9\times8.85\times10^{-14}}\left(\frac{10^{18}N_D}{10^{18}+N_D}\right)\right]^{1/2}$$

$$\Rightarrow 1.755\times10^{16} = \frac{N_D}{1+\dfrac{N_D}{10^{18}}}$$

We can select n-type doping concentration of $N_D = 1.755\times10^{16}\text{cm}^{-3}$ for the junction.

6. The built-in potential is calculated from:

$$\phi_l = V_t \ln\frac{p_n^n p}{n_i^2} = 0.0259\ln\frac{10^{16}\times5\times10^{16}}{10^{20}} = 0.76\ \text{V}$$

The depletion layer width is obtained from:

$$x_d = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)(\phi_l - V_a)}$$

the electric field from:

$$\varepsilon(x=0) = -\frac{2(\phi_l - V_a)}{x_d}$$

and the potential across the n-type region equals:

$$\phi_n = \frac{qN_d x_n^2}{2\varepsilon_s}$$

where

$$x_n = x_d\frac{N_a}{N_a+N_d}$$

Also:

$$\phi_n = \frac{(\phi_l - V_a)N_a}{N_a+N_d}$$

This yields the following numeric values:

	$V_a = 0$ V	$V_a = 0.5$ V	$V_a = -2.5$ V
x_d	0.315 μm	0.143 μm	0.703 μm
ε	40 kV/cm	18 kV/cm	89 kV/cm
ϕ_n	0.105 V	0.0216 V	0.522 V

7. From Eq. 12 and Eq. 35, we can obtain the $1/C^2$ versus V relationship for doping concentration of $10^{15}, 10^{16}$ or $10^{17}\,\mathrm{cm}^{-3}$, respectively.

For $N_D = 10^{15}\,\mathrm{cm}^{-3}$,

$$\frac{1}{C_j^2} = \frac{2(V_{bi} - V)}{q\varepsilon_s N_B} = \frac{2\times(0.837 - V)}{1.6\times10^{-19}\times11.9\times8.85\times10^{-14}\times10^{15}} = 1.187\times10^{16}(0.837 - V)$$

For $N_D = 10^{16}\,\mathrm{cm}^{-3}$,

$$\frac{1}{C_j^2} = \frac{2(V_{bi} - V)}{q\varepsilon_s N_B} = \frac{2\times(0.896 - V)}{1.6\times10^{-19}\times11.9\times8.85\times10^{-14}\times10^{16}} = 1.187\times10^{15}(0.896 - V)$$

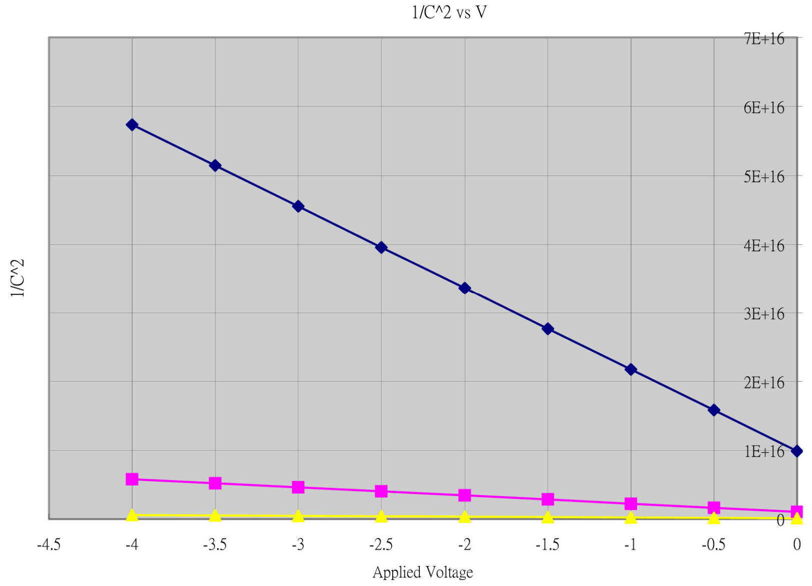
For $N_D = 10^{17}\,\mathrm{cm}^{-3}$,

$$\frac{1}{C_j^2} = \frac{2(V_{bi} - V)}{q\varepsilon_s N_B} = \frac{2\times(0.956 - V)}{1.6\times10^{-19}\times11.9\times8.85\times10^{-14}\times10^{17}} = 1.187\times10^{14}(0.956 - V)$$

When the reversed bias is applied, we summarize a table of $1/C_j^2$ vs V for various N_D values as following,

V	$N_D = 1\mathrm{E}15$	$N_D = 1\mathrm{E}16$	$N_D = 1\mathrm{E}17$
−4	5.741E+16	5.812E+15	5.883E+14
−3.5	5.148E+16	5.218E+15	5.289E+14
−3	4.555E+16	4.625E+15	4.696E+14
−2.5	3.961E+16	4.031E+15	4.102E+14
−2	3.368E+16	3.438E+15	3.509E+14
−1.5	2.774E+16	2.844E+15	2.915E+14
−1	2.181E+16	2.251E+15	2.322E+14
−0.5	1.587E+16	1.657E+15	1.728E+14
0	9.935E+15	1.064E+15	1.134E+14

Hence, we obtain a series of curves of $1/C^2$ versus V as following,



The slopes of the curves is positive proportional to the values of the doping concentration.

The interceptions give the built-in potential of the *p-n* junctions.

8. For a 1-sided P^+ N junction, the capacitance change with voltage yields the doping density at the lightly-doped junction. In this case, the N-region, i.e., $N_D(x)$.

The formula is: $N_D(x) = - C^3/[q\varepsilon_s A^2 (dC/dV)]$. The corresponding values of position "x" are given by: $x = (q\varepsilon_s A)/c$. From the given values of capacitance C at the corresponding voltage V, one obtains:

Voltage Difference (V2-V1)	Capacitances C1, C2 (pF)	$C=(C_1+C_2)/2$	$N_D(x)$ cm^{-3}	x (cm)
0.6 – 0.3 = 0.3	273, 257	265	3.3747×10^{16}	3.0868×10^{-7}
1.5 – 0.6 = 0.9	257, 230	243.5	4.6544×10^{16}	3.3593×10^{-7}
2.1 – 1.5 = 0.6	230, 219	224.5	5.969×10^{16}	3.6436×10^{-7}
2.7 – 2.1 = 0.6	219, 210	214.5	6.3633×10^{16}	3.8135×10^{-7}
3.3 – 2.7 = 0.7	210, 204	207	1×10^{17}	3.9517×10^{-7}

9. From Eq. 35, we can obtain

$$\frac{1}{C_j^2} = \frac{2(V_{bi}-V)}{q\varepsilon_s N_B} \Rightarrow N_D = \frac{2(V_{bi}-V_R)}{q\varepsilon_s} C_j^2$$

$$\because V_R \gg V_{bi} \Rightarrow N_D \cong \frac{2(V_R)}{q\varepsilon_s} C_j^2 = \frac{2\times 4}{1.6\times 10^{-19}\times 11.9\times 8.85\times 10^{-14}} \times (0.85\times 10^{-8})^2$$

$$\Rightarrow N_d = 3.43\times 10^{15}\,\text{cm}^{-3}$$

We can select the n-type doping concentration of 3.43×10^{15} cm^{-3}.

10. From Eq. 56,

$$G = -U\left[\frac{\sigma_p \sigma_n \upsilon_{th} N_t}{\sigma_n \exp\left(\frac{E_t - E_i}{kT}\right) + \sigma_p \exp\left(\frac{E_i - E_t}{kT}\right)}\right] n_i$$

$$= \left[\frac{10^{-15}\times 10^{-15}\times 10^{7}\times 10^{15}}{10^{-15}\exp\left(\frac{0.02}{0.0259}\right) + 10^{-15}\exp\left(\frac{-0.02}{0.0259}\right)}\right]\times 9.65\times 10^{9} = 3.89\times 10^{16}$$

and

$$W = \sqrt{\frac{2\varepsilon_s(V_{bi}+V)}{qN_A}} = \sqrt{\frac{2\times 11.9\times 8.85\times 10^{-14}\times(0.717+0.5)}{1.6\times 10^{-19}\times 10^{15}}} = 12.66\times 10^{-5}\,\text{cm} = 1.266\ \mu\text{m}$$

Thus

$$J_{gen} = qGW = 1.6\times 10^{19}\times 3.89\times 10^{16}\times 12.66\times 10^{-5} = 7.879\times 10^{-7}\,\text{A/cm}^2.$$

11. The gentle slope is probably due to carrier generation in the depletion region, while the abrupt current rise is due to junction avalanche breakdown.

12. $J = J_p(x_n) + J_n(-x_p) = J_s(e^{qV/kT} - 1)$

$$\Rightarrow \frac{J}{J_s} = e^{\frac{V}{0.0259}} - 1$$

$$\Rightarrow 0.95 = {}^{\frac{V}{0.0259}} - 1$$

$$\Rightarrow V = 0.017\ \text{V}.$$

13. Assume $\tau_g = \tau_p = \tau_n = 10^{-6}$s, $D_n = 21$ cm^2/sec, and $D_p = 10$ cm^2/sec

(a) The saturation current calculation.

From Eq. 55a and $L_p = \sqrt{D_p \tau_p}$, we can obtain

$$J_s = \frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n} = qn_i^2 \left(\frac{1}{N_D} \sqrt{\frac{D_p}{\tau_{p0}}} + \frac{1}{N_A} \sqrt{\frac{D_n}{\tau_{n0}}} \right)$$

$$= 1.6 \times 10^{-19} \times (9.65 \times 10^9)^2 \left(\frac{1}{10^{18}} \sqrt{\frac{10}{10^{-6}}} + \frac{1}{10^{16}} \sqrt{\frac{21}{10^{-6}}} \right)$$

$$= 6.87 \times 10^{12} \text{ A/cm}^2$$

And from the cross-sectional area $\text{A} = 1.2 \times 10^{-5}\ \text{cm}^2$, we obtain

$$I_s = A \times J_s = 1.2 \times 10^{-5} \times 6.87 \times 10^{-12} = 8.244 \times 10^{-17} \text{A.}$$

(b) The total current density is

$$J = J_s \left(e^{\frac{qV}{kt}} - 1 \right)$$

Thus

$$I_{0.7V} = 8.244 \times 10^{-17} \left(e^{\frac{0.7}{0.0259}} - 1 \right) = 8.244 \times 10^{-17} \times 5.47 \times 10^{11} = 4.51 \times 10^{-5} \text{A}$$

$$I_{-0.7V} = 8.244 \times 10^{-17} \left(e^{\frac{-0.7}{0.0259}} - 1 \right) = 8.244 \times 10^{-17} \text{A}\,.$$

14. The built-in voltage $V_{bi} = (kT/q)\ln[(N_D N_A)/\, n_i^2]$

$\rightarrow V_{bi} = 25.9 \times 10^{-3} \ln[10^{18} 10^{16})/(1.5 \times 10^{10})^2] = 0.813$ Volt.

$W = 2\ \mu\text{m} = [(2\ \varepsilon_{Si}/q)(V_{bi} + V_{RB})(1/N_A + 1/N_D)]^{½}$.

Solving for V_{RB} leads to: 29.53 Volts reverse bias.

15. From Eq. 59, and assume $D_p = 10\ \text{cm}^2/\text{sec}$, we can obtain

$$J_R \cong q\sqrt{\frac{D_p}{\tau_p}} \frac{n_i^2}{N_D} + \frac{qn_i W}{\tau_g}$$

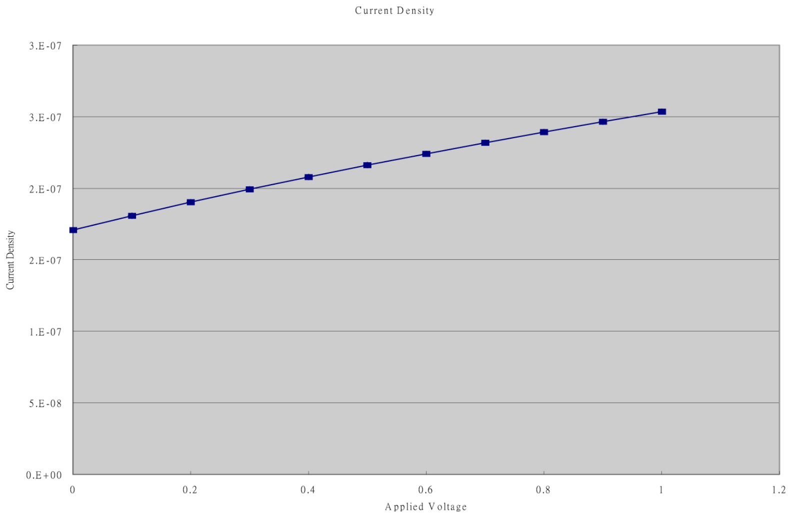
$$= 1.6 \times 10^{-19} \sqrt{\frac{10}{10^{-6}}} \frac{(9.65 \times 10^9)^2}{10^{15}} + \frac{1.6 \times 10^{-19} \times 9.65 \times 10^9}{10^{-6}} \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-14} \times (V_{bi} + V_R)}{1.6 \times 10^{-19} \times 10^{15}}}$$

$$V_{bi} = 0.0259 \ln \frac{10^{19} \times 10^{15}}{(9.65 \times 10^9)^2} = 0.834 \text{ V}$$

Thus

$$J_R = 5.26\times10^{-11} + 1.872\times10^{-7}\sqrt{0.834+V_R}$$

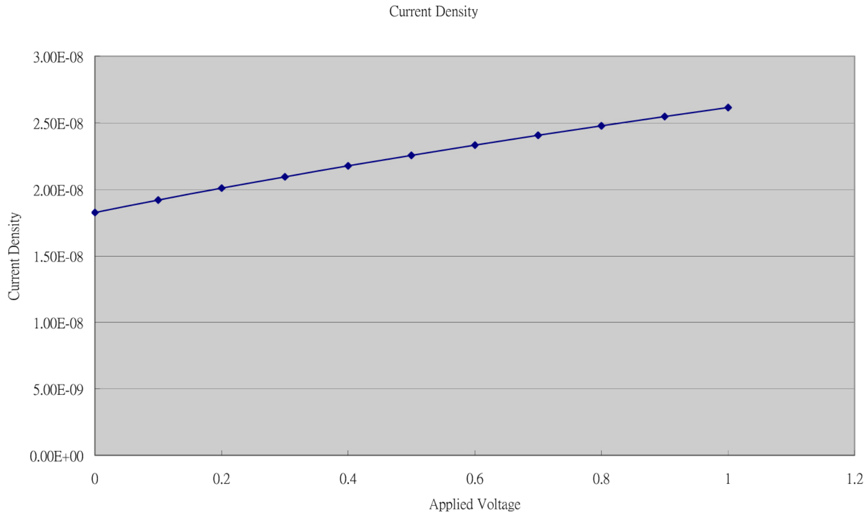
VR	Js
0	1.713E-07
0.1	1.755E-07
0.2	1.941E-07
0.3	2,129E-07
0.4	2.316E-07
0.5	2.503E-07
0.6	2.691E-07
0.7	2.878E-07
0.8	3.065E-07
0.9	3.252E-07
1	3.439E-07



When $N_D = 10^{17}\,\text{cm}^{-3}$, we obtain

$$V_{bi} = 0.0259\ln\frac{10^{19}\times10^{17}}{(9.65\times10^{9})^{2}} = 0.953\text{ V}$$

$$J_R = 5.26\times10^{-13} + 1.872\times10^{-8}\sqrt{0.956+V_R}$$



16. From $J = J_s\left(e^{\frac{qV}{kt}} - 1\right)$

we can obtain

$$\frac{V}{0.0259} = \ln\left[\left(\frac{J}{J_s}\right) + 1\right] \Rightarrow V = 0.0259 \times \ln\left[\left(\frac{10^{-3}}{8.244\times10^{-17}}\right) + 1\right] = 0.78\ \text{V}.$$

17. From Eq. 39,

$$Q_p = q\int_{x_n}^{\infty}(p_n - p_{n0})dx$$
$$= q\int_{x_n}^{\infty} p_{n0}(q^{qV/kT} - 1)e^{-(x-x_n)/L_p}dx$$

The hole diffusion length is larger than the length of neutral region

$$Q_p = q\int_{x_n}^{x_n'}(p_n - p_{n0})dx$$
$$= q\int_{x_n}^{x_n'} p_{no}(e^{qV/kT} - 1)e^{-(x-x_n)/L_p}dx$$
$$= qp_{no}(-L_p)\left(e^{\frac{qV}{kT}} - 1\right)\left(e^{-\frac{x_n'-x_n}{L_p}} - e^{-\frac{x_n-x_n}{L_p}}\right)$$
$$= 1.6\times10^{-19}\times\frac{(9.65\times10^9)^2}{10^{16}}(-5\times10^{-4})\left(e^{\frac{1}{0.0259}} - 1\right)\left(e^{-\frac{1}{5}} - e^{-\frac{0}{5}}\right)$$
$$= 8.784\times10^{-3}\ \text{C/cm}^2.$$

18. From Fig. 26, the critical field at breakdown for a Si one-sided abrupt junction is about 2.8×10^5 V/cm. Then from Eq. 85, we obtain

$$V_B(\text{breakdown voltage}) = \frac{\mathrm{E}_c W}{2} = \frac{\varepsilon_s \mathrm{E}_c^2}{2q}(N_B)^{-1}$$

$$= \frac{11.9 \times 8.85 \times 10^{-14} \times (2.8 \times 10^{5})^{2}}{2 \times 1.6 \times 10^{-19}} (10^{15})^{-1}$$
$$= 258 \text{ V}$$

$$W = \sqrt{\frac{2\varepsilon_s (V_{bi} - V)}{qN_B}} \cong \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-14} \times 258}{1.6 \times 10^{-19} \times 10^{15}}} = 1.843 \times 10^{-3} \text{ cm} = 18.43 \text{ μm}$$

When the n-region is reduced to $5 \mu\text{m}$, the punch-through will take place first.

From Eq. 87, we can obtain

$$\frac{V_B'}{V_B} = \frac{\text{shaded area in Fig.29 insert}}{(\text{E}_\text{c}\text{W}_\text{m})/2} = \left(\frac{W}{W_m}\right)\left(2 - \frac{W}{W_m}\right)$$

$$V_B' = V_B \left(\frac{W}{W_m}\right)\left(2 - \frac{W}{W_m}\right) = 258 \times \left(\frac{5}{18.43}\right)\left(2 - \frac{5}{18.43}\right) = 121 \text{ V}$$

Compared to Fig. 29, the calculated result is the same as the value under the conditions of $W = 5 \mu\text{m}$ and $N_B = 10^{15} \text{cm}^{-3}$.

19. We can use following equations to determine the parameters of the diode.

$$J_F = q\sqrt{\frac{D_p}{\tau_p}} \frac{n_i^2}{N_D} e^{qV/kT} + \frac{qWn_i}{2\tau_r} e^{qV/2kT} \cong q\sqrt{\frac{D_p}{\tau_p}} \frac{n_i^2}{N_D} e^{qV/kT}$$
$$V_B = \frac{E_c W}{2} = \frac{\varepsilon_s E_c^2}{2q} (N_D)^{-1}$$

$\Rightarrow$

$$AJ_F = Aq\sqrt{\frac{D_p}{\tau_p}} \frac{n_i^2}{N_D} e^{qV/kT} \Rightarrow A \times 1.6 \times 10^{-19} \times \sqrt{\frac{D_p}{10^{-7}}} \frac{(9.65 \times 10^9)^2}{N_D} e^{\frac{0.7}{0.0259}} = 2.2 \times 10^{-3}$$
$$V_B = \frac{\text{E}_c W}{2} = \frac{\varepsilon_s \text{E}_c^2}{2q} (N_D)^{-1} \Rightarrow 130 = \frac{11.9 \times 8.85 \times 10^{-14} \text{E}_c^2}{2 \times 1.6 \times 10^{-19}} (N_D)^{-1}$$

Let $\text{E}_c = 4 \times 10^5 \text{ V/cm}$, we can obtain $N_D = 4.05 \times 10^{15} \text{cm}^{-3}$.

The mobility of minority carrier hole is about 500 at $N_D = 4.05 \times 10^{15} \text{cm}^{-3}$.

$\therefore D_p = 0.0259 \times 500 = 12.95 \text{ cm}^2/\text{s}$

Thus, the cross-sectional area A is $8.6 \times 10^{-5} \text{cm}^2$.

20. $a = \frac{2 \times 10^{18}}{2 \times 10^{-4}} = 10^{22} \text{cm}^{-4}$

$$V_B = \frac{2\text{E}W}{3} = \frac{4\text{E}_c^{3/2}}{3}\left[\frac{2\varepsilon_s}{q}\right]^{1/2} (a)^{-1/2}$$

$$= \frac{4\mathrm{E}_c^{3/2}}{3}\left[\frac{2\times 11.9\times 8.85\times 10^{-14}}{1.6\times 10^{-19}}\right]^{1/2}\times (10^{22})^{-1/2}$$

$$= 4.84\times 10^{-8}\mathrm{E}_c^{3/2}$$

The breakdown voltage can be determined by a selected E_c.

21. (a) The i-layer is easy to deplete, and assume the field in the depletion region is constant. From Eq. 84, we can obtain.

$$\int_0^W 10^4\left(\frac{\mathrm{E}}{4\times 10^5}\right)^6 dx = 1 \Rightarrow 10^4\left(\frac{\mathrm{E}}{4\times 105}\right)^6 \times 10^{-3} = 1 \Rightarrow \mathrm{E}_{critical} = 4\times 10^5\times (10)^{1/6} = 5.87\times 10^5 \text{ V/cm}$$

$$\therefore V_B = 5.87\times 10^5\times 10^{-3} = 587 \text{ V}$$

(b) From Fig. 26, the critical field is 5×10^5 V/cm.

$$V_B(\text{breakdown voltage}) = \frac{\mathrm{E}_c W}{2} = \frac{\varepsilon_s \mathrm{E}_c^2}{2q}(N_B)^{-1}$$

$$= \frac{12.4\times 8.85\times 10^{-14}\times (5\times 10^5)^2}{2\times 1.6\times 10^{-19}}(2\times 10^{16})^{-1}$$

$$= 42.8 \text{ V.}$$

22. To calculate the results with applied voltage of $V = 0.5$ V, we can use a similar calculation in Example 10 with (1.6–0.5) replacing 1.6 for the voltage. The obtained electrostatic potentials are 1.1 V and 3.4×10^{-4} V, respectively. The depletion widths are 3.821×10^{-5} cm and 1.274×10^{-8} cm, respectively.

Also, by substituting $V = -5$ V to Eqs. 90 and 91, the electrostatic potentials are 6.6 V and 20.3×10^{-4} V, and the depletion widths are 9.359×10^{-5} cm and 3.12×10^{-8} cm, respectively.

The total depletion width will be reduced when the heterojunction is forward-biased from the thermal equilibrium condition. On the other hand, when the heterojunction is reverse-biased, the total depletion width will be increased.

23. $E_g(0.3) = 1.424 + 1.247\times 0.3 = 1.789$ eV

$$V_{bi} = \frac{E_{g2}}{q} - \frac{\Delta E_C}{q} - (E_{F2} - E_{V2})/q - (E_{C1} - E_{F1})/q$$

$$= 1.789 - 0.21 - \frac{kT}{q}\ln\frac{4.7\times 10^{17}}{5\times 10^{15}} - \frac{kT}{q}\ln\frac{7\times 10^{18}}{5\times 10^{15}} = 1.273 \text{ V}$$

$$x_1 = \left[\frac{2N_A\varepsilon_1\varepsilon_2 v_{bi}}{qN_D(\varepsilon_1 N_D + \varepsilon_2 N_A)}\right]^{1/2} = \left[\frac{2\times 12.4\times 11.46\times 8.85\times 10^{-14}\times 1.273}{1.6\times 10^{-19}\times 5\times 10^{15}(12.4+11.46)}\right]^{1/2}$$

$$= 4.1\times 10^{-5} \text{ cm.}$$

Since $N_D x_1 = N_A x_2 \quad \therefore x_1 = x_2$

$$\therefore W = 2x_1 = 8.2\times 10^{-5} \text{ cm} = 0.82\ \mu\text{m.}$$

CHAPTER 4

1. (a) The common-base and common-emitter current gains is given by

$$\alpha_0 = \gamma\alpha_T = 0.997 \times 0.998 = 0.995$$

$$\beta_0 = \frac{\alpha_0}{1-\alpha_0} = \frac{0.995}{1-0.995}$$
$$= 199.$$

(b) Since $I_B = 0$ and $I_{Cp} = 10\times10^{-9}$ A, then I_{CBO} is 10×10^{-9} A. The emitter current is

$$I_{CEO} = (1+\beta_0)I_{CBO}$$
$$= (1+199)\cdot 10\times10^{-9}$$
$$= 2\times10^{-6}\,\text{A}.$$

2. For an ideal transistor,

$$\alpha_0 = \gamma = 0.999$$

$$\beta_0 = \frac{\alpha_0}{1-\alpha_0} = 999.$$

I_{CBO} is known and equals to 10×10^{-6} A. Therefore,

$$I_{CEO} = (1+\beta_0)I_{CBO}$$
$$= (1+999)\cdot 10\times10^{-6}$$
$$= 10\,\text{mA}.$$

3. The creation of traps would reduce the carrier lifetime parameter by facilitating more carrier capture and recombination events. In particular, there would now be more recombination of the injected holes in the base region during their transit towards the collector. Thus, the base transport factor a_T would reduce, and so would the current gain a0. Mathematically, $a_T = \text{sec h}[W/L_{pB}]$, with $L_{pB} = (D_B\,\tau_{pB})^{0.5}$. Hence, a decrease in carrier lifetime, will reduce L_{pB}, and thus, reduce a_T. Also, emitter efficiency $\gamma = 1/[1+(D_E N_B W)/(D_{pB} N_E L_E)]$. Since $L_E = (D_E\,\tau_E)^{0.5}$ the decrease in τ_E will reduce L_E, and hence, decrease γ as well.

4. (a) The emitter-base junction is forward biased. From Chapter 3 we obtain

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.0259\ln\left[\frac{5\times10^{18}\cdot 2\times10^{17}}{(9.65\times10^{9})^2}\right] = 0.956\text{ V}.$$

The depletion-layer width in the base is

$$W_1 = \left(\frac{N_A}{N_A + N_D}\right)(\text{Total depletion - layer width of the emitter - base junction})$$
$$= \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A}{N_D}\right)\left(\frac{1}{N_A + N_D}\right)(V_{bi} - V)}$$
$$= \sqrt{\frac{2\cdot 1.05\times 10^{-12}}{1.6\times 10^{-19}}\left(\frac{5\times 10^{18}}{2\times 10^{17}}\right)\left(\frac{1}{5\times 10^{18} + 2\times 10^{17}}\right)(0.956 - 0.5)}$$
$$= 5.364\times 10^{-6}\text{ cm} = 5.364\times 10^{-2}\,\mu\text{m}.$$

Similarly we obtain for the base-collector function

$$V_{bi} = 0.0259\ln\left[\frac{2\times 10^{17}\cdot 10^{16}}{(9.65\times 10^{9})^2}\right] = 0.795\text{ V}.$$

and

$$W_2 = \sqrt{\frac{2\cdot 1.05\times 10^{-12}}{1.6\times 10^{-19}}\left(\frac{10^{16}}{2\times 10^{17}}\right)\left(\frac{1}{10^{16} + 2\times 10^{17}}\right)(0.795 + 5)}$$
$$= 4.254\times 10^{-6}\text{ cm} = 4.254\times 10^{-2}\,\mu\text{m}.$$

Therefore the neutral base width is

$$W = W_{\text{B}} - W_1 - W_2 = 1 - 5.364\times 10^{-2} - 4.254\times 10^{-2} = 0.904\ \mu\text{m}.$$

(b) Using Eq. 13*a*

$$p_n(0) = p_{no}e^{qV_{EB}/kT} = \frac{n_i^2}{N_D}e^{qV_{EB}/kT} = \frac{(9.65\times 10^{9})^2}{2\times 10^{17}}e^{0.5/0.0259} = 2.543\times 10^{11}\text{cm}^{-3}.$$

5. In the emitter region

$$D_E = 52\text{ cm/s } L_E = \sqrt{52.10^{-8}} = 0.721\times 10^{-3}\text{ cm}$$
$$n_{EO} = \frac{(9.65\times 10^{9})^2}{5\times 10^{18}} = 18.625.$$

In the base region

$$D_p = 40\text{ cm/s } L_p = \sqrt{D_p\tau_p} = \sqrt{40\cdot 10^{-7}} = 2\times 10^{-3}\text{ cm}$$
$$p_{no} = \frac{n_i^2}{N_D} = \frac{(9.65\times 10^{9})^2}{2\times 10^{17}} = 465.613.$$

In the collector region

$$D_C = 115 \text{ cm/s } L_C = \sqrt{115 \cdot 10^{-6}} = 10.724 \times 10^{-3} \text{ cm}$$

$$n_{CO} = \frac{(9.65 \times 10^9)^2}{10^{16}} = 9.312 \times 10^3.$$

The current components are given by Eqs. 20, 21, 22, and 23:

$$I_{Ep} = \frac{1.6 \times 10^{-19} \cdot 0.2 \times 10^{-2} \cdot 40 \cdot 465.613}{0.904 \times 10^{-4}} e^{0.5/0.0259} = 1.596 \times 10^{-5} \text{ A}$$

$$I_{Cp} \cong I_{Ep} = 1.596 \times 10^{-5} \text{ A}$$

$$I_{En} = \frac{1.6 \times 10^{-19} \cdot 0.2 \times 10^{-2} \cdot 52 \cdot 18.625}{0.721 \times 10^{-3}} (e^{0.5/0.0259} - 1) = 1.041 \times 10^{-7} \text{ A}$$

$$I_{Cn} = \frac{1.6 \times 10^{-19} \cdot 0.2 \times 10^{-2} \cdot 115 \cdot 9.312 \times 10^3}{10.724 \times 10^{-3}} = 3.196 \times 10^{-14} \text{ A}$$

$$\mathrm{I}_{BB} = I_{Ep} - I_{Cp} = 0.$$

6. (a) The emitter, collector, and base currents are given by

$$I_E = I_{Ep} + I_{En} = 1.606 \times 10^{-5} \text{ A}$$

$$I_C = I_{Cp} + I_{Cn} = 1.596 \times 10^{-5} \text{ A}$$

$$I_B = I_{En} + I_{BB} - I_{Cn} = 1.041 \times 10^{-7} \text{ A}.$$

(b) We can obtain the emitter efficiency and the base transport factor:

$$\gamma = \frac{I_{Ep}}{I_E} = \frac{1.596 \times 10^{-5}}{1.606 \times 10^{-5}} = 0.9938$$

$$\alpha_T = \frac{I_{Cp}}{I_{Ep}} = \frac{1.596 \times 10^{-5}}{1.596 \times 10^{-5}} = 1.$$

Hence, the common-base and common-emitter current gains are

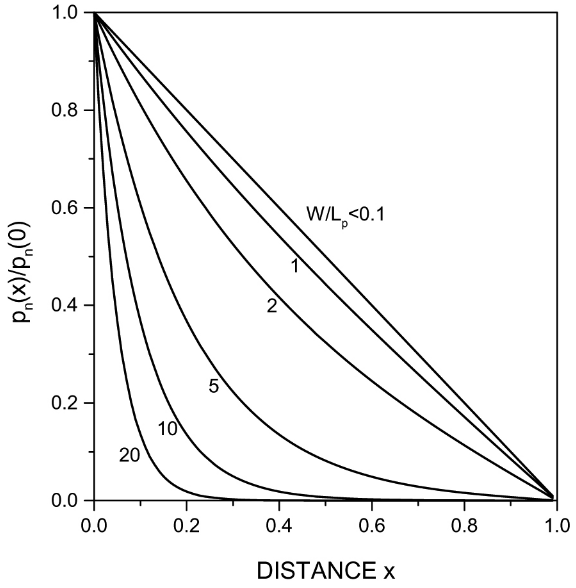
$$\alpha_0 = \gamma \alpha_T = 0.9938$$

$$\beta_0 = \frac{\alpha_0}{1 - \alpha_0} = 160.3.$$

(c) To improve γ, the emitter has to be doped much heavier than the base.

To improve α_T, we can make the base width narrower.

7. We can sketch $p_n(x)/p_n(0)$ curves by using a computer program:



In the figure, we can see when $W/L_p < 0.1$ ($W/L_p = 0.05$ in this case), the minority carrier distribution approaches a straight line and can be simplified to Eq. 15.

8. The forward bias at the emitter-base junction tends to inject electrons through the base into the collector. Since the collector is floating, the terminal current is zero. The injected electrons will accumulate. Hence, an electric field will be set up to pull back these injected electrons to attain a steady-state. This field points from the base to the collector. So the (p) base-(n)collector gets forward biased.

9. The total excess minority carrier charge can be expressed by

$$\begin{aligned} Q_B &= qA\int_0^W [p_n(x) - p_{no}]dx \\ &= qA\int_0^W \left[p_{no}e^{qV_{EB}/kT}\left(1-\frac{x}{W}\right)\right]dx \\ &= qAp_{no}e^{qV_{EB}/kT}\left(x-\frac{x^2}{2W}\right)\Bigg|_0^W \\ &= \frac{qAWp_{no}e^{qV_{EB}/kT}}{2} \\ &= \frac{qAWp_n(0)}{2}. \end{aligned}$$

From Fig. 6, the triangular area in the base region is $\frac{Wp_n(0)}{2}$. By multiplying this value by q and the cross-sectional area A, we can obtain the same expression as Q_B. In Problem 4,

$$Q_B = \frac{1.6\times10^{-19}\cdot 0.2\times10^{-2}\cdot 0.904\times10^{-4}\cdot 2.543\times10^{11}}{2}$$
$$= 3.678\times10^{-15}\ \text{C.}$$

10. In Eq.27,

$$\begin{aligned}
I_C &= a_{21}(e^{qV_{EB}/kT}-1)+a_{22} \\
&\cong \frac{qAD_p p_n(0)}{W} \\
&= \frac{2D_p}{W^2}\frac{qAQp_n(0)}{2} \\
&= \frac{2D_p}{W^2}Q_B.
\end{aligned}$$

Therefore, the collector current is directly proportional to the minority carrier charge stored in the base.

11. The base transport factor is

$$\alpha_T \cong \frac{I_{Cp}}{I_{Ep}} = \frac{\dfrac{1}{\sinh\left(\dfrac{W}{L_p}\right)}\left[(e^{qV_{EB}/kT}-1)+\cosh\left(\dfrac{W}{L_p}\right)\right]}{\cosh\left(\dfrac{W}{L_p}\right)\left[(e^{qV_{EB}/kT}-1)+\dfrac{1}{\cosh\left(\dfrac{W}{L_p}\right)}\right]}.$$

For $W/L_p \ll 1, \cosh(W/L_p)\cong 1$. Thus,

$$\begin{aligned}
\alpha_T &= \frac{1}{\sinh\left(\dfrac{W}{L_p}\right)\cdot\coth\left(\dfrac{W}{L_p}\right)} \\
&= \operatorname{sech}\left(\frac{W}{L_p}\right) \\
&= 1-\frac{1}{2}\left(\frac{W}{L_p}\right)^2 \\
&= 1-(W^2/2L_p^2).
\end{aligned}$$

12. $\gamma = [1 + (D_E / D_B)(W_B / L_E)(N_B / N_E)]^{-1} = [1 + (1)\{0.1L_B / (0.5L_B)\}(N_B / 10N_B)]^{-1}$

$$\gamma = [1 + 0.02]^{-1} = 50/51$$
$$= 0.98039.$$

Also: $\alpha_T = 1 - W_B^2 / \{2\,L_B^2\} = 1\ - (1/2)\{(0.1L_B)/(L_B)\}^2 = 0.995.$

13. The common-emitter current gain is given by

$$\beta_0 \equiv \frac{\alpha_0}{1-\alpha_0} = \frac{\gamma\alpha_T}{1-\gamma\alpha_T}.$$

Since $\gamma \cong 1$,

$$\begin{aligned}\beta_0 &\cong \frac{\alpha_T}{1-\alpha_T} \\ &= \frac{1-(W^2/2L_p^2)}{1-[1-(W^2/2L_p^2)]} \\ &= (2L_p^2/W^2)-1.\end{aligned}$$

If $W/L_p \ll 1$, then $\beta_0 \cong 2L_p^2/W^2$.

14. $L_p = \sqrt{D_p \tau_p} = \sqrt{100 \cdot 3\times10^{-7}} = 5.477\times10^{-3}\ \text{cm} = 54.77\ \mu\text{m}$

Therefore, the common-emitter current gain is

$$\begin{aligned}\beta_0 &\cong 2L_p^2/W^2 = \frac{2(54.77\times10^{-4})^2}{(2\times10^{-4})^2} \\ &= 1500.\end{aligned}$$

15. In the emitter region,

$$\begin{aligned}\mu_{pE} &= 54.3 + \frac{407}{1+0.374\times10^{-17}\cdot3\times10^{18}} = 87.6 \\ D_E &= 0.0259\cdot87.6 = 2.26\ \text{cm/s}.\end{aligned}$$

16. (a) The total number of impurities in the neutral base region is

$$\begin{aligned}Q_G &= \int_W^0 N_{AO} e^{-x/l} dx = N_{AO} l (1 - e^{-W/l}) \\ &= 2\times10^{18}\cdot3\times10^{-5}(1-e^{-8\times10^{-5}/3\times10^{-5}}) = 5.583\times10^{13}\ \text{cm}^{-2}.\end{aligned}$$

(b) Average impurity concentration is

$$\begin{aligned}\frac{Q_G}{W} &= \frac{5.583\times10^{13}}{8\times10^{-15}} \\ &= 6.979\times10^{17}\ \text{cm}^{-3}.\end{aligned}$$

17. For $N_A = 6.979\times10^{17}\,\text{cm}^{-3}, D_n = 7.77\text{cm}^3/\text{s}$, and

$$L_n = \sqrt{D_n\tau_n} = \sqrt{7.77\cdot10^{-6}} = 2.787\times10^{-3}\text{ cm}$$

$$\alpha_T \cong 1-\frac{W^2}{2L_n^2} = 1-\frac{(8\times10^{-5})^2}{2(2.787\times10^{-3})^2} = 0.999588$$

$$\gamma = \frac{1}{1+\dfrac{D_E}{D_n}\dfrac{Q_G}{N_E L_E}} = \frac{1}{1+\dfrac{1}{7.77}\cdot\dfrac{5.583\times10^{13}}{10^{19}\cdot10^{-4}}} = 0.99287.$$

Therefore,

$$\alpha_0 = \gamma\alpha_T = 0.99246$$

$$\beta_0 = \frac{\alpha_0}{1-\alpha_0} = 131.6.$$

18. The mobility of an average impurity concentration of $6.979\times10^{17}\text{cm}^{-3}$ is about $300\text{ cm}^2/\text{Vs}$. The average base resistivity $\bar{\rho}_B$ is given by

$$\bar{\rho}_B = \frac{1}{q\mu_n(Q_G/W)} = 0.0299\ \Omega\text{-cm}.$$

Therefore,

$$R_B = 5\times10^{-3}(\bar{\rho}_B/W) = 5\times10^{-3}\cdot(0.0299/8\times10^{-5}) = 1.869\ \Omega.$$

For a voltage drop of kT/q,

$$I_B = \frac{kT}{qR_B} = 0.0139\text{ A}.$$

Therefore,

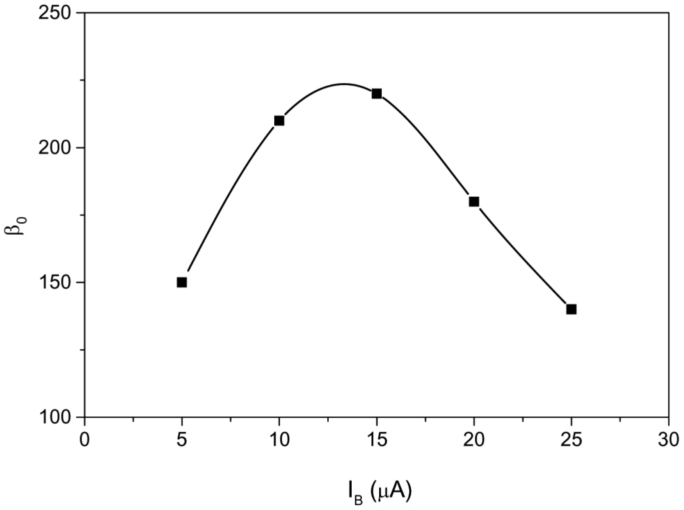
$$I_C = \beta_0 I_B = 131.6\cdot0.0139 = 1.83\text{ A}.$$

19. From Fig. 10*b* and Eq. 35, we obtain

$I_B(\mu\text{A})$	$I_C(\text{mA})$	$\beta_0 = \dfrac{\Delta I_C}{\Delta I_B}$
0	0.20	—
5	0.95	150
10	2.00	210
15	3.10	220
20	4.00	180
25	4.70	140

β_0 is not a constant. At low I_B, because of generation-recombination current, β_0 increases with increasing I_B. At high I_B, V_{EB} increases with I_B, this in turn causes a reduction of V_{BC} since $V_{EB}+V_{BC}=V_{EC}=5\text{ V}$. The reduction of V_{BC} causes a widening of the neutral base region, therefore β_0 decreases.

The following chart shows β_0 as function of I_B. It is obvious that β_0 is not a constant.



20. Comparing the equations with Eq. 32 gives

$$I_{FO}=a_{11},\ \alpha_R I_{RO}=a_{12}$$

$$\alpha_F I_{FO}=a_{21},\text{ and } I_{RO}=a_{22}.$$

Hence,

$$\alpha_F=\frac{a_{21}}{a_{11}}=\frac{1}{1+\frac{W}{L_E}\cdot\frac{D_E}{D_p}\cdot\frac{n_{EO}}{p_{no}}}$$

$$\alpha_R=\frac{a_{12}}{a_{22}}=\frac{1}{1+\frac{W}{L_C}\cdot\frac{D_C}{D_p}\cdot\frac{n_{CO}}{p_{no}}}.$$

21. In the collector region,

$$L_C=\sqrt{D_C\tau_C}=\sqrt{2\cdot 10^{-6}}=1.414\times 10^{-3}\text{ cm}$$

$$n_{CO}=n_i^2/N_C=(9.65\times 10^9)^2/5\times 10^{15}=1.863\times 10^4\text{cm}^{-3}.$$

From Problem 20, we have

$$\alpha_F = \frac{1}{1+\frac{W}{L_E}\cdot\frac{D_E}{D_p}\cdot\frac{n_{EO}}{p_{no}}} = \frac{1}{1+\frac{0.5\times10^{-4}}{10^{-3}}\cdot\frac{1}{10}\cdot\frac{9.31}{9.31\times10^{-2}}}$$
$$= 0.99995$$

$$\alpha_R = \frac{1}{1+\frac{W}{L_C}\cdot\frac{D_C}{D_p}\cdot\frac{n_{CO}}{p_{no}}} = \frac{1}{1+\frac{0.5\times10^{-4}}{1.414\times10^{-3}}\cdot\frac{2}{10}\cdot\frac{1.863\times10^{4}}{9.31\times10^{-2}}}$$
$$= 0.876$$

$$I_{FO} = a_{11} = qA\left(\frac{D_p p_{no}}{W} + \frac{D_E n_{EO}}{L_E}\right)$$
$$= 1.6\times10^{-19}\cdot5\times10^{-4}\cdot\left(\frac{10\cdot9.31\times10^{2}}{0.5\times10^{-4}} + \frac{1\cdot9.31}{10^{-4}}\right)$$
$$= 1.49\times10^{-14}\text{ A}$$

$$I_{RO} = a_{22} = qA\left(\frac{D_p p_{no}}{W} + \frac{D_C n_{CO}}{L_C}\right)$$
$$= 1.6\times10^{-19}\cdot5\times10^{-4}\cdot\left(\frac{10\cdot9.31\times10^{2}}{0.5\times10^{-4}} + \frac{2\cdot1.863\times10^{4}}{1.414\times10^{-3}}\right)$$
$$= 1.7\times10^{-14}\text{ A.}$$

The emitter and collector currents are

$$I_E = I_{FO}(e^{qV_{EB}/kT}-1) + \alpha_R I_{RO}$$
$$= 1.715\times10^{-4}\text{ A}$$

$$I_C = \alpha_F I_{FO}(e^{qV_{EB}/kT}-1) + I_{RO}$$
$$= 1.715\times10^{-4}\text{ A.}$$

Note that these currents are almost the same (no base current) for $W/L_p \ll 1$.

22. Referring Eq. 11, the field-free steady-state continuity equation in the collector region is

$$D_C\left[\frac{d^2 n_C(x')}{dx'^2}\right] - \frac{n_C(x') - n_{po}}{\tau_C} = 0.$$

The solution is given by $(L_C = \sqrt{D_C \tau_C})$

$$n_C(x') = C_1 e^{x'/L_C} + C_2 e^{-x'/L_C}.$$

Applying the boundary condition at $x' = \infty$ yields

$$C_1 e^{\infty/L_C} + C_2 e^{-\infty/L_C} = 0.$$

Hence $C_1 = 0$. In addition, for the boundary condition at $x' = 0$,

$$C_2 e^{-0/L_C} = C_2 = n_C(0)$$

$$n_C(0) = n_{CO}(e^{qV_{CB}/kT} - 1).$$

The solution is

$$n_C(x) = n_{CO}(e^{qV_{CB}/kT} - 1)e^{-x'/L_C}.$$

The collector current can be expressed as

$$\begin{aligned} I_C &= A\left(-qD_p \left.\frac{dp_n}{dx}\right|_{x=W}\right) + A\left(-qD_C \left.\frac{dp_C}{dx'}\right|_{x'=0}\right) \\ &= qA\frac{D_p p_{no}}{W}(e^{qV_{EB}/kT} - 1) - qA\left(\frac{D_p p_{no}}{W} + \frac{D_C n_{CO}}{L_C}\right)(e^{qV_{CB}/kT} - 1) \\ &= a_{21}(e^{qV_{EB}/kT} - 1) - a_{22}(e^{qV_{CB}/kT} - 1). \end{aligned}$$

23. Neutral base width $W = X_B - x_{n,depl}{}^{EB} - x_{n,depl}{}^{CB}$.

Here: $x_{n,depl}{}^{EB} = [(2\varepsilon/q)N_A{}^E/\{N_D{}^B(N_A{}^E + N_D{}^B)\}(V_{bi}{}^{EB} - V_{EB}{}^{FB})]$, and

$x_{n,depl}{}^{CB} = [(2\varepsilon/q)N_A{}^C/\{N_D{}^B(N_A{}^C + N_D{}^B)\}(V_{bi}{}^{CB} - V_{CB}{}^{RB})]$

Now: $V_{bi}{}^{EB} = 25.9\times10^{-3}\mathrm{Ln}[(5\times10^{18}\times10^{16})/(10^{10})^2] = 0.8766$ V,

$V_{bi}{}^{CB} = 25.9\times10^{-3}\mathrm{Ln}[(10^{15}\times10^{16})/(10^{10})^2] = 0.656$ V.

$x_{n,depl}{}^{EB} = [\{2\times11.9\times8.85\times10^{-14}/(1.6\times10^{-19})\}5\times10^{18}/\{10^{16}(5\times10^{18}+10^{16})\}(0.8766-0.5)]$,

→ $x_{n,depl}{}^{EB} = 2.22\times10^{-5}$cm.

$x_{n,depl}{}^{CB} = [\{2\times11.9\times8.85\times10^{-14}/(1.6\times10^{-19})\}10^{15}/\{10^{16}(10^{15}+10^{16})\}(0.656+5.0)]$,

→ $x_{n,depl}{}^{CB} = 2.6\times10^{-5}$cm. Hence: $W = 10^{-4} - 2.22\times10^{-5} - 2.6\times10^{-5} = 5.182.6\times10^{-5}$cm.

Emitter efficiency: $\gamma = 1/\{1 + (D_E W\, N_B)/(D_p{}^B\, L_E\, N_E)\}$.

But: $L_B = [D_B \tau_B]^{½} = [10\times10^{-7}]^{½} = 10^{-3}$cm.

$L_E = [D_E\ \tau_E]^{½} = [2\times10^{-8}]^{½} = 1.41\times10^{-4}$cm.

$L_C = [35\times10^{-6}]^{½} = 5.916\times10^{-3}$cm.

$\gamma = 1/\{1 + (2\times5.182.6\times10^{-5}\times10^{16})/(10\times1.41\times10^{-4}\times5\times10^{18})\} = 0.00085.$

$\alpha_T = 1 - W^2/(2\,L_p{}^2) = 1 - (5.18\times10^{-5})^2/(2\times\{10^{-3}\}^2) = 0.9986.$

$\alpha_o = \alpha_T\, \gamma = 0.99845.$

$\beta_0 = \alpha_o/(1-\alpha_o) = 0.99845/(1.0-0.99845) \sim 644.$

$a_{11} = qA[(Dp^B\, n_i^2)/(N_B W) + (Dn^E n_i^2)/(L_E N_E)$

$a_{11} = 1.6\times10^{-19}\times0.03[(10\times10^{20})/(10^{16}\times5.18\times10^{-5}) + (2\times10^{20})/(1.41\times10^{-4}\times5\times10^{18})]$

$a_{11} = 9.27\times10^{-12}$ Amps.

Similarly, $a_{12} = q\,A\,Dp^B\, n_i^2/(N_B W) = 1.6\times10^{-19}\times0.03\times10\times10^{20}/(10^{16}\times5.18\times10^{-5})$

$a_{12} = 9.264\times10^{-12}$ Amps.

Next: $a_{21} = a_{12} = 9.264\times10^{-12}$ Amps.

Finally, $a_{22} = q\,A[(Dp^B\, n_i^2)/(N_B W) + (Dn^C\, n_i^2)/(L_C N_C)$

$a_{22} = 1.6\times10^{-19}\times0.03[(10\times10^{20})/(10^{16}\times5.18\times10^{-5}) + (35\times10^{20})/(5.916\times10^{-3}\times10^{15})].$

$a_{22} = 1.21\times10^{-11}$ Amps.

24. Using Eq. 44, the base transit time is given by

$$\tau_B = W^2/2D_p = \frac{(0.5\times10^{-4})^2}{2\times10} = 1.25\times10^{-10}\,\text{s}.$$

We can obtain the cutoff frequency :

$$f_T \cong 1/2\pi\tau_B = 1.27\ \text{GHz}.$$

From Eq. 41, the common-base cutoff frequency is given by:

$$f_\alpha \cong f_T/\alpha_0 = \frac{1.27\times10^9}{0.998} = 1.275\ \text{GHz}.$$

The common-emitter cutoff frequency is

$$f_\beta = (1-\alpha_0)f_\alpha = (1-0.998)\times1.275\times10^9 = 2.55\ \text{MHz}.$$

Note that f_β can be expressed by

$$f_\beta = (1-\alpha_0)f_\alpha = (1-\alpha_0)/\alpha_0\times f_T\,\frac{1}{\beta_0}\,f_T.$$

25. Neglect the time delays of emitter and collector, the base transit time is given by

$$\tau_B = \frac{1}{2\pi f_T} = \frac{1}{2\pi\times5\times10^9} = 31.83\times10^{-12}\ \text{s}.$$

From Eq. 44, W can be expressed by

$$W = \sqrt{2D_p\tau_B}.$$

Therefore,

$$W = \sqrt{2\times 10\times 31.83\times 10^{-12}}$$
$$= 2.52\times 10^{-5} \text{ cm}$$
$$=0.252\ \mu\text{m}.$$

The neutral base width should be $0.252\ \mu\text{m}$.

26. Stored base charge $Q_B = (V_{CC}/R_L)\tau_B$, where.

$\tau_B = W^2/(2D_p) = (0.5\times 10^{-4})^2/(2\times 10) = 1.25\times 10^{-10}\text{s}$. Hence,

$Q_B = [5/(10\times 10^3)]\times 1.25\times 10^{-10} = 6.25\times 10^{-14}$ Coulomb.

Next, storage delay time $t_s = \tau_p L\text{n}[Q_B(t_2)/Q_B]$, where τ_p = minority lifetime $= 10^{-7}\text{s}$.

But: $Q_B(t_2) = I_B\tau_p[1-\exp(-t_2/\tau_p)]$. Thus:

$Q_B(t_2) = 2\times 10^{-6}\times 10^{-7}\times[1-\exp(-10^{-6}/10^{-7})]$.

i.e., $Q_B(t_2) = 2\times 10^{-13}\text{C}$. Finally, $t_s = 10^{-7} L\text{n}[2\times 10^{-13}/6.25\times 10^{-14}] = 1.16\times 10^{-7}\text{s}$.

27. $\Delta E_g = 9.8\%\times 1.12 \cong 110$ meV.

$$\beta_o \sim \exp\left(\frac{\Delta E_g}{kT}\right)$$

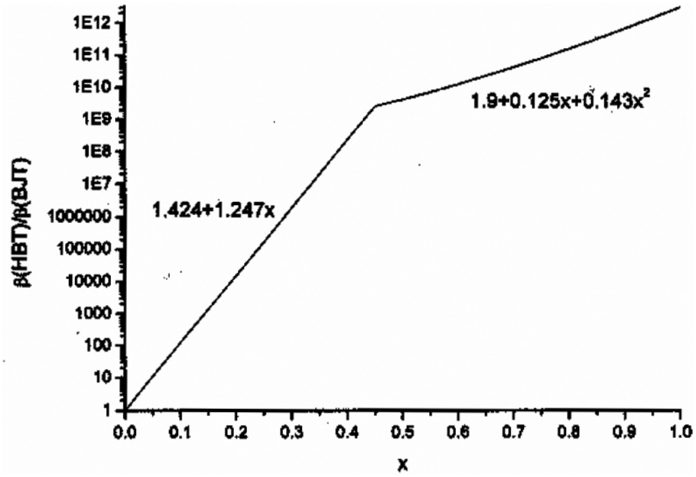
$$\therefore \frac{\beta_o(100°\text{C})}{\beta_o(0°\text{C})} = \exp\left(\frac{110\text{ meV}}{373\,k} - \frac{110\text{ meV}}{273\,k}\right) = 0.29.$$

28. $$\therefore \frac{\beta_o(\text{HBT})}{\beta_o(\text{BJT})} = \exp\left(\frac{E_{gE}-E_{gB}}{kT}\right) = \exp\left[\frac{E_{gE}(x)-1.424}{0.0259}\right]$$

where

$$E_{gE}(x) = 1.424 + 1.247x,\ x \le 0.45$$
$$= 1.9 + 0.125x + 0.143x,\ 0.45 < x \le 1.$$

The plot of $\beta_0(\text{HBT})/\beta_0(\text{BJT})$ is shown in the following graph.



Note that $\beta_0\text{(HBT)}$ increases exponentially when x increases.

29. The impurity concentration of the $n1$ region is 10^{14}cm^{-3}. The avalanche breakdown voltage (for $W > W_m$) is larger than 1500 $\text{V}(W_m > 100\ \mu\text{m})$. For a reverse block voltage of 120 V, we can choose a width such that punch-through occurs, i.e.,

$$V_{PT} = \frac{qN_D W^2}{2\varepsilon_s}.$$

Thus,

$$W = \left(\frac{2\varepsilon_s V_{PT}}{qN_D}\right) = 3.96\times10^{-3}\ \text{cm}.$$

When switching occurs,

$$\alpha_1 + \alpha_2 \cong 1.$$

That is,

$$\alpha_1 = 0.5\sqrt{\frac{L_p}{W}}\ln\left(\frac{J}{J_0}\right)$$
$$= 1 - 0.4 = 0.6$$
$$\ln\left(\frac{J}{J_0}\right) = \frac{0.6}{0.5}\sqrt{\frac{25\times10^{-4}}{39.6\times10^{-4}}}$$
$$= 1.51.$$

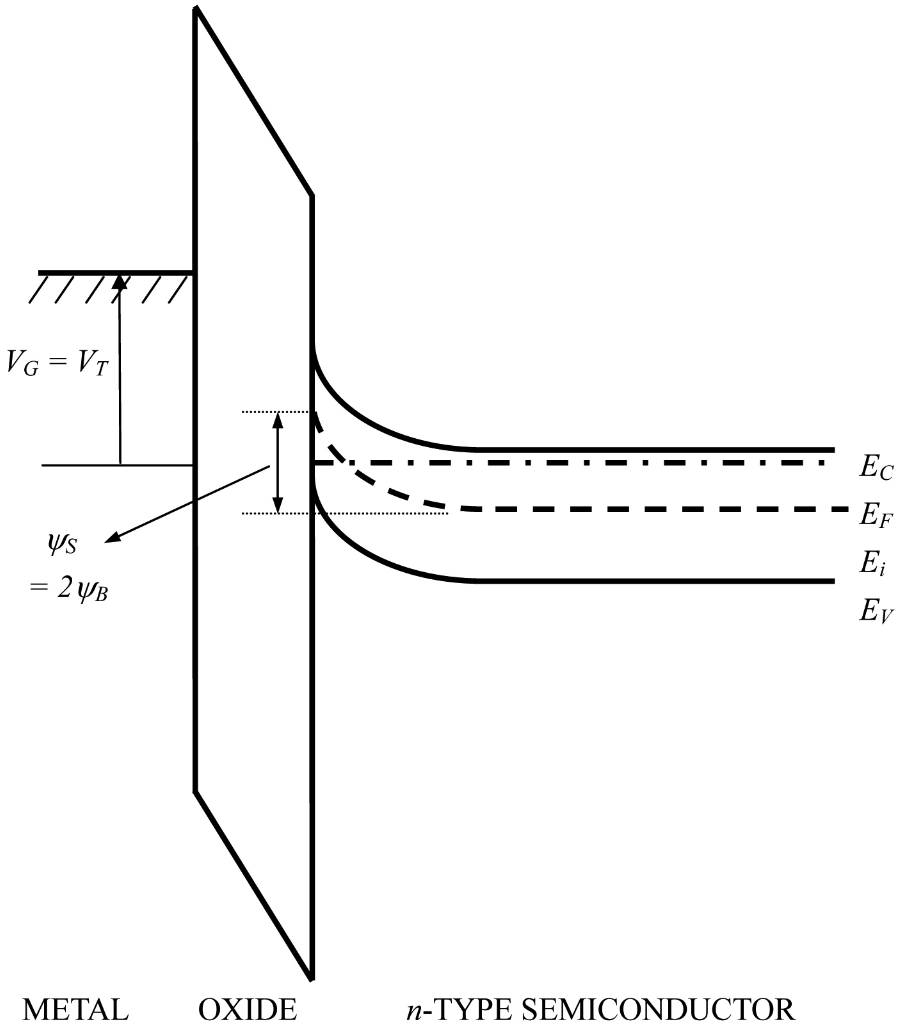
Therefore,

$$J \cong 4.5J_0 = 2.25\times10^{-5}\,\text{A/cm}^2$$

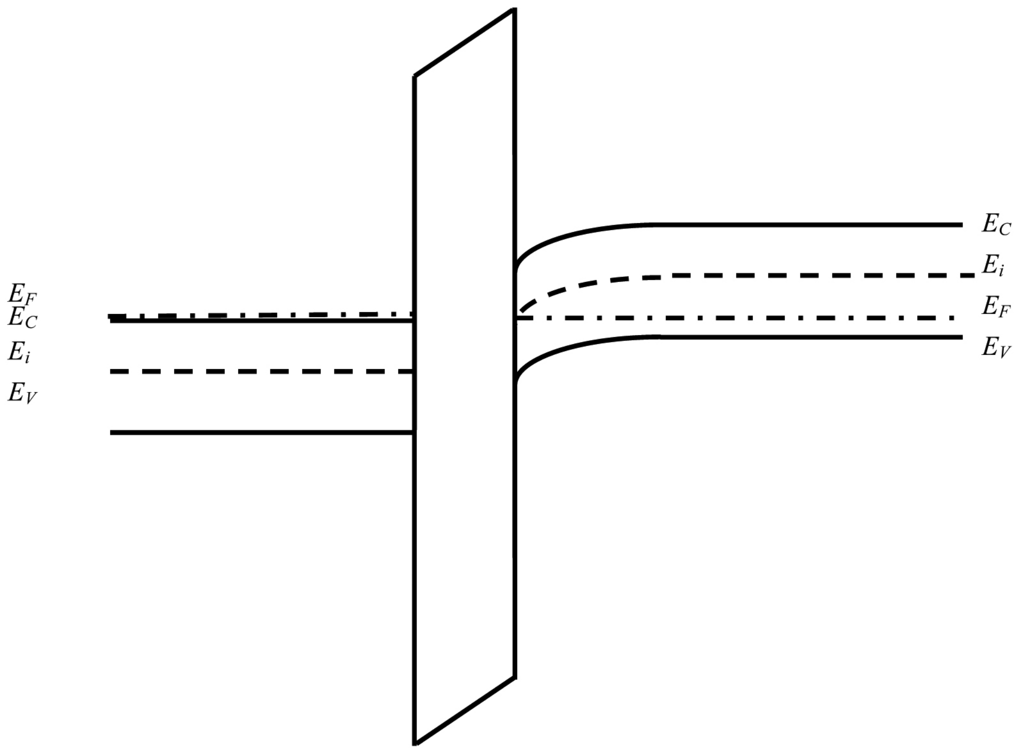
$$\text{Area} = \frac{I_s}{J} = \frac{1\times10^{-3}}{2.25\times10^{-5}} = 44.4\ \text{cm}^2.$$

CHAPTER 5

1.



2.



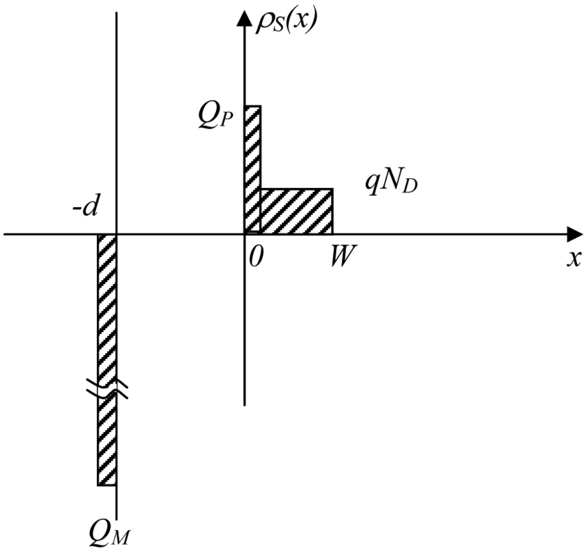
n^+ POLYSILICON OXIDE p-TYPE SEMICONDUCTOR

3. The flatband voltage equals the work function difference since there is no charge in the oxide or at the oxide-semiconductor interface.

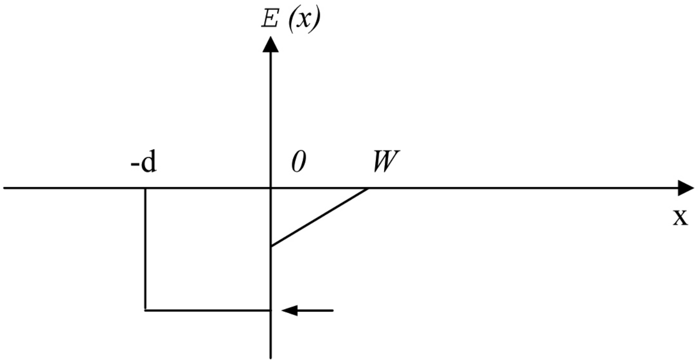
$$V_{FB} = \Phi_{MS} = \Phi_M - \chi - \frac{E_g}{2q} - V_t \ln\frac{N_a}{n_i}$$

$$= 4.1 - 4.05 - 0.56 - 0.026 \times \ln\frac{10^{17}}{10^{10}} = -0.93\text{ V}$$

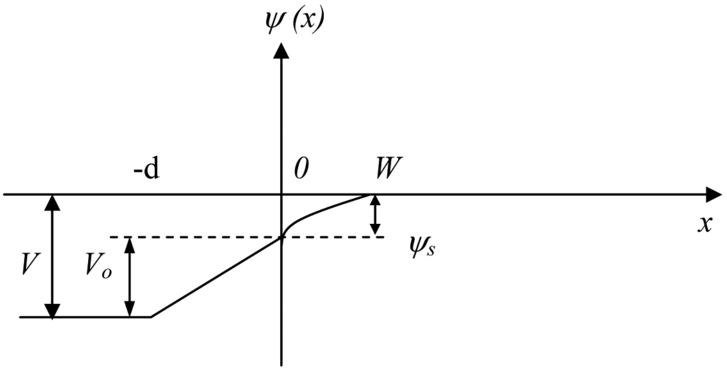
4.



(a)



(b)



(c)

5. $$W_m = 2\sqrt{\frac{\varepsilon_s kT \ln\left(\frac{N_A}{n_i}\right)}{q^2 N_A}}$$

$$= 2\sqrt{\frac{11.9\times 8.85\times 10^{-14}\times 0.026\ln\left(\frac{5\times 10^{16}}{9.65\times 10^{9}}\right)}{1.6\times 10^{-19}\times 5\times 10^{16}}}$$

$$= 1.5\times 10^{-5}\text{ cm} = 0.15\ \mu\text{m}.$$

6. The electric field E at the semiconductor and oxide sides of the junction are given by: $E_s = (q\,N_A\ W)/\varepsilon_s$, and $E_{ox} = (q\,N_A\ W)/\varepsilon_{ox}$.

Now W = depletion width $= [(2\varepsilon V_S^{inv})/(q\,N_A)]^{1/2}$, where V_S^{inv} is the voltage at inversion.

Thus: $V_S^{inv} = 2(kT/q)\ \text{Ln}(N_A/n_i) = 2\times 25.9\times 10^{-3}\times \text{Ln}\ (5\times 10^{17}/1.45\times 10^{10}) = 0.636$ V.

So: $W = [(2\times 8.85\times 10^{-14}\times 11.9\times 0.636)/(1.6\times 10^{-19}\times 5\times 10^{15})]^{1/2} = 0.41\ \mu$m.

Hence: $E_S = (1.6\times 10^{-19}\times 5\times 10^{15}\times 0.41\times 10^{-4})/(8.85\times 10^{-14}\times 11.9) = 31.14$ kV/cm.

Also: : $E_{ox} = (1.6\times 10^{-19}\times 5\times 10^{15}\times 0.41\times 10^{-4})/(8.85\times 10^{-14}\times 3.9) = 95$ kV/cm.

7. $$C_{\min} = \frac{\varepsilon_{ox}}{d + (\varepsilon_{ox}/\varepsilon_S)W_m}$$

$W_m = 0.15\mu\text{m}$ From Prob. 5

$$\therefore C_{\min} = \frac{3.9\times 8.85\times 10^{-14}}{8\times 10^{-7} + \frac{3.9}{11.9}\times 1.5\times 10^{-5}} = 6.03\times 10^{-8}\text{ F/cm}^2.$$

8. $$\psi_B = \frac{kT}{q}\ln\frac{N_A}{n_i} = 0.026\ln\frac{10^{17}}{9.65\times 10^{9}} = 0.42\text{ V}$$

$$E_s = \frac{qN_A W}{\varepsilon_S}\ \text{at intrinsic}\ \psi_S = \psi_B$$

$$W = \sqrt{\frac{2\varepsilon_S\psi_S}{qN_A}} = \sqrt{\frac{2\times 11.9\times 8.85\times 10^{-14}\times 0.42}{1.6\times 10^{-19}\times 10^{17}}} = 0.74\times 10^{-5}\text{cm}$$

$$E_S = \frac{1.6\times 10^{-19}\times 10^{17}\times 0.74\times 10^{-5}}{11.9\times 8.85\times 10^{-14}} = 1.11\times 10^{5}\text{ V/cm}$$

$$E_o = E_S\frac{\varepsilon_S}{\varepsilon_{ox}} = 1.11\times 10^{5}\times\frac{11.9}{3.9} = 3.38\times 10^{5}\text{ V/cm}$$

$$V = V_o + \psi_S = E_o d + \psi_S = (3.38\times 10^{5}\times 5\times 10^{-7}) + 0.42.$$
$$= 0.59\text{ V}.$$

9. $C_{max}/C_{min} = 1+(\varepsilon_{ox}/\varepsilon_S)$ (W_m/d). Since W_m does not depend on the oxide thickness, and all other parameters are the same for the two devices, one gets:

$[1+(\varepsilon_{ox}/\varepsilon_S)(W_m/d_1)] = 3$ and $[1+(\varepsilon_{ox}/\varepsilon_S)(W_m/d_2)] = 2$

$\to [(3.9/11.9)(W_m/d_1)] = 2$ and $[(3.9/11.9)(W_m/d_2)] = 1$.

Solving: $d_1/d_2 = ½$.

10. $$Q_{ot} = \frac{1}{d}\int_0^{10^{-4}} yq(10^{17})dy = \frac{1.6\times10^{-19}}{10^{-6}}\left[\frac{1}{2}\times10^{17}\times(10^{-6})^2\right]$$

$$= 8\times10^{-9}\,\text{C/cm}^2$$

$$\Delta V_{FB} = \frac{Q_{ot}}{C_o} = \frac{8\times10^{-9}}{3.45\times10^{-7}} = 2.32\times10^{-2}\text{ V.}$$

11. $$Q_{ot} = \frac{1}{d}\int_0^{d} y\rho_{ot}(y)dy$$

$$\rho_{ot} = q\times5\times10^{11}\delta(x),\ \text{where } \delta(x) = \begin{cases}\infty, & y = 5\times10^{-7}\\ 0, & y \neq 5\times10^{-7}\end{cases}$$

$$\therefore Q_{ot} = \frac{1}{10^{-6}}\times5\times10^{11}\times5\times10^{-7}\times1.6\times10^{-19}$$

$$= 4\times10^{-8}\text{ C/cm}^2$$

$$\therefore \Delta V_{FB} = \frac{Q_{ot}}{C_o} = \frac{4\times10^{-8}}{3.45\times10^{-7}} = 0.12\text{ V}$$

$$= \frac{1}{10^{-6}}\times1.6\times10^{-19}\times\frac{1}{3}\times5\times10^{23}(10^{-6})^3.$$

12. $\Delta V_{FB} = \frac{Q_{ot}}{C_o} = \frac{q}{C_o}\times\frac{d}{d}\times N_m$, where N_m is the area density of Q_m.

$$\Rightarrow N_m = \frac{\Delta V_{FB}\times C_o}{q} = \frac{0.3\times3.45\times10^{-7}}{1.6\times10^{-19}} = 6.47\times10^{11}\text{ cm}^{-2}.$$

13. $$Q_{ot} = \frac{1}{d}\int_0^{10-6} y(q\times5\times10^{23}\times y)dy$$

$$= 2.67\times10^{-8}\text{ C/cm}^2$$

$$\therefore \Delta V_{FB} = \frac{Q_{ot}}{C_o} = \frac{2.67\times10^{-8}}{3.45\times10^{-7}} = 7.74\times10^{-2}\text{ V.}$$

14. Since $V_D \ll (V_G - V_T)$, the first term in Eq. 35 can be approximated as

$$\frac{Z}{L}\mu_n C_o (V_G - 2\psi_B - \frac{V_D}{2})V_D.$$

Performing Taylor's expansion on the 2nd term in Eq. 35, we obtain

$$(V_D + 2\psi_B)^{3/2} - (2\psi_B)^{3/2} \cong (2\psi_B)^{3/2} + \frac{3}{2}(2\psi_B)^{1/2}V_D - (2\psi_B)^{3/2} = \frac{3}{2}(2\psi_B)^{1/2}V_D$$

Equation 33 can now be re-written as

$$I_D \cong \frac{Z}{L}\mu_n C_o \left[(V_G - 2\psi_B - \frac{V_D}{2})V_D - \frac{2}{3}\frac{\sqrt{2\varepsilon_S q N_A}}{C_o} \times \frac{3}{2}\sqrt{2\psi_B}V_D \right]$$

$$\cong \frac{Z}{L}\mu_n C_o \left\{ V_G - \left[2\psi_B + \frac{\sqrt{2\varepsilon_S q N_A (2\psi_B)}}{C_o} \right] - \frac{V_D}{2} \right\} V_D$$

$$\cong (\frac{Z}{L})\mu_n C_o (V_G - V_T - \frac{V_D}{2})V_D$$

where $V_T = 2\psi_B + \dfrac{\sqrt{2\varepsilon_S q N_A (2\psi_B)}}{C_o}$.

15. The threshold voltage equals:

$$V_T = V_{FB} + 2\phi_F + \frac{\sqrt{4\varepsilon_s q N_a \phi_F}}{C_{ox}}$$

$$= -0.93 + 2\times 0.42$$

$$+ \frac{\sqrt{4\times 11.9\times 8.85\times 10^{-14}\times 1.6\times 10^{-19}\times 10^{17}\times 0.42}}{3.9\times 8.85\times 10^{-14}/20\times 10^{-7}}$$

$$= 0.88\ \text{V}$$

16. $$\psi = \frac{kT}{q}\ln(\frac{N_A}{n_i}) = 0.026\ln\left[\frac{5\times 10^{16}}{9.65\times 10^{-9}}\right] = 0.40\ \text{V}$$

$$\text{K} \equiv \frac{\sqrt{\varepsilon_s q N_A}}{C_o} = \frac{\sqrt{11.9\times 8.85\times 10^{-14}\times 1.6\times 10^{-19}\times 5\times 10^{-16}}}{3.45\times 10^{-7}}$$

$$= 0.27$$

$$\therefore V_{Dsat} \cong V_G - 2\psi_B + K^2\left(1 - \sqrt{1 + \frac{2V_G}{K^2}}\right)$$

$$= 5 - 0.8 + (0.27)^2\left[1 - \sqrt{1 + \frac{10}{(0.27)^2}}\right]$$

$$= 3.42\ \text{V}$$

$$I_{Dsat} = \frac{Z\mu_n C_o}{2L}(V_G - V_T)^2 = \frac{10\times 800\times 3.45\times 10^{-7}}{2\times 1}(5 - 0.7)^2$$

$$= 2.55\times 10^{-2}\ \text{A}.$$

17. The device is operated in linear region, since $V_D = 0.1\text{ V} < (\text{V}_\text{G} - V_T) = 0.5\text{ V}$

Therefore, $g_d = \frac{\partial I_D}{\partial V_D}\Big|_{V_G = const} = \frac{Z}{L}\mu_n C_o (V_G - V_T)$

$$= \frac{5}{0.25} \times 500 \times 3.45 \times 10^{-7} \times 0.5$$

$$= 1.72 \times 10^{-3}\text{ S.}$$

18. $g_m = \frac{\partial I_D}{\partial V_G}\Big|_{V_D = const.} = \frac{Z}{L}\mu_n C_o V_D$

$$= \frac{5}{0.25} \times 500 \times 3.45 \times 10^{-7} \times 0.1$$

$$= 3.45 \times 10^{-4}\text{S.}$$

19. $V_T = V_{FB} + 2\psi_B + \frac{2\sqrt{\varepsilon_S q N_A \psi_B}}{C_o}$, $\psi_\text{B} = 0.026 \ln\left(\frac{10^{17}}{9.65 \times 10^9}\right)$

$$V_{FB} = \phi_{ms} - \frac{Q_f}{C_o} = -\frac{E_g}{2} - \psi_B - \frac{1.6 \times 10^{-19} \times 5 \times 10^{10}}{3.45 \times 10^{-7}}$$

$$= -0.56 - 0.42 - 0.02 = -1\text{ V}$$

$$\therefore V_T = -1 + 0.84 + \frac{2\sqrt{11.9 \times 8.85 \times 10^{-14} \times 10^{17} \times 0.42 \times 1.6 \times 10^{-19}}}{3.45 \times 10^{-7}}$$

$$= -1 + 0.84 + 0.49$$

$$= 0.33\text{ V}$$

(ϕ_{ms} can also be obtained from Fig. 8 to be -0.98 V).

20. $0.7 = 0.33 + \frac{qF_B}{3.45 \times 10^{-7}}$

$$F_B = \frac{0.37 \times 3.45 \times 10^{-7}}{1.6 \times 10^{-19}} = 8 \times 10^{11}\text{ cm}^{-2}.$$

21. The oxide capacitance equals:

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}}{2 \times 10^{-6}} = 173\text{ nF/cm}^2$$

The flatband capacitance equals:

$$C_{FB} = \frac{1}{\frac{1}{C_{ox}} + \frac{L_D}{\varepsilon_s}}$$

$$= \frac{1}{\frac{1}{173 \times 10^{-9}} + \frac{1.3 \times 10^{-6}}{11.9 \times 8.85 \times 10^{-14}}} = 142\text{ nF/cm}^2$$

where the Debye length is obtained from:

$$L_D = \sqrt{\frac{\varepsilon_s V_t}{qN_a}} = \sqrt{\frac{11.9\times 8.85\times 10^{-14}\times 0.0259}{1.6\times 10^{-19}\times 10^{17}}} = 13\text{ nm}$$

The high frequency capacitance in inversion equals:

$$C_{HF,inv} = \frac{1}{\frac{1}{C_{ox}}+\frac{x_{d,T}}{\varepsilon_s}}$$

$$= \frac{1}{\frac{1}{173\times 10^{-9}}+\frac{1.05\times 10^{-5}}{11.9\times 8.85\times 10^{-14}}} = 63\text{ nF/cm}^2$$

and the depletion layer width at threshold equals:

$$x_{d,T} = \sqrt{\frac{2\varepsilon_s(2\phi_F)}{qN_a}}$$

$$= \sqrt{\frac{2\times 11.9\times 8.85\times 10^{-14}\times 2\times 0.419}{1.6\times 10^{-19}\times 10^{17}}} = 105\text{ nm}$$

22. $$V_{FB} = \Phi_{MS} = \Phi_M - \chi - \frac{E_g}{2q} - V_t \ln\frac{N_a}{n_i}$$

$$= 4.1 - 4.05 - 0.56 - 0.026\times \ln\frac{10^{17}}{10^{10}} = -0.93\text{ V}$$

$$\phi_{ms} = -\frac{E_g}{2} + \psi_B = -0.56 + 0.42 = -0.14\text{ V}$$

$$V_T = \phi_{ms} - \frac{Q_f}{C_o} - 2\psi_B - \frac{2\sqrt{\varepsilon_s q N_D \psi_B}}{C_o}$$

$$= -0.14 - 0.02 - 0.84 - \frac{2\sqrt{11.9\times 8.85\times 10^{-14}\times 1.6\times 10^{-19}\times 10^{17}\times 0.42}}{3.45\times 10^{-7}}$$

$$= -1.49\text{ V.}$$

23. $$-0.7 = -1.49 + \frac{qF_B}{3.45\times 10^{-7}}$$

$$F_B = \frac{0.79\times 3.45\times 10^{-7}}{1.6\times 10^{-19}} = 1.7\times 10^{12}\text{ cm}^{-2}.$$

24. $\psi_B = 0.026\ln\left(\dfrac{10^{17}}{9.65\times10^{9}}\right) = 0.42\text{ V}$

$$V_T = \phi_{ms} - \frac{qQf}{C_o} + 2\psi_B + \frac{2\sqrt{\varepsilon_s qN_A\psi_B}}{C_o}$$

$$= -0.98 - \frac{1.6\times10^{-19}\times10^{11}}{C_o} + 0.84 + \frac{2\sqrt{11.9\times8.85\times10^{-14}\times10^{17}\times0.42\times1.6\times10^{-19}}}{C_o}$$

$$= -0.14 + \frac{15.2\times10^{-8}}{C_o}$$

$$C_o = \frac{3.9\times8.85\times10^{-14}}{d} = \frac{3.45\times10^{-13}}{d}$$

$$V_T > 20 \Rightarrow \frac{d\times15.2\times10^{-8}}{3.45\times10^{-13}} > 20.14$$

$\therefore d < 4.57\times10^{-5}\text{cm} = 0.457\ \mu\text{m}.$

25. $\Delta V_T = \dfrac{\sqrt{2q\varepsilon_s N_A}}{C_o}\left(\sqrt{2\psi_B + V_{BS}} - \sqrt{2\psi_B}\right)$

$$\psi_B = 0.026\ln(\frac{10^{17}}{9.65\times10^{9}}) = 0.42\text{ V}$$

$$C_o = \frac{3.9\times8.85\times10^{-14}}{5\times10^{-7}}6.9\times10^{-7}\text{F/cm}^2$$

$\Delta V_T = 0.1\text{ V}$ if we want to reduce I_D at $V_G = 0$ by one order of magnitude, since the subthreshold swing is 100 mV/decade.

$$0.1 = \frac{\sqrt{2\times1.6\times10^{-19}\times4.9\times8.85\times10^{-14}\times10^{17}}}{6.9\times10^{-7}}\left(\sqrt{0.84+V_{BS}} - \sqrt{0.84}\right)$$

$\therefore V_{BS} = 0.83\text{ V}.$

26. $V_T = 0.5\text{ V at } I_d = 0.1\ \mu\text{A}$

$$\text{Subthreshold swing} = \left(\frac{\log I_D\big|_{V_G=V_T} - \log I_D\big|_{V_G=0}}{V_T - 0}\right)^{-1}$$

$$0.1 = \frac{0.5}{-7 - \log I_D\big|_{V_G=0}}$$

$\log I_D\big|_{V_G=0} = -12 \quad \therefore I_D\big|_{V_G=0} = 1\times10^{-12}\text{ A}.$

CHAPTER 6

1. Scaling factor $\kappa = 10$

Switching energy $= \frac{1}{2}(C \cdot A)V^2$

$$C' = \frac{\varepsilon_{ox}}{d} = \kappa C$$

$$A' = \frac{A}{\kappa^2}$$

$$V' = \frac{V}{\kappa}$$

$\therefore$ scaling factor for switching energy: $\kappa \cdot \frac{1}{\kappa^2} \cdot \frac{1}{\kappa^2} = \frac{1}{\kappa^3} = \frac{1}{1000}$

A reduction of one thousand times.

(a) $\therefore$ Switching energy $= \frac{1}{2}(C \cdot A)V^2 = \frac{1}{2000}$

(b) $P\tau \sim \frac{1}{k^2}\frac{1}{k} \times 1 = \frac{1}{k^3} \times 1 = \frac{1}{1000} \times 1 = 1$ mJ

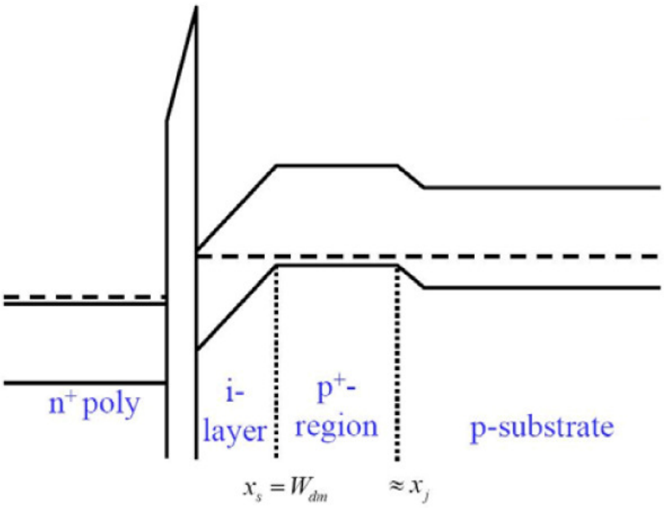
2. $\mathrm{C_{ox} = \varepsilon_{ox}/t_{ox} = 3.9 * 8.85 \times 10^{-14} / (200 \times 10^{-8}) = 1.726 \times 10^{-7}\ F/cm^2}$.

$\mathrm{\Delta V_T = -[q\ N_A\ W_m\ r_j / (C_{ox}\ L)][\{1 + 2\ W_m/r_j\}^{½} - 1]}$.

$\mathrm{\Delta V_T = -\{[1.6 \times 10^{-19} * 3 \times 10^{16} * 0.18 \times 10^{-4} (0.3/1.0)]/[1.726 \times 10^{-7}]\}[\{1 + 2 * 0.18/0.3\}^{½} - 1]}$

$\mathrm{\rightarrow \Delta V_T = -0.0726\ V}$.

3.



4. The junction built-in voltage $= 25.9\times10^{-3}\,\mathrm{Ln}[10^{19}*10^{16}/(1.5\times10^{10})^2]$=0.874 V.

The zero-bias source-substrate pn junction width X_{d0} is: $[(2\varepsilon_s\,V_{bi})/(qN_A)]^{½}$

Thus: $X_{d0} = [(2*11.7*8.85\times10^{-14}0.874)/(1.6\times10^{-19}\,10^{16})]^{½} = 0.336\,\mu\mathrm{m}$.

Let the reverse-biased drain-substrate pn junction width be X_d.

So: $X_d = [\{2\varepsilon_s(V_{bi}+V_{DS})\}/(qN_A)]^{½}$

At punch-through: : $X_{d0} + X_d = L \rightarrow 0.336 + X_d = 1.2 \rightarrow\ X_d = 0.864\,\mu\mathrm{m}$.

Using: $X_d = [\{2\varepsilon_s(V_{bi}+V_{DS})\}/(qN_A)]^{½}$ leads to: $V_{bi} + V_{DS} = 5.77$ V.

Thus: $V_{DS} = 5.77 - 0.874 = 4.9$ V.

5. L=0.7 µm, $W = 7\,\mu\mathrm{m}$, $t_{ox} = 17.5\,\mathrm{nm}$, $N_A = 7.14\times10^{15}\,\mathrm{cm}^{-3}$, and voltage = 2.1 V.

6. Pros:

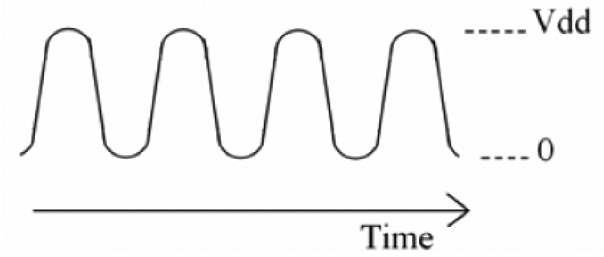
1. Higher operation speed.
2. High device density

Cons:

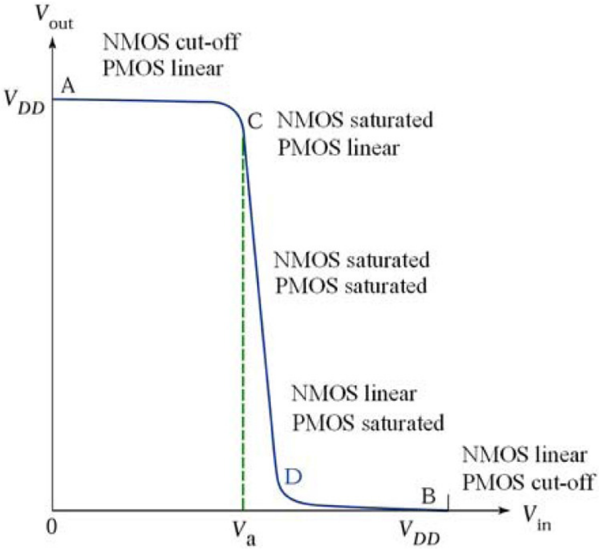
1. More complicated fabrication flow.
2. High manufacturing cost.

7. The rise time if the minimum geometry CMOS will be substantially greater than the fall time due to the fact that the hole mobility is much smaller (about one-third) than the electron mobility.

8. (a)



(b)



(c) At point C, the voltage $\mathrm{Va} = \mathrm{Vin}$ (input voltage), the NMOS is just becoming saturated from the linear region. Since NMOS is in the linear region and PMOS in the saturation region, the drain current of NMOS is equal to the drain current of the PMOS, that is

$\mathrm{I_{DN}} = \mathrm{I_{DP}}$

Therefore,

$$20\left[(V_\mathrm{a}-2)^2-\frac{(V_\mathrm{a}-2)^2}{2}\right]=\frac{20}{2}(5-V_\mathrm{a}-1)^2$$

$$20(V_\mathrm{a}-2)^2=20(4-V_\mathrm{a})^2 \Rightarrow V_\mathrm{a}=3\mathrm{V}$$

9. The maximum width of the surface depletion region for bulk MOS

$$W_m = 2\sqrt{\frac{\varepsilon_s kT\ln(N_A/n_i)}{q^2 N_A}}$$

$$=2\sqrt{\frac{11.9\times 8.85\times 10^{-14}\times 0.026\times\ln(5\times 10^{17}/9.65\times 10^{9})}{1.6\times 10^{-19}\times 5\times 10^{17}}}$$

$$=4.9\times 10^{-6}\,\mathrm{cm}$$

$$=49\ \mathrm{nm}$$

For FD-SOI, $d_{si} \le W_m = 49$ nm.

10. $$\Delta V_T = \frac{qN_A\Delta d_{Si}}{C_o}$$

Thus, the range of V_T is from $(0.18-0.046)=0.134$ V to (0.18+0.046)=0.226 V.

$$=\frac{1.6\times 10^{-19}\times 5\times 10^{17}\times 5\times 10^{-7}}{8.63\times 10^{-7}}$$

$$=46\ \mathrm{mV}$$

11. $$V_T = V_{FB}+2\psi_B+\frac{qN_A d_{si}}{C_o}$$

$$V_{FB}=\phi_{ms}=-\frac{E_g}{2}-\frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right)=-\frac{1.12}{2}-0.026\ln\left(\frac{5\times 10^{17}}{9.65\times 10^{9}}\right)=-1.02\ \mathrm{V}$$

$$2\psi_B = 2\times\frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right)=0.92\ \mathrm{V}$$

$$\mathrm{C_o}=\frac{3.9\times 8.85\times 10^{-14}}{4\times 10^{-7}}=8.63\times 10^{-7}\ \mathrm{F/cm^2}$$

$$\therefore V_T=-1.02+0.92+\frac{1.6\times 10^{-19}\times 5\times 10^{17}\times 3\times 10^{-6}}{8.63\times 10^{-7}}$$

$$=-0.1+0.28$$

$$=0.18\ \mathrm{V.}$$

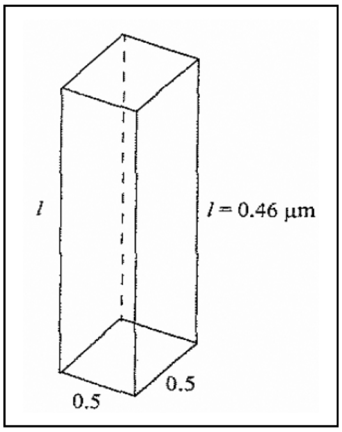
12. $$C=\frac{\varepsilon_{\text{ox}}}{\text{t}_{\text{ox}}}=\frac{3.9\times8.85\times10^{-14}}{50\times10^{-8}}=6.9\times10^{-7}\ \text{F/cm}^2$$

$$\text{Area}=(0.5\times0.5)+4(0.5\times l)=0.25+2l\ \mu\text{m}^2$$

$$Q=CV\quad C=\frac{Q}{V}=\frac{10^5\times1.6\times10^{-19}}{2}=8\times10^{-15}\ \text{F}$$

$$8\times10^{-15}=6.9\times10^{-7}(0.25+2l)\times10^{-8}=6.9\times10^{-15}(0.25+2l)$$

$$\therefore l=\frac{1.16-0.25}{2}=0.46\ \mu\text{m}$$



13. $$I=C\frac{dV}{dt}=5\times10^{-14}\times\frac{2.5}{4\times10^{-3}}=3.1\times10^{-11}\text{A}$$

14. The planar capacitor

$$\text{C=A}^{\varepsilon}\,{}^{ox}\!/_{d}=\frac{(1\times10^{-4})^2 3.9\times8.86\times10^{-14}}{1\times10^{-6}}=3.45\times10^{-15}\ \text{F}$$

For the trench capacitor

$$A=4\times7\ \mu\text{m}^2+1\ \mu\text{m}^2=29\ \mu\text{m}^2$$

$$C=29\times3.45\times10^{-15}\ \text{F}=100\times10^{-15}\ \text{F.}$$

15. $$g_o=\frac{Z}{L}\mu_p C_i(V_o-V_T)$$

$$4\times10^{-5}=A(-5-V_T)\qquad\therefore V_T=7V$$

$$1\times10^{-5}=A(-5+2)$$

$$\Delta V=7-(-2)=9\text{V.}$$

16. $$Q_{FG}=C_{FG\cdot CG}\times\Delta V$$

$$n_{FG}=\frac{C_{FG\cdot CG}\times\Delta V}{q}=\frac{2.59\times10^{-15}\times0.5}{1.6\times10^{-19}}=8094\ \text{electrons.}$$

17. $$\varepsilon_1 = \frac{V_G}{d_1 + d_2(\varepsilon_1 / \varepsilon_2)} + \frac{Q}{\varepsilon_1 + \varepsilon_2(d_1 / d_2)}.$$

$$J_1 = \sigma g_1 = 10^{-7}\left\{\frac{10\times 10^7}{10+100\left(\frac{4}{10}\right)} + \frac{Q}{\left[4+10\left(\frac{10}{100}\right)8.85\times 10^{-14}\right]}\right\} = 0.2 - 2.26\times 10^5\left|Q\right|$$

$$when\ \mathrm{J}_1 = 0 \quad \left|Q\right| = \frac{0.2}{2.26\times 10^5} = 8.84\times 10^7\ Couls$$

$$\Delta V_T = \frac{Q}{\varepsilon_2/d_2} = \frac{8.85\times 10^7}{10\times 8.85\times 10^{-4}/10^{-5}} = 10\ \mathrm{V}$$

18.

	Cell size	Write one byte rate	Rewrite cycle	Keep data without power	Applications
SRAM	Large	Fast	unlimited	No	Embedded in ligic chips
DRAM	Mid	Fast	unlimited	No	Stand-alone chips and embedded
Flash	Small	Slow	limited	Yes	Nonvoltaile storage stand-alone

19. $$V_T = V_{FB} + 2\psi_B + \frac{\sqrt{\varepsilon_s q N_A \psi_B}}{C_0}$$

$$C_0 = 3.9\times\frac{8.854\times 10^{-14}}{10^{-5}} = 3.45\times 10^{-8}\ \mathrm{F/cm^2}$$

$$\psi_B = 0.026\ln\left(\frac{10^{17}}{9.65\times 10^9}\right) = 0.42\ \mathrm{V}$$

$$V_{FB} = \phi_{ms} - \frac{Q_f}{2} = -\frac{E_g}{2} - \psi_B - 0$$

$$= -0.56 - 0.42 = -0.98\ \mathrm{V}$$

$$\therefore V_T = -0.98 + 0.42 + \frac{2\sqrt{11.9\times 8.85\times 10^{-14}\times 10^{17}\times 0.42\times 1.6\times 10^{-19}}}{3.45\times 10^{-8}}$$

$$= -0.98 + 0.42 + 4.9$$

$$= 4.34\ \mathrm{V}$$

20. $$\Delta V_{FB} = -\frac{Q_f}{C_0} = -\frac{5\times10^{11}\times1.6\times10^{-19}}{3.45\times10^{-8}} = -2.32\text{ V}$$

$$\begin{aligned} V_T &= 4.34 - 2.32 \\ &= 2.02\text{ V}. \end{aligned}$$

CHAPTER 7

1. From Eq.1, the theoretical barrier height is

$$\phi_{Bn} = \phi_m - \chi = 4.55 - 4.01 = 0.54 \text{ eV}$$

We can calculate V_n as

$$V_n = \frac{kT}{q}\ln\frac{N_C}{N_D} = 0.0259\ln\left(\frac{2.86\times10^{19}}{2\times10^{16}}\right) = 0.188 \text{ V}$$

Therefore, the built-in potential is

$$V_{bi} = \phi_{Bn} - V_n = 0.54 - 0.188 = 0.352 \text{ V}.$$

2. (a) From Eq.11

$$\frac{d(1/C^2)}{dV} = \frac{(6.2-4.6)\times10^{14}}{-2-0} = -2.3\times10^{14}(\text{cm}^2/\text{F})^2/\text{V}$$

$$ND = \frac{2}{q\varepsilon_s}\left[\frac{-1}{d(1/C^2)/dV}\right] = 4.7\times10^{16} \text{ cm}^{-3}$$

$$V_n = \frac{kT}{q}\ln\frac{N_C}{N_D} = 0.0259\ln\left(\frac{4.7\times10^{17}}{4.7\times10^{16}}\right) = 0.06 \text{ V}$$

From Fig.6, the intercept of the GaAs contact is the built-in potential V_{bi}, which is equal to 0.7 V. Then, the barrier height is

$$\phi_{Bn} = V_{bi} + V_n = 0.76 \text{ V}$$

(b) $J_s = 5 \times 10^{17}$ A/cm^2

$A^* = 8$ A/K^2-cm^2 for n– type GaAs

$J_s = A^*T^2e^{-q\phi_{Bn}/kT}$

$$\phi_{Bn} = \frac{kT}{q}\ln\left(\frac{A^*T^2}{J_s}\right) = 0.0259 \ln\left[\frac{8\times(300)^2}{5\times10^{-7}}\right] = 0.72 \text{ eV}$$

The barrier height from capacitance is 0.04 V or 5% larger.

(c) For $V = -1$ V

$$W = \sqrt{\frac{2\varepsilon_s(V_{bi}+V_R)}{qN_D}} = \sqrt{\frac{2\times1.09\times10^{-12}(0.7+1)}{1.6\times10^{-19}\times4.7\times10^{16}}}$$

$$= 2.22\times10^{-5} \text{ cm} = 2.22 \text{ μm}$$

$$\text{E}_m = \frac{qN_DW}{\varepsilon_s} = 1.43\times10^5 \text{ V/cm}$$

$$C = \frac{\varepsilon_s}{W} = 5.22\times10^{-8}\ \text{F/cm}^2.$$

3. The current-voltage expression for a Schottky barrier diode is given by:

   $I = I_s\,[\exp\{qV/(kT)\} - 1]$, where $I_s = A\,A^*\,T^2 \exp[-q\Phi_b/(kT)]$.

   Under the condition: $kT/q \ll V$, the above equation can be re-written as:

   $\text{Ln}(I/T^2) = \text{Ln}(A\,A^*) - q\,(\Phi_b - V)/(kT)$. So plot of $\text{Ln}[I/T^2]$ versus $1/T$ at constant bias V has a slope of $-\,q\,(\Phi_b - V)/k$ and an intercept of $\text{Ln}(A\,A^*)$.

   For the data given: $V = 0.4$ Volt and area $A = 0.001\ \text{cm}^2$. Plotting $\text{Ln}[I/T^2]$ versus $1000/T$ yields slope of: -1.7642 and intercept of: -.9584.

   Hence: slope $= -1.7642 = -\,q\,(\Phi_b - V)/k = -1.6\times10^{-19}\,(\Phi_b - 0.4)/1.38\times10^{-23}$

   Consequently: $\Phi_b = 0.751$ V.

   Intercept $= -.9584 = \text{Ln}(A\,A^*) = \text{Ln}(0.001\,A^*)$ ➔ $A^* = 110.0525\ \text{A/cm}^2\,\text{K}^2$.

4. The barrier height is

$$\phi_{Bn} = \phi_m = \chi = 4.65 - 4.01 = 0.64\ \text{V}$$

$$V_n = \frac{kT}{q}\ln\frac{N_C}{N_D} = 0.0259\times\ln\left(\frac{2.86\times10^{19}}{3\times10^{16}}\right) = 0.177\ \text{V}$$

   The built-in potential is

$$V_{bi} = \phi_{Bn} - V_n = 0.64 - 0.177 = 0.463\ \text{V}$$

   The depletion width is

$$W = \sqrt{\frac{2\varepsilon_s (V_{bi} - V_R)}{qN_D}} = \sqrt{\frac{2\times11.9\times8.85\times10^{-14}}{1.6\times10^{-19}\times3\times10^{16}}\times0.463} = 0.142\ \mu\text{m}$$

   The maximum electric field is

$$|\mathrm{E}_m| = |\mathrm{E}(x=0)| = \frac{qN_D}{\varepsilon_s}W = \frac{(1.6\times10^{19})\times(3\times10^{16})\times(1.42\times10^{-5})}{11.9\times(8.85\times10^{-14})} = 6.54\times10^{4}\ \text{V/cm}.$$

5. The unit of C needs to be changed from μF to F/cm², so

$$1/C^2 = 1.74\times10^{15} - 2.12\times10^{15}\,V_a\,(\text{cm}^2/\text{F})^2$$

   Therefore, we obtain the built-in potential at $1/C^2 = 0$

$$V_{bi} = \frac{1.57\times10^{15}}{2.12\times10^{15}} = 0.74\ \text{V}$$

   From the given relationship between C and V_a, we obtain

$$\frac{d\left(\dfrac{1}{C^2}\right)}{dV_a} = -2.12\times10^{15} \quad (\mathrm{cm}^2/\mathrm{F})^2/\mathrm{V}$$

From Eq.11

$$ND = \frac{2}{q\varepsilon_s}\left[\frac{-1}{d(1/C^2)/dV}\right]$$

$$= \frac{2}{1.6\times10^{-19}\times11.9\times8.85\times10^{-14}}\left(\frac{1}{2.12\times10^{15}}\right)$$

$$= 5.6\times10^{15}\ \mathrm{cm}^{-3}$$

$$V_n = \frac{kT}{q}\ln\frac{N_C}{N_D} = 0.0259\ln\left(\frac{2.86\times10^{19}}{5.6\times10^{16}}\right) = 0.161\ \mathrm{V}$$

We can obtain the barrier height

$$\phi_{Bn} = V_{bi} + V_n = 0.74 + 0.161 = 0.901\ \mathrm{V}.$$

6. The built-in potential is

$$V_{bi} = \phi_{Bn} - \frac{kT}{q}\ln\frac{N_C}{N_D}$$

$$= 0.8 - 0.0259\ln\left(\frac{2.86\times10^{19}}{1.5\times10^{16}}\right)$$

$$= 0.8 - 0.195$$

$$= 0.605\ \mathrm{V}$$

Then, the work function is

$$\phi_m = \phi_{Bn} + \chi$$

$$= 0.8 + 4.01$$

$$= 4.81\ \mathrm{V}.$$

7. The barrier height equals: $\varphi_B = \Phi_M - \chi = 4.5 - 4.05 = 0.45$ V.

Now the built-in potential φ_i equals: $\varphi_B - (kT/q)\ \mathrm{Ln}[N_C/N_D]$

$= 0.45 - 0.0259*\mathrm{Ln}\ [2.82\times10^{19}/10^{17}] = 0.3$ Volt.

8. The saturation current density is

$$J_s = A^* T^2 \exp\left(\frac{-q\phi_{Bn}}{kT}\right)$$

$$= 110 \times (300)^2 \times \exp\left(\frac{-0.8}{0.0259}\right)$$

$$= 3.81 \times 10^{-7} \text{ A/cm}^2$$

The injected hole current density is

$$J_{po} = \frac{qD_p n_i{}^2}{L_p N_D} = \frac{1.6\times10^{-19}\times12\times(9.65\times10^9)^2}{1\times10^{-3}\times1.5\times10^{16}} = 1.19\times10^{-11} \text{ A/cm}^2$$

$$\frac{\text{Hole current}}{\text{Electron current}} = \frac{J_{po}(e^{qV/kT}-1)}{J_s(e^{qV/kT}-1)}$$

$$= \frac{J_{po}}{J_s} = \frac{1.18\times10^{-11}}{3.81\times10^{-3}} = 3\times10^{-5}.$$

9. The difference between the conduction band and the Fermi level is given by

$$V_n = 0.0259 \ln\left(\frac{4.7\times10^{17}}{1\times10^{17}}\right) = 0.04 \text{ V}.$$

The built-in potential barrier is then

$$V_{bi} = 0.9 - 0.04 = 0.86 \text{ V}$$

For a depletion mode operation, V_T is negative. Therefore, From Eq.38a

$$V_T = 0.86 - V_P < 0$$

$$V_P = \frac{qa^2 N_D}{2\varepsilon_s} = \frac{1.6\times10^{-19} a^2 \times10^{17}}{2\times12.4\times8.85\times10^{-14}} > 0.86$$

$$\frac{1.6\times10^{-2}}{2.19\times10^{-12}} a^2 > 0.86$$

$$a > 1.08\times10^{-5} \text{ cm} = 0.108 \text{ μm}.$$

10. From Eq.33 we obtain

$$g_m = \frac{I_P}{2V_p^2}\sqrt{\frac{V_P}{V_G+V_{bi}}}V_D$$

$$= \frac{Z\mu_n\varepsilon_s}{aL}\sqrt{\frac{V_P}{V_{bi}}}V_D$$

$$V_P = \frac{qN_D a^2}{2\varepsilon_s} = \frac{1.6\times10^{-19}\times7\times10^{16}\times(3\times10^{-5})^2}{2\times12.4\times8.85\times10^{-14}} = 4.62 \text{ V}$$

$$\therefore g_m = \frac{5\times10^{-4}\times4500\times12.4\times8.85\times10^{-14}}{0.3\times10^{-4}\times1.5\times10^{-4}}\sqrt{\frac{4.62}{0.84}}\times1$$

$$= 1.28\times10^{-3} \text{ S} = 1.28 \text{ mS}.$$

11. (a) Now: $q\ V_{bi} + E_C = E_F + q\ \varphi_{Bn}$ ➔ $V_{bi} = (E_F - E_C)/q + \varphi_{Bn}$.
But: $(E_C - E_F)/q = (k_B\ T/q)\ \mathrm{Ln}[N_C / N_D] = 25.9 \text{ x } 10^{-3}\ \mathrm{Ln}[4.7 \text{ x } 10^{17} / 10^{17}] = 0.039$ V.
➔ $V_{bi} = 0.9 - 0.039 = 0.861$ V.

Hence: $W|_{(VG = 0)} = [(2\ \varepsilon_s\ V_{bi})/(q\ N_D)]^{0.5}$

➔ $W|_{(VG = 0)} = [(2 \times 12.9 \times 8.85 \times 10^{-14} \times 0.861)/(1.6 \times 10^{-19} \text{ x } 10^{17})]^{0.5}$

➔ $W|_{(VG = 0)} = 0.119 \text{ x } 10^{-4}$ cm < a.

So channel exists at $V_G = 0$, and MESFET is a normally ON device.

(b) $I_{D\,sat}$ [at (V_G= 0)] = $[(Z\ \varepsilon_s\ \mu_n\ \varepsilon_0)/(2aL)]\ (V_G - V_{th})^2|_{(Vg = 0)}$

➔ $I_{D\,sat}$ [at (V_G= 0)] = $[(Z\ \varepsilon_s\ \mu_n\ \varepsilon_0)/(2aL)]\ (V_{th})^2$

Now V_{th} = threshold voltage = $V_{bi} - (a^2\ q\ N_D)/(2\ \varepsilon_s)$

➔ $V_{th} = 0.861 - (0.2 \times 10^{-4})^2\ 1.6 \times 10^{-19} \times 10^{17} /(2 \times 8.85 \times 10^{-14} \times 12.9) = -1.942$ V.

Hence, $I_{D\,sat} = 10 \times 10^{-4} \times 5000 \times 8.85 \times 10^{-14} \times 12.9\ (-1.942)^2 /(2 \times 0.2 \times 10^{-4} \times 10^{-4})$.

So: $I_{D\,sat} = 5.38$ mA.

(c) Cut-off frequency: $f_T = g_m /(2\ \pi\ C_G) \sim [2\ \mu_n\ q\ N_D\ a^2\ ]/(\pi\ \varepsilon_s\ L^2)$.

Putting the values: $f_T = 2 \times 5000 \times 1.6 \times 10^{-19} \times 10^{17}\ (0.2 \times 10^{-4})^2/[\pi \times .85 \times 10^{-14} \times (10^{-4})^2]$.

➔ $f_T = 1.75 \times 10^{12}$ Hz.

12. From Eq.31b, the pinch-off voltage is

$$V_P = \frac{qN_D a^2}{2\varepsilon_s} = \frac{1.6\times10^{-19}\times10^{17}(2\times10^{-5})}{2\times12.4\times8.85\times10^{-14}} = 0.364 \text{ V}$$

The threshold voltage is

$$V_T = V_{bi} - V_p = 0.8 - 0.364 = 0.436 \text{ V}$$

and the saturation current is given by Eq. 39

$$\begin{aligned} I_{Dsat} &= \frac{Z\varepsilon_s\mu_n}{2aL}(V_G - V_T)^2 \\ &= \frac{50\times10^{-4}\times12.4\times8.85\times10^{-14}\times4500}{2\times(0.5\times10^{-4})\times(1\times10^{-4})}(0-0.436)^2 = 4.7\times10^{-4} \text{ A.} \end{aligned}$$

13. $$0.85 - 0.0259\ln\left(\frac{4.7\times10^{17}}{N_D}\right) - \frac{1.6\times10^{-19}\times N_D}{2\times12.4\times8.85\times10^{-14}}a^2 = 0$$

For $N_D = 4.7 \times 10^{16}$ cm^{-3}

$$a = \left(0.85 - 0.0259 \ln \frac{4.7\times10^{17}}{N_D}\right)^{1/2} \frac{(3.7\times10^3)}{\sqrt{N_D}}$$

$$= 1.52\times10^{-5}\text{ cm} = 0.152\ \mu\text{m}$$

For $= N_D = 4.7\times10^{17}\text{cm}^{-3}$

$$a = 0.496 = 10^{-5}\text{cm} = 0.0496\,\mu\text{m}$$

14. Built-in voltage $V_{bi} = (k_B T/q)\,\mathrm{Ln}[N_A/n_i]$

$\Rightarrow V_{bi} = 25.9\times10^{-3}\mathrm{Ln}[(8\times10^{17})(2\times10^{15})/(1.45\times10^{10})^2] = 0.7682\text{ V}.$

Total depletion width $W = [(2\,\varepsilon_s V \{ N_A + N_D \})/(q\,N_A N_D)]^{0.5}$

For pinch-off, one needs: $x_n = a$, where $x_n = W N_A/(N_A + N_D) = a$.

Using expression for W and equating: $W N_A/(N_A + N_D) = a$ leads to:

$V = q\,a^2 (N_A + N_D)\,N_D/(2\,N_A\,\varepsilon_s) = V_p$ (by definition.)

Hence: $V_p = [1.6\times10^{-19}\times(3\times10^{-4})^2\,]\,/\,[2.\times8.85\times10^{-14}\times11.9] = 13.7$ V.

<u>Comment</u>: Since $V_p = V_{Dsat} + V_G + V_{bi}$, here we have:

$$V_{Dsat} + V_G = 13.7 - 0.7682 = 12.93.$$

So as long as $(V_D + V_G) < 12.93$ V, the channel will remain open.

15. From Eq.48 the pinch-off voltage is

$$V_P = \phi_{Bn} - \frac{\Delta E_c}{q} - V_T$$

$$= 0.89 - 0.23 - (-0.5)$$

$$= 0.62\text{ V}$$

and then,

$$V_P = \frac{qN_D d_1^2}{2\varepsilon_s} = \frac{1.6\times10^{-19}\times3\times10^{18}\times d_1^2}{2\times12.3\times8.85\times10^{-14}} = 0.62\text{ V}$$

$$d_1 = 1.68\times10^{-6}\text{ cm}$$

$$d_1 = 16.8\text{ nm}$$

Therefore, this thickness of the doped AlGaAs layer is 16.8 nm.

16. $V_P = (q\,N_D\,d_1^2)/(2\varepsilon_s) = 1.6\times10^{-19}\times10^{18}\,(50\times10^{-7})^2/(2\times12.3\times8.85\times10^{-4}) = 1.836$ V.

The threshold voltage $V_T = \varphi_{Bn} - \Delta E_C/q - V_P = 0.85 - 0.23 - 1.836 = -1.216$ Volt.
Therefore, the device is a normally on MODFET.
The two-dimensional electron gas at the source for $V_G = 0$ is:
$n_s = [12.3\times8.85\times10^{-14}/(1.6*10^{-19}*(50 + 4 + 8)*10^{-7}) * [-1 - (-1.216)]$
$= 2.352\times10^{11}\text{ cm}^2.$

17. The pinch-off voltage is

$$V_P = \frac{qN_D d_1^2}{2\varepsilon_s} = \frac{1.6\times10^{-19}\times5\times10^{17}\times(50\times10^{-7})^2}{2\times12.3\times8.85\times10^{-14}} = 1.84\text{ V}$$

The barrier height is

$$\phi_{Bn} = V_T + \frac{\Delta E_c}{q} + V_P = -1.3 + 0.25 + 1.84 = 0.79\text{ V}$$

The 2DEG concentration is

$$n_s = \frac{12.3\times8.85\times10^{-14}}{1.6\times10^{-19}\times(50+10+8)}\times[0-(-1.3)] = 1.29\times10^{12}\text{ cm}^{-2}.$$

18. The pinch-off voltage is

$$\begin{aligned} V_P &= \frac{qN_D}{2\varepsilon_s}d_1^2 \\ &= \frac{1.6\times10^{-19}\times3\times10^{18}}{2\times12.3\times8.85\times10^{-14}}(35\times10^{-7})^2 = 2.7\text{ V} \end{aligned}$$

the threshold voltage is

$$\begin{aligned} V_T &= \phi_{Bn} - \frac{\Delta_{EC}}{q} - V_p \\ &= 0.89 - 0.24 - 2.7 \\ &= -2.05\text{ V} \end{aligned}$$

Therefore, the two-dimensional electron gas is

$$n_s = \frac{12.3\times8.85\times10^{-14}}{1.6\times10^{-19}\times(35+8)\times10^{-7}}\times[0-(-2.05)] = 3.2\times10^{12}\text{ cm}^{-2}.$$

19. The pinch-off voltage is

$$V_P = \frac{qN_D d_1^2}{2\varepsilon_s} = 1.5 = \frac{1.6\times10^{-19}\times1\times10^{18}\times d_1^2}{2\times12.3\times8.85\times10^{-4}}$$

The thickness of the doped AlGaAs is

$$d_1 = \sqrt{\frac{1.5\times2\times12.3\times8.85\times10^{-14}}{1.6\times10^{-19}\times10^{18}}} = 4.45\times10^{-8}\text{ cm} = 44.5\text{ nm}$$

$$V_T = \phi_{Bn} - \frac{\Delta E_C}{q} - V_P = 0.8 - 0.23 - 1.5 = -0.93\text{ V}.$$

CHAPTER 8

1. The differential negative resistance is: $|(V_v - V_p)/(I_V - I_p)|$

$= |(0.6–0.15)/(2\text{mA}–20\text{mA})| = 25$ Ohms.

2. $V_{bi} \cong (E_g/q) + V_n + V_p = 1.42 + 0.03 + 0.03 = 1.48$ V

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} + V)}$$

$$= \sqrt{\frac{2\times 1.16\times 10^{-12}}{1.6\times 10^{-19}}\left(\frac{10^{19} + 10^{19}}{10^{19}\times 10^{19}}\right)(1.48 - 0.25)}$$

$$= 1.89\times 10^{-6}\ \text{cm=18.9 nm}$$

$$C = \frac{\varepsilon_s}{W} = \frac{1.16\times 10^{-12}}{1.89\times 10^{-6}} = 6.13\times 10^{-7}\ \text{F/cm}^2.$$

3. The frequency f = $v_S/[2(W- x_A)]$. So minimum frequency ~ $v_S/(2W)$.

$$\Rightarrow 10^7/(2^*10^{-3}) = 5^*10^9 \text{ Hz.}$$

4. (a) $R_{SC} = \frac{1}{I}\int_0^W \Delta \mathrm{E}\, dx = \frac{1}{I}\int_0^W \frac{Ix}{A\varepsilon_S v_S} dx = \frac{W^2}{2A\varepsilon_S v_S}$

$$= \frac{(12\times 10^{-4})^2}{2(5\times 10^{-4})1.05\times 10^{-12}\times 10^7} = 137\ \Omega$$

(b) The breakdown voltage for $ND = 10^{15}$ cm^{-3} and $W = 12$ μm is 250 V (Refer to Chapter 4).

The voltage due to R_{SC} is

$$IR_{SC} = (10^3 \times 5\times 10^{-4})\times 137 = 68.5 \text{ V}$$

The total applied voltage is then 250 + 68.5 = 318.5 V.

5. (a) The dc input power is 100V(10^{-1}A) = 10W. For 25% efficiency, the power dissipated as heat is 10W(1–25%) = 7.5 W.

$$\Delta T = 7.5\text{W} \times (10°\text{C/W}) = 75°\text{C}$$

(b) ΔV_B = (60mV/ °C) x 75° C = 4.5 V

The breakdown voltage at room temperature is (100–4.5) = 95.5V .

6. (a) For a uniform breakdown in the avalanche region, the maximum electric field is $E_m = 4.4 \times 10^5$ V/cm. The total voltage at breakdown across the diode is

$$V_B = \mathsf{E}_m x_A + \left(\mathsf{E}_m - \frac{qQ}{\varepsilon_S} \right)(W - x_A)$$

$$= 4.4\times10^5 (0.4\times10^{-4}) + \left(4.4\times10^5 - \frac{1.6\times10^{-19}\times1.5\times10^{12}}{1.09\times10^{-12}} \right)(3-0.4)\times10^{-4}$$

$$= 17.6 + 57.2 = 74.8 \text{ V}$$

(b) The average field in the drift region is

$$\frac{57.2}{(3-0.4)\times10^{-4}} = 2.2\times10^5 \text{ V/cm}$$

This field is high enough to maintain velocity saturation in the drift region.

(c) $f = \dfrac{v_s}{2(W - x_A)} = \dfrac{10^7}{2(3-0.4)10^{-4}} = 19 \text{ GHz}.$

7. $f = v_{\text{sat}} /(2\text{L})$ - > L = $10^7/(2{*}20 \times 10^9) = 2.5 \times 10^{-4}$ cm.

So, avalanche region length x_A = 5% of L = $0.05{*}2.5 \times 10^{-4} = 1.25 \times 10^{-5}$ cm and length of drift region x_D = 95% of L = 2.375×10^{-4} cm.

From Poisson's equation, the electric field profile E(x) in avalanche region is:

$\text{E}(x) = \text{E}_{\text{max}} - q{*}\text{N}_{\text{high}}\, x/(\varepsilon_s)$ for avalanche region: $0 < x < x_A$, and

$\text{E}(x) = \text{E}_{\text{max}} - q{*}\text{N}_{\text{high}}\, x_A /(\varepsilon_s) - q{*}\text{N}_{\text{low}} (\text{x}-\text{x}_\text{A})/(\varepsilon_\text{s})$ for drift region: $x_A < x < \text{L}$.

Since E(x=L) goes to zero: $E_{max} - q{*}N_{high}\, x_A /(\varepsilon_s) - q{*}N_{low}\, (L-x_A)/(\varepsilon_s) = 0$.

Given that: $\text{E}_{\text{max}} = 450 \times 10^3$ V/cm, $n_{\text{low}} = 5 \times 10^{15}$/cc, $\varepsilon_s = 12.9{*}8.85 \times 10^{-14}$

One obtains: $N_{\text{low}} = 1.618 \times 10^{17}$

8. (a) In the p layer

$$\mathrm{E}_1(x) = \mathsf{E}_m - \frac{qN_1 x}{\varepsilon_s} \qquad 0 \le x \le b = 3\ \mu\text{m}$$

$$\mathrm{E}_2(x) = \mathsf{E}_m - \frac{qN_1 b}{\varepsilon_s} \qquad b \le x \le W = 12\ \mu\text{m}$$

$\mathrm{E}_2(x)$ should be larger than 10^5 V/cm for velocity saturation

$$\therefore \mathsf{E}_m - \frac{qN_1 b}{\varepsilon_s} \ge 10^5$$

or $\mathrm{E}_\mathrm{m} \ge 10^5 + \dfrac{qN_1 b}{\varepsilon_s} = 10^5 + 4.66\times10^{-11}\ \mathrm{N}_1.$

This equation coupled the plot of E_m versus N in Chapter 3 gives

$N1 = 7 \times 10^{15}$ cm^{-3} for $\mathrm{E}_\mathrm{m} = 4.2 \times 10^5$ V/cm

$$\therefore V_B = \frac{(\mathcal{E}_m - \mathcal{E}_2)b}{2} + \mathcal{E}_2 W = \frac{(4.2-1)\times 10^5 \times 3\times 10^{-4}}{2} + 10^5 \times 9 \times 10^{-4}$$

$$= 138 \text{ V}$$

(b) Transit time $t = \dfrac{W-b}{v_s} = \dfrac{(12-3)\times 10^{-4}}{10^7} = 9\times 10^{-15} = 90$ ps.

9. (a) For transit-time mode, we require $n_0 L \geq 10^{12}$ cm^{-2}.

$$n_0 \approx 10^{12} / L = 10^{12} / 1\times 10^{-4} = 10^{16} \text{ cm}^{-3}$$

(b) $t = L/v = 10^{-4} / 10^7 = 10^{-11}$ s = 10 ps

(c) The threshold field for InP is 10.5 kV/cm; the corresponding applied voltage is

$$V = \left(\frac{10.5\times 10^3}{2}\right)(1\times 10^{-4}) = 0.525 \text{ V}$$

The current is

$$I = JA = (q\mu_n n_0 \mathcal{E})A = (1.6\times 10^{-19} \times 4600 \times 10^{16} \times 5.25\times 10^3)\times 10^{-4} = 3.86 \text{ A}$$

The power dissipated in the device is then

$$P = IV = 2.02 \text{ W}.$$

10. Using: $n_0 L \geq 10^{12}$ one gets $L \geq 10^{12}/10^{15} = 10^{-3}$ cm.

Next, using: $f = 1/\tau = v_D/L \rightarrow f = 1.5 \times 10^7 / 10^{-3} = 1.5 \times 10^{10}$ Hz.

So time between pulses $T = 1/f = 6.667 \times 10^{-11}$ second.

11. Prior to reaching the cavity region, the domain moves at some velocity (say) v_{dom}. When this domain reaches the cavity region, carriers move around the cavity. This “detour” effectively increases the local electron density. This larger density causes an enhancement in the electric field within the domain as the result of the Poisson equation.

This increased field correspondingly increases the domain velocity. As a result, the domain moves quicker towards the anode. The net results are: (i) the current oscillations are still maintained, and (ii) the frequency of oscillation is slightly increased.

12. The energy E_n for infinitely deep quantum well is

$$E_n = \frac{h^2}{8m^* L^2} n^2$$

$$\left| \frac{\Delta E_n}{\Delta L} \right| = \left| \frac{h^2 n^2}{8m^*} (-2)(L^{-3}) \right| = \frac{2E_1}{L}$$

$$\Delta E_n = \frac{2E_1}{L} \Delta L$$

$$\therefore \Delta E_1 = 3\,\text{meV}, \quad \Delta E_2 = 11\,\text{meV}.$$

13. From Fig. 14 we find that the first excited energy is at 280 meV and the width is 0.8 meV. For same energy but a width of 8 meV, we use the same well thickness of 6.78 nm for GaAs, but the barrier thickness must be reduced to 1.25 nm for AlAs.

    The resonant-tunneling current is related to the integrated flux of electrons whose energy is in the range where the transmission coefficient is large. Therefore, the current is proportional to the width ΔEn, and sufficiently thin barriers are required to achieve a high current density.

CHAPTER 9

1. $P(x) = P_o(1 - R)\exp(-\alpha x)$

   $x = (-1/\alpha)\ln[P(x)/P_o(1 - R)] = (-1/4 \times 10^4)\ln[1/(2 \times 0.9)] = 0.15\ \mu m$.

2. Incident power = 15 mW. Of this 0.1 (= 10%) is reflected. So net power entering device = 0.9 * 15 mW = 13.5 mW.

   Of this, 55% is dissipated as heat.
   This then equals: 13.5 mW * 0.55 = 7.425 mW.
   So thermal energy dissipated per second to the lattice = 7.425 mJ per second.

   Power used for carrier generation = 13.5 mW * 0.45 = 6.075 mW.

   Now power through device P(x) decreases with distance "x" according to Beer-Lambert law. Thus: $P(x) = P(x=0)\exp(-\alpha x)$ where α is the absorption coefficient of $4\times10^4\ cm^{-1}$ (value as given) .

   Differential power lost per differential length dx is: $dP(x) = [dP(x)/dx]\ dx$

   Here: $dP(x)/dx = - P(x=0)\ \alpha \exp(-\alpha x)$.
   So: $dP(x) = - P(x=0)\ \alpha \exp(-\alpha x)\ dx$.

   So power used for carrier generation over this differential length dx = $|dP(x)|$ = magnitude of power lost.

   So total power used in generating carriers = $\int dP(x)\ dx$.
   Thus, total power used for carrier generation = $[{}_0\int^L P(x=0)\ \alpha \exp(-\alpha x)\ dx$
   $= \{P(x=0)\ [1-\exp(-\alpha L)]$.

   But as already determined (above), the power used for carrier generation = 13.5 mW * 0.45 = 6.075 mW.

   Thus: $6.075\ mW = 13.5\ mW\ [1- \exp(-4\times10^4 L)] \Rightarrow L = 1.4945 \times 10^{-5}\ cm$.

3. $h\nu\ (0.6\mu m) = 1.24/0.6 = 2.07\ eV$ (From Eq.9)

   $\alpha\ (0.6\ \mu m) = 3 \times 10^4\ cm^{-1}$

   The net incident power on the sample is the total incident power minus the reflected power, or 10 mW.

   $$10^{-2}(1 - e^{-3\times10^4 W}) = 5\times10^{-3}$$

   $$W = 0.231\ \mu m.$$

   The portion of each photon's energy that is converted to heat is

   $$\frac{h\nu - E_g}{h\nu} = \frac{2.07 - 1.42}{2.07} = 31.4\%$$

   The amount of thermal energy dissipated to the lattice per second is

   31.4% × 5 = 1.57 mW.

4. From Eqs. 11 and 12, the spectral half-width of the intensity is

   $2\Delta\lambda = (2/hc)\ \lambda^2 kT = (2/hc)(550 \times 10^{-9})2 \times k \times 293 = 18\ nm$.

5. The critical angle $\theta_c = \sin^{-1}(n_1/n_2) = \sin^{-1}(1/3.4) = 0.298532$ radians.

So: $\cos(\theta_c) = 0.95577$.

Hence electrical-to-optical conversion efficiency = 4*3.4* (l–0.95577)/(1+3.4)^2 = 0.031.

Electrical power = $1.8 * 80 \times 10^{-3} = 0.144$

Hence, optical power produced: 0.144*0.031 W = 4.464 mW.

Of this, 50% of the photons would be going towards the top region -> 2.232 mW.

So, the power reaching the reaching the surface = 2.232*exp[-αW].

Here $\alpha W = 5 \times 10^3 * 3 \times 10^{-4} = 1.5$.

So power reaching surface = 2.232*exp[−1.5] = 0.4977 mW.

6. $$f_T = \frac{1}{2\pi\tau}$$

$$\frac{1}{\tau} = \frac{10^{19}}{10^9} + \frac{1}{10^{-7}} = 10^{10} + 10^7 = 10^{10}$$

$$\tau = 10^{-10} S$$

$$f_T = \frac{1}{2\pi \times 10^{-10}} = 1.6 \text{ GHz}$$

7. From Eq. 13 $P(\omega) = \dfrac{p(0)}{\sqrt{1+(\omega\tau)^2}}$

$$\frac{P(\omega)}{p(0)} = \frac{1}{\sqrt{1+(\omega\tau)^2}} = \frac{1}{2}$$

$$4 = 1 + (\omega\tau)^2$$

$$3 = (2\pi f\tau)^2$$

$$f = \frac{\sqrt{3}}{2\pi\tau} = \frac{\sqrt{3}}{2\pi \times 10^{-10}} = 0.28 \times 10^{10} \text{ Hz}$$

8. Energy (in eV) corresponding to the 680 nm wavelength = hc/(qλ). This in MKS units is: $(6.626 \times 10^{-34} * 3 \times 10^8)/(1.6 \times 10^{-19} * 680 \times 10^{-9}) = 1.827$ eV.

Thus 1.42 + 1.2x = 1.827 -> x = 0.3391.

9. $80 \times 1.8 \times 0.716 = 103.1$ mW.

$0.5 \times 103.1 \times \exp(-5 \times 10^5 \times 3 \times 10^6) = 11.5$ mW

10. $11.5 \times 4\bar{n}_1\,\bar{n}_2 / (\bar{n}_1 + \bar{n}_2)^2 \times 4\bar{n}_2\,\bar{n}_3 / (\bar{n}_2 + \bar{n}_3)^2 = 11.5 \times 4 \times 3.38 \times 1.6/(1.6+3.38)^2 \times 4 \times 1.6 /(1+1.6)^2 = 9.5$ mW.

11. From Eq. 1

$$\lambda = \frac{c}{\nu} = \frac{hc}{h\nu} = \frac{1.24}{E_g(\text{eV})}\,\mu\text{m},$$

$$\frac{d\lambda}{dT} = \frac{hc}{E_g^2}\left(\frac{dE_g}{dT}\right) = -\frac{2.8}{10}\,\text{nm/°K} = -0.28\times10^{-3}\,\mu\text{m/°K}$$

$$\frac{dE_g}{dT} = \frac{E_g^2}{hc}\left(\frac{d\lambda}{dT}\right) = \frac{(1.42)^2}{1.24}(0.28\times10^{-3})\text{=}4.55\times10^{-4}\ \text{eVK}$$

12. (a) For GaAs, $\bar{n}_2 = 3.66$ at $\lambda = 0.8$ μm and for air, $\bar{n}_1 = 1.0$.

The critical angle is

$$\theta_c = \sin^{-1}\left(\frac{\bar{n}_1}{\bar{n}_2}\right) = \sin^{-1}\left(\frac{1}{3.66}\right) = 15.9°$$

The fraction of photons that will not experience total internal reflection is

$$\frac{2\times15.9}{360} = 0.0883 = 8.83\%$$

(b) Fresnel loss:

$$R = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right) = \left(\frac{3.66-1}{3.66+1}\right)^2 = 0.326$$

$$0.0883(1-0.326) = 0.0595 = 5.95\%$$

13. (a) From Eq. 30 $|\Delta\lambda| \cong \frac{\lambda^2}{2\bar{n}L}$.

$$\Delta\lambda \cong \frac{(1300\ \text{nm})}{2\times3.4\times300\times10^3\ \text{nm}} = 0.828\ \text{nm}$$

(b) $\nu = \frac{c}{\lambda}$

$$\left|\frac{\text{d}\nu}{\text{d}\lambda}\right| = \frac{c}{\lambda^2}$$

$$\Delta\nu = \frac{c}{\lambda^2}\Delta\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda = \frac{c}{\lambda^2}\frac{\lambda^2}{2\bar{n}L} = \frac{c}{2\bar{n}L}$$

$$\Delta\nu = \frac{c}{2\bar{n}L} = \frac{3\times10^{10}\ \text{cm/s}}{2\times3.4\times300\times10^{-4}\ \text{cm}} = 147\ \text{GHz}$$

14. From Eq. 22

$$R = \left(\frac{3.39-1}{3.39+1}\right)^2 = 0.296$$

(a) The mirror loss

$$\frac{1}{L}\ln\left(\frac{1}{R}\right) = \frac{1}{300\times10^{-4}}\ln\left(\frac{1}{0.296}\right) = 40.58\ \text{cm}^{-1}.$$

(b) The threshold current reduction is

$$\frac{J_{th}(R=0.296)-J_{th}(R1=0.296,R_2=0.9)}{J_{th}(R=0.296)}$$

$$\approx\frac{\left[\alpha+\frac{1}{L}\ln\left(\frac{1}{R}\right)-\left[\alpha+\frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)\right]\right]}{\alpha+\frac{1}{L}\ln\left(\frac{1}{R}\right)}$$

$$=\frac{40.58-\frac{1}{2\cdot 300\times10^{-4}}\ln\left(\frac{1}{0.296\cdot 0.90}\right)}{10+40.58}$$

$$=36.6\%.$$

15. The gain is given by:

$$\gamma(\upsilon)=K\sqrt{hv-E_{gap}}\cdot[f_c(E_2)-f_v(E_1)]\cdot$$

Hence for $E_{gap} < h\gamma < E_{Fn} - E_{Fp}$: $f_c(E_2) \sim 1$, $f_v(E_1) \sim 0$. So gain is positive.

For case $h\gamma < E_{gap}$: gain = 0. Finally, for $h\gamma > E_{Fn} - E_{Fp}$, one has: $f_c(E_2) \sim 0$, $f_v(E_1) \sim 1$. So gain < 0, i.e., a net loss.

16. From Eq. 23

$$\sin\theta_c=\frac{\overline{n}_1}{\overline{n}_2}\Rightarrow\overline{n}_1=\overline{n}_2\cdot\sin\theta_c$$

From Eq. 26

$$\Gamma\cong1-\exp(-C\Delta\overline{n}d)=1-\exp(-8\times10^5\cdot3.6(1-\sin\theta_c)\cdot1\times10^{-4})$$

For $\theta c = 84°$ $n_1 = 3.58$ $\Gamma_1 = 0.794$

$\theta_c = 78°$ $n_2 = 3.52$ $\Gamma_2 = 0.998$.

17. From Eq. 28 we have

$$m\lambda=2\overline{n}\ L.$$

Differentiating the above equation with respect to λ, we obtain

$$\lambda\frac{dm}{d\lambda}+m=2L\frac{d\overline{n}}{d\lambda}.$$

Substituting $2\overline{n}L/\lambda$ for m and letting $dm/d\lambda = -\Delta m/\Delta\lambda$, yield

$$\lambda\left(\frac{-\Delta m}{\Delta\lambda}\right)+\frac{2\overline{n}L}{\lambda}=2L\frac{d\overline{n}}{d\lambda}$$

$$\therefore\Delta\lambda=\frac{\lambda^2\Delta m}{2\overline{n}L\left[1-\left(\frac{\lambda}{\overline{n}}\right)\left(\frac{d\overline{n}}{d\lambda}\right)\right]}$$

and

$$\Delta\lambda=\frac{(0.89)^2\times1}{2(3.58)300\left[1-\left(\frac{0.89}{3.58}\right)(2.5)\right]}=9.7\times10^{-4}\ \mu\text{m}=0.97\text{ nm}.$$

18. Using $\Delta\lambda = \lambda^2/(2nL)$, here n = 3.6, and L = 75 μm, and $\lambda = (hc)/(q*E_{gap})$ $= (6.626 \times 10^{-34} * 3 \times 10^8)/(1.6 \times 10^{-19} * 1.42) = 8.74912 \times 10^{-7}$ meter $= 8.74912 \times 10^{-5}$ cm. So: $\Delta\lambda = (8.74912 \times 10^{-5})^2/(2*3.6*75 \times 10^{-4}) = 1.41754 \times 10^{-7}$ cm = 0.14175 A.

19. Using $J_{th} = (g_0\Gamma/\beta)+(1/\beta)[\alpha + \{1/(2L\}\mathrm{Ln}(\{1/(R_1R_2)\}]$, one gets:

2000 = (100*0.9/0.1) + (1/0.1)[100 + {1/(2L)} Ln{1/(0.5*0.9)}].

Solving for L:

L = [Ln{1/0.45}]/20 ~ 0.4 mm.

The gain = $\{1/(2L\}\mathrm{Ln}(\{1/(R_1R_2)\}) = (1/0.08)\ \mathrm{Ln}(1/.45) = 9.981$.

20. From the equation, we have for m = 0:

$$\lambda_B^2 \pm 4\bar{n}L\lambda_B \mp 4\bar{n}L\lambda_o = 0 \qquad (25a)$$

which can be solved as

$$\lambda_B = \pm\left(\frac{-4\bar{n}L \pm \sqrt{16\bar{n}^2L^2 + 16\bar{n}L\lambda_o}}{2}\right) \qquad (25b)$$

There are several variations of ± in this solutions. Take the solution which is the only practical one, i.e., $\lambda_B \approx \lambda_o$, gives $\lambda_B = 1.3296$ or 1.3304 μm.

$$\Lambda \cong \frac{1.33}{2\times 3.4} = 0.196\,\mu\text{m}.$$

21. The threshold current in Fig. 38b is given by

$$I_{th} = I_0 \exp(T/110).$$

Therefore

$$\xi \equiv \frac{1}{I_{th}}\frac{dI_{th}}{dT} = \frac{1}{110} = 0.0091(°\text{C})^{-1}.$$

If $T_0 = 50$ °C, the temperature coefficient becomes

$$\xi = \frac{1}{50} = 0.02\ (°\text{C})^{-1}.$$

which is larger than that for $T_0 = 110$ °C. Therefore the laser with $T_0 = 50$ °C is worse for high-temperature operation.

CHAPTER 10

1. $\lambda = 0.8\ \mu m$

R(ideal) =(100%) × 0.8 × 10^{-4}/1.24=0.645 A/W

The photon energy for 0.8 μm is 1.24/0.8 = 1.55 eV

(l) GaAs bandgap is 1.42 eV, since photon energy is larger than the bandgap, we have $\eta = 0.645$ A/W.

(2) For A10.34Ga0.66As the bandgap is 1.424 + 1.247 × 0.34 = 1.85 eV (Eq. 58 in Ch. 12). The photons with 1.55 eV energy have insufficient energy to generate electrons and holes $\therefore\ \eta = 0$

(3) For the heterojunction cell, photons will be absorbed in the 1.42 eV region, but will not be absorbed in the 1.85 eV regions $\therefore\ \eta = 0.645$ A/W

(4) For the tandem cell, photons wil1 not be absorbed in the upper 1.85 eV cell, but will be absorbed in the lower 1.42 eV cell. However, since the cells are in series, the top cell wil1 block the current generated by the bottom cell once the current is larger than the top cell's dark saturation current va1ue.

The bottom cell wil1 be forced to open-circuit, dissipating the generated current $\therefore \eta = 0$

2. Incident optical power $P_{opt} = \pi R^2 = \pi(0.04)^2 * 0.3 \times 10^{-3} = 1.5079 \times 10^{-6}$ W.

Responsivity $R = I_{ph} / P_{opt} = 5 \times 10^{-7}/1.5079\text{x}10^{-6} = 0.33157$ A/W.

Quantum efficiency = $R[hc/(q\lambda)] = 0.33157*(6.623 \times 10^{-34} * 3 \times 10^{8})/(1.6 \times 10^{-19} * 800 \times 10^{-9})$

= 0.514689 ~ 51.4%.

3. (a) $\Delta I = q\ (\mu n + \mu p)\ \Delta nEA$

$$\therefore \Delta n = electron - hole\ pairs = \frac{\Delta I}{q(\mu_n + \mu_p)\mathrm{E}A}$$

$$= \frac{2.83\times10^{-3}}{1.6\times10^{-19}(3600+1700)(10/0.6)(2\times1\times10^{-2})} = 10^{13}\mathrm{cm}^{-3}$$

(b) $\tau = \dfrac{2.83\times10^{-3}}{23.6} = 120\ \mu s$

(c) $\Delta n(t) = \Delta n \exp(-t/\tau) = 10^{13} \exp\left(-\dfrac{10^{-3}}{1.2\times10^{-4}}\right) = 2.5\times10^{9}\ \mathrm{cm}^{-3}$.

4. From Eq. 6

$$Ip = q\left(0.85\cdot\frac{10^{-6}}{3\times q}\right)\cdot\left(\frac{3000\cdot 6\times10^{-10}\cdot 5000}{10\times10^{-4}}\right)$$

$$= 2.55\times10^{-6} = 2.55\ \mu\text{A}$$

and from Eq. 8

$$\text{Gain} = \frac{\mu_n\tau\varepsilon}{L} = \frac{3000\cdot 6\times10^{-10}\cdot 5000}{10\times10^{-4}} = 9.$$

5. $\mathcal{R} = \eta\lambda/1.24 = 0.168 \times 1.1/1.24 = 0.149$ A/W.

$I_{opt} = 0.149 \times 5 \times 10^{-6} = 0.745\ \mu$A.

6. $\eta_{ext} = 0.9 \times 0.8 \times [\exp(-10^6 \times 10^{-6}) - \exp(-10^6 \times 2 \times 10^{-6})] = 0.168$.

7. From Eq. 9

$$\eta = \left(\frac{I_P}{q}\right)\cdot\left(\frac{P_{opt}}{h\nu}\right)^{-1} = \left(\frac{I_P}{P_{opt}}\right)\cdot\left(\frac{h\nu}{q}\right) = R\cdot\left(\frac{h\nu}{q}\right)$$

The wavelength λ of light is related to its frequency ν by $\nu = c/\lambda$, where c is the velocity of light in vacuum. Therefore $h\nu/q = hc/\lambda q$ and $h = 6.625 \times 10^{-34}$ J-s, $c = 3.0 \times 10^{10}$ cm/s, q = 1.6 x 10–19 coul, 1 eV = 1.6×10^{-19} J.

Therefore, $h\nu/q = 1.24/\lambda$ (μm)

Thus, $\eta = (R \times 1.24)/\lambda$ and $R = (\eta\lambda)/1.24$.

8. Thermal equilibrium current = $q(n\,\mu_n + p\,\mu_p)$ (V/L) A ~ $qn\,\mu_n$ (V/L) A

→ $1.6 \times 10^{-19} * 5 \times 10^{15}$ 1200 $[3/(120 \times 10^{-4})]$ $5 \times 10^{-4} = 0.12$ A.

Steady-state excess carrier density $\Delta n \sim G_L\,\tau_{n0} = 10^{21} * 5 \times 10^{-7}\ \text{cm}^{-3} = 5 \times 10^{14}\ \text{cm}^{-3}$

Photocurrent $I_{ph} = q\,A\,[\Delta n\,\mu_n + \Delta p\,\mu_p]\,(V/L) = q\,A\,\Delta n\,[\mu_n + \mu_p]\,(V/L)$

$= 1.6 \times 10^{-19} * 5 \times 10^{14}\,[1200 + 400]\,[3/(120 \times 10^{-4})]\,5 \times 10^{-4} = 0.016$ A.

Photocurrent gain = 0.012/0.016 = 7.5

9. From Fig. 5, Ch. 9, the absorption coefficient at 900 nm is ~3 × 102 cm^{-1}, and the absorption depth is ~33 μm. We can assume that absorption and hence photogeneration occurs over the entire i layer. The field in the i-Si layer is

$$\mathcal{E} = (100\text{V})/(20 \times 10^{-4}\ \text{cm}) = 5 \times 104\ \text{V cm}^{-1}$$

At this field the electron drift velocity is very near its saturation at 10^7 cm/s, whereas the hole drift velocity is about 7×10^6 cm/s as shown in Fig. 22, Ch. 2. The transit time t_r of holes across the *i*-Si layer is

$$tr = (20 \times 10^{-4} \text{ cm})/(7 \times 10^6 \text{ cm/s}) = 2.86 \times 10^{-10} \text{ s or } 0.3 \text{ ns}.$$

10. The optimum occurs when the depletion layer is chosen so that the transit time is of the order of one-half the modulation period

$$\frac{\tau}{2} = t = \frac{w}{\upsilon}$$

If $\upsilon_s = 10^7$ cm/s

$$W = \upsilon\left(\frac{\tau}{2}\right) = \upsilon\frac{1}{2f} = \frac{10^7}{2\times10\times10^9} = 5\times10^{-4} = 5 \text{ µm}.$$

11. Energy at earth's mean distance/unit area/unit time = $(r_s/d_{es})^2$ energy at sun's surface/unit area/unit time = $(r_s/d_{es})^2\sigma T^4$

Absorbing energy of the Earth = radiating energy of the Earth

$(r_s/d_{es})^2\sigma Ts^4\pi r_e^2 = \sigma Te^4 4\pi r_e^2$

$T_e = T_s(r_s/d_{es})^{1/2}/2^{1/2} = 289.38$ K

(b) The absorbing energy of the Earth decreases to 0.7, and the radiating energy of the Earth decreases to 0.6. The earth is hotter.

$0.7(r_s/d_{es})^2\sigma Ts^4\pi r_e^2 = 0.6\sigma Te'^4 4\pi r_e^2$

$T_e' = Te(0.7/0.6)^{1/4} = 300.75$ K

(c) The increase of 2 °C is roughly from a doubling of atmospheric CO_2.

$302.75 = T_e(0.7/\varepsilon')^{1/4}$

$\varepsilon' = 0.584$

12. (a) For a photodiode, only a narrow wavelength range centered at the optical signal wavelength is important; whereas for a solar cell, high spectral response over a broad solar wavelength range are required.

(b) Photodiode are small to minimize junction capacitance, while solar cells are large-area device.

(c) An important figure of merit for photodiodes is the quantum efficiency (number of electron-hole pairs generated by incident photon), whereas the main concern for solar cells is the power conversion efficiency (power delivered to the load per incident solar energy).

13. $L_n = [D_n \tau_{n0}]^{½} = [25*5 \times 10^{-7}]^{½} = 35.4$ µm ; $L_p = [D_p \tau_{p0}]^{½} = [10*10^{-7}]^{½} = 10$ µm.

$J_s = [qD_n n_{p0}/L_n + qD_n n_{p0}/L_n]$

$= 1.6 \times 10^{-19}*(1.5 \times 10^{10})^2 [25/(35.4 \times 10^{-4} * 5 \times 10^{18}) + 10/(10 \times 10^{-4} * 10^{16})]$

$= 3.6 \times 10^{-11}$ A/cm^2

Hence: $V_{oc} = V_t \text{Ln}[1 + J_L/J_s] = 25.9 \times 10^{-3} \text{Ln}[1 + 15 \times 10^{-3} /3.6 \times 10^{-11}] = 0.514$ V.

14 P_{max}=(0.8)(90)(0.75)=54 mW

15. The maximum power is generated for: $dP/dV_a = 0$. This is:

$I_s (e^{Vm/Vt} - 1) - I_{ph} +(V_m/V_t) I_s e^{Vm/Vt} = 0$, where V_m is the voltage corresponding to the maximum power point. This voltage can be obtained by solving the following transcendental equation:

$V_m = V_t \text{Ln}[\{1 + I_{ph}/I_s\}/\{1+ V_m/V_t\}]$.

Using iteration and a starting value of $V_m = 0.5$ V, one obtains the following successive values: $V_m = 0.5, 0.542, 0.54$.

The value $I_m \sim I_s (V_m/V_t) e^{Vm/Vt} = 10^{-12} (0.54/0.0259) \exp(0.54/0.0259) = 0.02365$

The efficiency η equals: $\eta = | V_m I_m/ P_{in}| = 0.54 * 0.02365/0.1 = 13\%$.

Hence fill factor = $(V_m I_m)/(V_{oc} I_{sc}) = 83\%$.

16. From Fig. 15, we draw a load line with a slope 1/5Ω to intersect the I-V curve at about I = 80mA and V = 0.4V. The power delivered to the load is

$P_{load} = 0.08 \times 0.4 = 0.032$ W

The input power is

$P_{in} = 600 \times 4 \times 10^{-4} = 0.24$ W

The efficiency is

$\eta = 0.032/0.24 = 0.13 = 13\%$.

17. From Fig. 40

The output powers for $R_s = 0$ and $R_s = 5\ \Omega$ can be obtained from the area

$P_1 (R_s = 0) = 95 \text{ mA} \times 0.375\text{V} = 35.6$ mW,

$P_2 (R_s = 5\ \Omega) = 50 \text{ mA} \times 0.18\text{V} = 9.0$ mW

$\therefore$ For $R_s = 0$ $P_1/ P_1 = 100\%$

For $R_s = 5\ \Omega$ $P_2/ P_1 = 9/35.6 = 25.3\%$.

18. From Eq. 8, Ch. 9,

For amorphous Si, $W = \frac{-1}{\alpha}\ln\frac{\Phi(W)}{\Phi_0} = \frac{-1}{\alpha}\ln(0.1) = 2.3\times10^{-5}$ cm=0.23 μm.

For CIGS, the thickness is 2.3 μm.

19. From Eq. 38 and 39

$$I = I_s(e^{qV/kT} - 1) - I_L$$

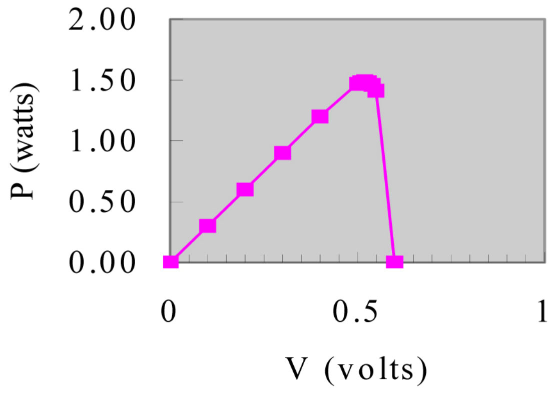
and

$$V_{oc} = \frac{kT}{q}\ln\left(\frac{I_L}{I_s}+1\right) \cong \frac{kT}{q}\ln\left(\frac{I_L}{I_s}\right)$$

$I_s = I_L e^{-qV/kT} = 3 \cdot e^{-0.6/0.02585} = 2.493 \times 10^{-10}$ A

$I = 3 - 2.493 \times 10^{-10} \cdot e^{V/0.02585}$ and $P = I\,V$

V	I	P
0	3.00	0.00
0.1	3.00	0.30
0.2	3.00	0.60
0.3	3.00	0.90
0.4	3.00	1.20
0.5	2.94	1.47
0.51	2.91	1.48
0.52	2.86	1.49
0.53	2.80	1.48
0.54	2.71	1.46
0.55	2.57	1.41
0.6	0.00	0.00



∴Maximum power output = 1.49 W

Fill Factor

$$FF = \frac{I_m V_m}{I_L V_{oc}} = \frac{P_m}{I_L V_{oc}} = \frac{1.49}{0.6\times3}$$

20. The efficiencies are 14.2% (1 sun), 16.2% (10-sun), 17.8% (100-sun), and 18.5% (1000-sun).

Solar cells needed under 1-sun condition

$$= \frac{\eta(\text{concentration}) \times P_{in}(\text{concentration})}{\eta(1-sun) \times P_{in}(1-sun)}$$

$$= \frac{16.2\% \times 10}{14.2\% \times 1} = 11.4 \quad \text{cells} \quad \text{for } 10-\text{sun}$$

$$= \frac{17.8\% \times 10}{14.2\% \times 1} = 125 \quad \text{cells} \quad \text{for } 100-\text{sun}$$

$$= \frac{18.5\% \times 10}{14.2\% \times 1} = 1300 \quad \text{cells} \quad \text{for } 100-\text{sun.}$$

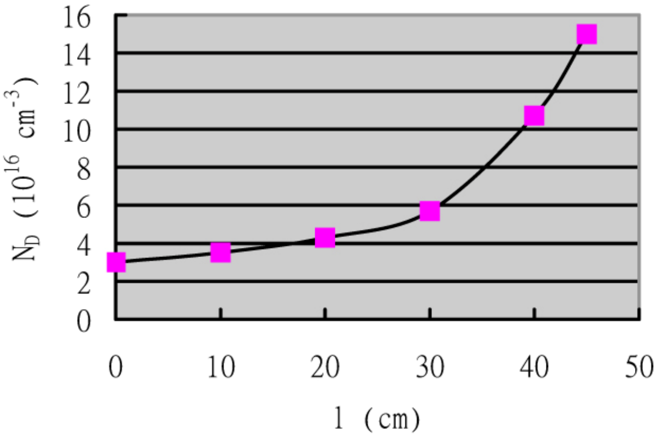
CHAPTER 11

1. $C_0 = 10^{17}\,\text{cm}^{-3}$

$k_0(\text{As in Si}) = 0.3$

$$C_S = k_0 C_0 (1 - M/M_0)^{k0-1}$$
$$= 0.3\times10^{17}(1-x)^{-0.7} = 3\times10^{16}/(1-l/50)^{0.7}$$

X	0	0.2	0.4	0.6	0.8	0.9
l(cm)	0	10	20	30	40	45
C_S(cm-3)	3×10^{16}	3.5×10^{16}	4.28×10^{16}	5.68×10^{16}	1.07×10^{17}	1.5×10^{17}



2. Volume of the melt $= 10\times1000/2.53\ \text{cm}^3 = 3.95257\times10^3\,\text{cm}^3$.

Number of boron atoms = Weight of boron * Avogadro number/(Atomic Weight)

➔ $(10^{-3}\,\text{grams} * 6.02\times10^{23}\,\text{atoms/mol})/(10.8\ \text{gram/mol}) = 5.574\times10^{19}\,\text{atoms}$.

So initial concentration of boron $= 5.574\times10^{19}\,\text{atoms}/3.95257\times10^3\,\text{cm}^3$

Thus, initial concentrationof boron $= 1.41\times10^{16}$ atoms/cc.

$C_s = C_o k[1 - M/M_o]^{k-1}$. So in this case: $C_s = 1.41\times10^{16*}0.8[1-0.5]^{0.8-1}$

➔ 1.2957×10^{16}/cc.

3. $k_0 = 0.8$ for boron in silicon

$M/M_0 = 0.5$

The density of Si is $2.33\ \text{g/cm}^3$.

The acceptor concentration for $\rho = 0.01\ \Omega-\text{cm}$ is $9\times10^{18}\,\text{cm}^{-3}$.

The doping concentration CS is given by

$$C_s = k_0 C_0 \left(1 - \frac{M}{M_0}\right)^{k_0 - 1}$$

Therefore

$$C_0 = \frac{C_s}{k_0\left(1 - \frac{M}{M_0}\right)^{k_0-1}} = \frac{9\times10^{18}}{0.8(1-0.5)^{-0.2}}$$

$$= 9.8\times10^{18}\ \mathrm{cm}^{-3}$$

The amount of boron required for a 10 kg charge is

$$\frac{10{,}000}{2.338}\times9.8\times10^{18} = 4.2\times10^{22}\ \text{boron atoms}$$

So that

$$10.8\ \text{g/mole}\times\frac{4.2\times10^{22}\ \text{atoms}}{6.02\times10^{23}\,\text{atoms/mole}} = 0.75\ \text{g boron.}$$

4. (a) The molecular weight of boron is 10.81.

The boron concentration can be given as

$$n_b = \frac{\text{number of boron atoms}}{\text{volume of silicon wafer}}$$

$$= \frac{5.41\times10^{-3}\text{g}/10.81\text{g}\times6.02\times10^{23}}{10.0^2\times3.14\times0.1}$$

$$= 9.78\times10^{18}\ \text{atoms/cm}^3$$

(b) The average occupied volume of everyone boron atoms in the wafer is

$$V = \frac{1}{n_b} = \frac{1}{9.78\times10^{18}}\text{cm}^3$$

We assume the volume is a sphere, so the radius of the sphere (r) is the average distance between two boron atoms. Then

$$r = \sqrt{\frac{3V}{4\pi}} = 2.9\times10^{-7}\text{cm.}$$

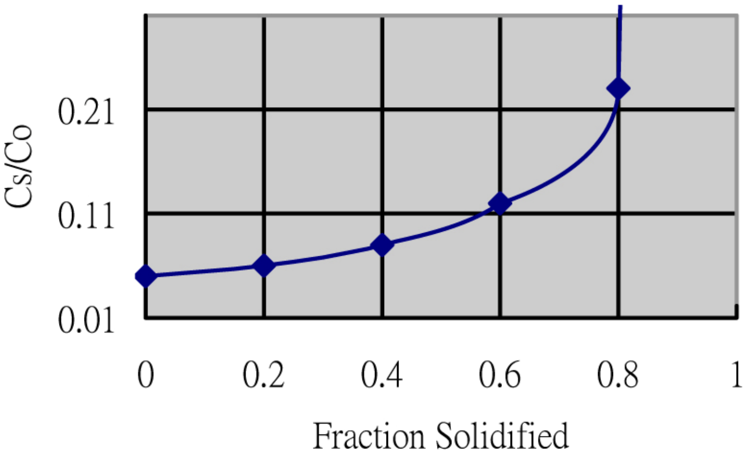
5. The segregation coefficient (k) for boron is 0.8, while that for phosphorous is 0.35. Since k for boron is close to 1, the concentration of boron atoms in the silicon ingot grown will tend to be (roughly) uniform as about $0.8*10^{17}\sim 8\times10^{16}\text{atoms/cc}$.

On the other hand, the k-value for phosphorous is much smaller. So the initial concentration of phosphorous in the silicon ingot will be $\sim 0.35^{*}8\times10^{16}\sim 2.8\times10^{16}\text{atoms/cc}$. So the initial portion of the wafer will have more

boron and will be p-type. Later, the concentration of phosphorous will tend to increase at a more rapid rate. At some fraction $f = M/M_o$, it would be greater than that of boron. In this case, the wafer will become n-type. Hence, a pn junction can form.
The fraction solidified after which the wafer begins to have n-type characteristic is given by the relation:

$$8\times10^{16*}0.35^{*}[1-f]^{0.35-1} > 10^{17*}0.8^{*}[1-f]^{0.8-1}.$$

6. We have



$$C_s / C_0 = k_0 \left(1 - \frac{M}{M_0}\right)^{k_0 - 1}$$

Fractional solidified	0	0.2	0.4	0.6	0.8	1.0
C_s/C_0	0.05	0.06	0.08	0.12	0.23	∞

7. The cross-sectional area of the seed is

$$\pi\left(\frac{0.55}{2}\right)^2 = 0.24 \text{ cm}^2$$

The maximum weight that can be supported by the seed equals the product of the critical yield strength and the seed's cross-sectional area:

$(2\times10^6)\times0.24 = 4.8\times10^5 \text{ g} = 480 \text{ kg}$

The corresponding weight of a 200-mm-diameter ingot with length l is

$$(2.33\text{g/cm}^3)\pi\left(\frac{20.0}{2}\right)^2 l = 480000 \text{ g}$$

$$\therefore l = 656 \text{ cm} = 6.56 \text{ m}.$$

8. The segregation coefficient of boron in silicon is 0.72. It is smaller than unity, so the solubility of B in Si under solid phase is smaller than that of the melt. Therefore, the excess B atoms will be thrown-off into the melt, then the concentration of B in the melt will be increased. The tail-end of the crystal is the last to solidify. Therefore, the concentration of B in the tail-end of grown crystal will be higher than that of seed-end.

9. The reason is that the solubility in the melt is proportional to the temperature, and the temperature is higher in the center part than at the perimeter. Therefore, the solubility is higher in the center part, causing a higher impurity concentration there.

10. The segregation coefficient of Ga in Si is 8×10^{-3}

From Eq. 18

$$C_s/C_0 = 1-(1-k)e^{-kx/L}$$

We have

$$\begin{aligned} x &= \frac{L}{k}\ln\left(\frac{1-k}{1-C_s/C_0}\right) \\ &= \frac{2}{8\times10^{-3}}\ln\left(\frac{1-8\times10^{-3}}{1-5\times10^{15}/5\times10^{16}}\right) \\ &= 250\ln(1.102) \\ &= 24 \text{ cm.} \end{aligned}$$

11. We have from Eq.18

$C_s = C_0[1-(1-k_e)\exp(-k_e x/L)]$

So the ratio $C_s/C_0 = [1-(1-k_e)\exp(-k_e x/L)]$

$$\begin{aligned} &= 1-(1-0.3)\cdot\exp(-0.3\times1) = 0.52 \text{ at } x/L = 1 \\ &= 0.38 \text{ at } x/L = 2. \end{aligned}$$

12. Using $C_s = C_o[1-(1-k_e)e^{-ke-x/L}]$, one gets in this case:

$5\times10^{15} = 2\times10^{16}[1-(1-k_e)e^{-ke2}]$. Solving for k_e leads to: $(1-k_e)e^{-ke2} = 0.75$

$k_e \sim 0.093$.

13. We have

$$\frac{M_s}{M_l} = \frac{\text{weight of GaAs at T}_\text{b}}{\text{weight of liquid at T}_\text{b}} = \frac{C_m - C_l}{C_s - C_m} = \frac{s}{l}$$

Therefore, the fraction of liquid remained f can be obtained as following

$$f = \frac{m_l}{m_s + m_l} = \frac{l}{s+l} \approx \frac{30}{16+30} = 0.65.$$

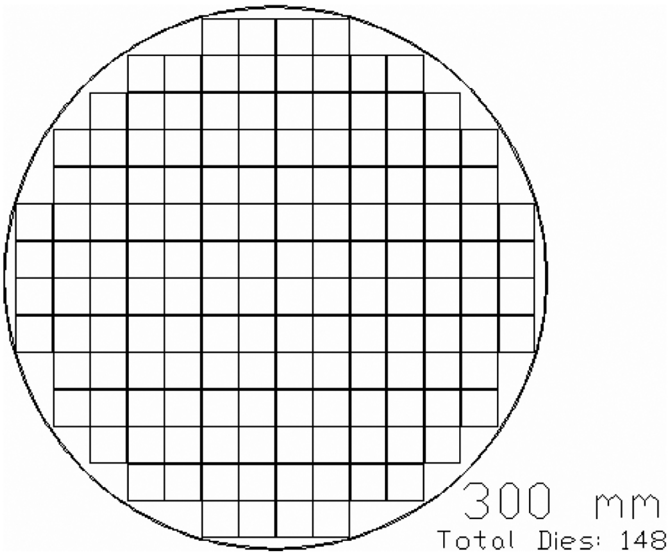
14. From the Fig.11, we find the vapor pressure of As is much higher than that of the Ga. Therefore, the As content will be lost when the temperature is increased. Thus the composition of liquid GaAs always becomes gallium rich.

15. $n_s = N\exp(-E_s/kT) = 5\times10^{22}\exp(2.3\,\text{eV/kT}) = 5\times10^{22}\exp\left[\frac{-88.8}{(T/300)}\right]$

$$\begin{aligned} &= 1.23\times10^{-16}\ \text{cm}^{-3} \approx 0 && \text{at } 27°\text{C} = 300\ \text{K} \\ &= 6.7\times10^{12}\ \text{cm}^{-3} && \text{at } 900°\text{C} = 1173\ \text{K} \\ &= 6.7\times10^{14}\ \text{cm}^{-3} && \text{at } 1200°\text{C} = 1473\ \text{K}. \end{aligned}$$

16. $37\times4 = 148$ chips

In terms of litho-stepper considerations, there are 500 μm space tolerance between the mask boundary of two dice. We divide the wafer into four symmetrical parts for convenient dicing, and discard the perimeter parts of the wafer. Usually the quality of the perimeter parts is the worst due to the edge effects.



17. $n_f = \sqrt{NN}\exp(-E_f/2kT)$

$$\begin{aligned} &= \sqrt{5\times10^{22}\times1\times10^{27}e^{-3.8eV/kT}}\times e^{-1.1eV/2kT} = 7.07\times10^{24}\times e^{-94.7/(T/300)} \\ &= 5.27\times10^{-17} \text{ at } 27°\text{C} = 300\ \text{K} \\ &= 2.14\times10^{14} \text{ at } 900°\text{C} = 1173\text{K}. \end{aligned}$$

18. $$v_{av} = \frac{\int_0^\infty v f_v dv}{\int_0^\infty f_v dv} = \sqrt{\frac{8kT}{\pi M}}$$

Where $f_v = \frac{4}{\sqrt{\pi}}\left(\frac{M}{2kT}\right)^{3/2} v^2 \exp\left(-\frac{Mv^2}{2kT}\right)$

M: Molecular mass

k: Boltzmann constant $= 1.38\times10^{-23}$ J/k

T: The absolute temperature

v: Speed of molecular

So that

$$v_{av} = \frac{2}{\sqrt{\pi}}\sqrt{\frac{2\times1.38\times10^{-23}\times300}{29\times1.67\times10^{-27}}} = 468 \text{ m/sec} = 4.68\times10^4 \text{cm/sec.}$$

19. $$\lambda = \frac{0.66}{P(\text{in Pa})}\text{cm}$$

$$\therefore P = \frac{0.66}{\lambda} = \frac{0.66}{150} = 4.4\times10^{-3} \text{ Pa.}$$

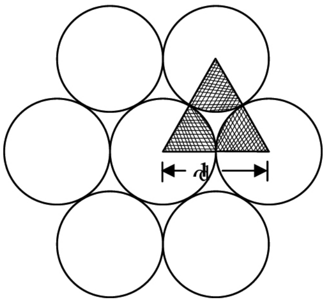
20. Using the formula: $t = N_s[MT]^{1/2}/(2.64\times10^{20}\text{P})$ with $T = 300$, $P = 10^{-4}, M = 28.0855$, yields: $t = 4.0784$. However, in this case, the process took 5 hours. This implies that not all silicon molecules "stuck" to the surface. The sticking fraction was: $4.0784/5 = 81.568\%$.

21. For close-packing arrange, there are 3 pie shaped sections in the equilateral triangle. Each section corresponds to 1/6 of an atom. Therefore

$$N_s = \frac{\text{number of atoms contained in the triangle}}{\text{area of the triangle}} = \frac{3\times\frac{1}{6}}{\frac{1}{2}d\times\frac{\sqrt{3}}{2}d}$$

$$= \frac{2}{\sqrt{3}d^2} = \frac{2}{\sqrt{3}(4.68\times10^{-8})^2}$$

$$= 5.27\times10^{14} \text{ atoms/cm}^2.$$



22. (a) The pressure at $970°\text{C}(=1243\text{K})$ is 2.9×10^{-1} Pa for Ga and 13 Pa for As_2. The arrival rate is given by the product of the imprringement rate and $\text{A}/\pi\text{L}^2$:

$$\text{Arrival rate} = 2.64\times10^{20}\left(\frac{P}{\sqrt{MT}}\right)\left(\frac{A}{\pi L^2}\right)$$

$$= 2.64\times10^{20}\left(\frac{2.9\times10^{-1}}{\sqrt{69.72\times1243}}\right)\left(\frac{5}{\pi\times12^2}\right)$$

$$= 2.9\times10^{15}\ \text{Ga molecules/cm}^2 - \text{s}$$

The growth rate is determined by the Ga arrival rate and is given by $(2.9\times10^{15})\times2.8/(6\times10^{14}) = 13.5\ \text{Å/s} = 810\ \text{Å/min}$.

(b) The pressure at $700°\text{C}$ for tin is 2.66×10^{-6} Pa. The molecular weight is 118.69. Therefore the arrival rate is

$$2.64\times10^{20}\left(\frac{2.66\times10^{-6}}{\sqrt{118.69\times973}}\right)\left(\frac{5}{\pi\times12^2}\right) = 2.28\times10^{10}\,\text{molecular/cm}^2\cdot\text{s}$$

If Sn atoms are fully incorporated and active in the Ga sublattice of GaAs, we have an electron concentration of

$$\left(\frac{2.28\times10^{10}}{2.9\times10^{15}}\right)\left(\frac{4.42\times10^{22}}{2}\right) = 1.74\times10^{17}\ \text{cm}^{-3}.$$

23. The x value is about 0.25, which is obtained from Fig. 26.

24. The lattice constants for InAs, GaAs, Si and Ge are 6.05, 5.65, 5.43, and 5.65 Å, respectively (Appendix F). Therefore, the f value for InAs-GaAs system is

$f = (5.65 - 6.05)/6.05 = -0.066$

And for Ge-Si system is

$f = (5.43 - 5.65)/5.65 = -0.39.$

CHAPTER 12

1. From Eq. 11 (with $\tau = 0$)

$$x^2 + Ax = Bt$$

From Figs. 6 and 7, we obtain $B/A = 1.5\ \mu\text{m/hr},\ B = 0.47\ \mu\text{m}^2/\text{hr}$, therefore $A = 0.31\ \mu\text{m}$. The time required to grow 0.45μm oxide is

$$t = \frac{1}{B}(x^2 + Ax) = \frac{1}{0.47}(0.45^2 + 0.31 \times 0.45) = 0.72 \text{ hr} = 44 \text{ min.}$$

2. After a window is opened in the oxide for a second oxidation, the rate constants are $B = 0.01\ \mu\text{m}^2/\text{hr},\ \text{A} = 0.116\ \mu\text{m}(B/A = 6 \times 10^{-2}\ \ \mu\text{m/hr})$.

If the initial oxide thickness is $20 \text{ nm} = 0.02\ \mu\text{m}$ for dry oxidation, the value of τ can be obtained as followed:

$$(0.02)^2 + 0.166(0.02) = 0.01(0 + \tau)$$

or

$$\tau = 0.372 \text{ hr.}$$

For an oxidation time of $20 \text{ min}(= 1/3 \text{ hr})$, the oxide thickness in the window area is

$$x^2 + 0.166x = 0.01(0.333 + 0.372) = 0.007$$

or

$$x = 0.0350\ \mu\text{m} = 35 \text{ nm (gate oxide).}$$

For the field oxide with an original thickness $0.45\ \mu\text{m}$, the effective τ is given by

$$\tau = \frac{1}{B}(x^2 + Ax) = \frac{1}{0.01}(0.45^2 + 0.166 \times 0.45) = 27.72 \text{ hr.}$$

$$x^2 + 0.166x = 0.01(0.333 + 27.72) = 0.28053$$

or $x = 0.4530\ \mu\text{m}$ (an increase of $0.003\mu\text{m}$ only for the field oxide).

3. Governing equation is: $x^2 + Ax = B(t + \tau)$. Using: $x = 2,\ A = 0.5,\ B = 0.203$, leads to: $t = 24.63$ hours.

4. $x^2 + Ax = B(t + \tau)$

$$\left(x + \frac{A}{2}\right)^2 - \frac{A^2}{4} = B(t + \tau)$$

$$\left(x + \frac{A}{2}\right)^2 = B\left[\frac{A^2}{4B} + (t + \tau)\right]$$

when $t \gg \tau, t \gg \frac{A^2}{4B}$,

then, $x^2 = \mathrm{Bt}$

similarly,

when $t \gg \tau, t \gg \frac{A^2}{4B}$,

then, $x = \frac{B}{A}(t+\tau)$

5. At 980°C(=1253K) and 1 atm, $B = 8.5\times10^{-3}\mu\mathrm{m}^2/\mathrm{hr}$, $B/A = 4\times10^{-2}\mu\mathrm{m/hr}$ (from Figs. 6 and 7). Since $A \equiv 2D/k$, $B/A = kC_0/C_1$, $C_0 = 5.2\times10^{16}$ molecules/cm^3 and $\mathrm{C}_1 = 2.2\times10^{22}\mathrm{cm}^{-3}$, the diffusion coefficient is given by

$$D = \frac{Ak}{2} = \frac{A}{2}\left(\frac{B}{A}\cdot\frac{C_1}{C_0}\right) = \frac{B}{2}\left(\frac{C_1}{C_0}\right)$$
$$= \frac{8.5\times10^{-3}}{2}\frac{2.2\times10^{22}}{5.2\times10^{16}}\mu\mathrm{m}^2/\mathrm{hr}$$
$$= 1.79\times10^{3}\mu\mathrm{m}^2/\mathrm{hr}$$
$$= 4.79\times10^{-9}\ \mathrm{cm}^2/\mathrm{s}.$$

6. (a) For $\mathrm{SiN_xH_y}$

$$\frac{\mathrm{Si}}{\mathrm{N}} = \frac{1}{x} = 1.2$$

$$\therefore x = 0.83$$

$$\text{atomic \% H} = \frac{100y}{1+0.83+y} = 20$$

$$\therefore y = 0.46$$

The empirical formula is $\mathrm{SiN_{0.83}H_{0.46}}$.

(b) $\rho = 5\times10^{28}e^{-33.3\times1.2} = 2\times10^{11}\ \Omega\text{-cm}$

As the Si/N ratio increases, the resistivity decreases exponentially.

7. Set $\mathrm{Ta_2O_5}$ thickness $= 3\mathrm{t}, \varepsilon_1 = 25$

$\mathrm{SiO_2}$ thickness $= t$, $\varepsilon_2 = 3.9$

$\mathrm{Si_3N_4}$ thickness $= t$, $\varepsilon_3 = 7.6$, area $= A$

then

$$\mathrm{C}_{\mathrm{Ta_2O_5}} = \frac{\varepsilon_1\varepsilon_0 A}{3t}$$

$$\frac{1}{\mathrm{C}_{\mathrm{ONO}}} = \frac{t}{\varepsilon_2\varepsilon_0 A} + \frac{t}{\varepsilon_3\varepsilon_0 A} + \frac{t}{\varepsilon_2\varepsilon_0 A}$$

$$\frac{1}{\mathrm{C}_{\mathrm{ONO}}} = \frac{\varepsilon_2\varepsilon_3\varepsilon_0 A}{(\varepsilon_2 + 2\varepsilon_3)t}$$

$$\frac{\mathrm{C}_{\mathrm{Ta_2O_5}}}{\mathrm{C}_{\mathrm{ONO}}} = \frac{\varepsilon_1(\varepsilon_2 + 2\varepsilon_3)}{3\varepsilon_2\varepsilon_3} = \frac{25(3.9 + 2\times 7.6)}{3\times 3.9\times 7.6} = 5.37.$$

8. Rate varies as $\exp[-E_a/(kT)]$. Here: $700^\circ\mathrm{C} = 973.15^\circ\mathrm{K}$, while $800^\circ\mathrm{C} = 1073.15^\circ\mathrm{K}$

   Hence deposition rate predicted at $800^\circ\mathrm{C}$ is:

   $$(1000)[\exp(-2*1.6\times10^{-19}/(1.38\times10^{-23}*1073.15)]/[\exp(-2*1.6\times10^{-19}/(1.38\times10^{-23}*973.15)].$$

   ➔ 9210.99 Å/minute predicted at $800^\circ\mathrm{C}$.

   If the measured rate is lower, it may be due to a low "sticking coefficient" , i.e., a value less that 100% to account for the fact that not all reacting molecules will deposit and stick onto the surface.

9. Let

   $\mathrm{Ta_2O_5}$ thickness $= 3\mathrm{t}, \varepsilon_1 = 25$

   $\mathrm{SiO_2}$ thickness $= t,\ \varepsilon_2 = 3.9$

   $\mathrm{Si_3N_4}$ thickness $= t,\ \varepsilon_3$=7.6,

   area = A

   then

   $$\frac{\varepsilon_1\varepsilon_0 A}{3t} = \frac{\varepsilon_2\varepsilon_0 A}{d}$$

   $$d = \frac{3\varepsilon_2 t}{\varepsilon_1} = 0.468t.$$

10. We can use energy-enhanced CVD methods such as using a focused energy source or UV lamp. Another method is to use boron doped P-glass which will reflow at temperatures less than 900°C.

11. The deposition rate can be expressed as

    $r = r_0 \exp(-Ea/kT)$

    where $E_a = 0.6$ eV for silane-oxygen reaction. Therefore for $T_1 = 698$ K

$$\frac{r(T_2)}{r(T_1)} = 2 = \exp\left[0.6\left(\frac{1}{kT_1} - \frac{1}{kT_2}\right)\right]$$

$$\ln 2 = \frac{0.6}{0.0259}\left[\left(\frac{300}{698} - \frac{300}{T_2}\right)\right]$$

$$\therefore T_2 = 1030 \text{ K} = 757°\text{C}.$$

12. Moderately low temperatures are usually used for polysilicon deposition, and silane decomposition occurs at lower temperatures than that for chloride reactions. In addition, silane is used for better coverage over amorphous materials such SiO_2.

13. There are two reasons. One is to minimize the thermal budget of the wafer, reducing dopant diffusion and material degradation. In addition, fewer gas phase reactions occur at lower temperatures, resulting in smoother and better adhering films. Another reason is that the polysilicon will have small grains. The finer grains are easier to mask and etch to give smooth and uniform edges. However, for temperatures less than $575°\text{C}$ the deposition rate is too low.

14. Atomic density = (Avogadro's No.)
(density/g/mole = 6×10^{23}(molecules/mole)[(3g/cm^3)/102g/mole)]

The surface density (#molecules/cm^2) is just (molecules/cm^3)$^{2/3}$ or approximately 7×10^{14} molecules/cm^2.

15. 3600/4 = 900 cycles. 0.34nm/cycle $\times$ 900cycles/hr = 306 nm/hr.

16. The flat-band voltage shift is

$$\Delta V_{FB} = 0.5 \ V \sim \frac{Q_{ot}}{C_0}$$

$$C_0 = \frac{\varepsilon_{ox}}{d} = \frac{3.9\times8.85\times10^{-14}}{500\times10^{-8}} = 6.9\times10^{-8} \text{ F/cm}^{-2}.$$

$\therefore$ Number of fixed oxide charge is

$$\frac{0.5\, C_0}{q} = \frac{0.5\times6.9\times10^{-8}}{1.6\times10^{-19}} = 2.1\times10^{11}\text{cm}^{-2}$$

To remove these charges, a $450°\text{C}$ heat treatment in hydrogen for about 30 minutes is required.

17. 20/0.25 = 80 sqs.

Therefore, the resistance of the metal line is

$5 \times 50 = 400\ \Omega$..

18. For $TiSi_2$

$30 \times 2.37 = 71.1$ nm

For $CoSi_2$

$30 \times 3.56 = 106.8$ nm.

19. For $TiSi_2$:

Advantage: low resistivity

It can reduce native-oxide layers

$TiSi_2$ on the gate electrode is more resistant to high-field-induced hot-electron degradation.

Disadvantage: bridging effect occurs.

Larger Si consumption during formation of $TiSi_2$

Less thermal stability

For $CoSi_2$:

Advantage: low resistivity

High temperature stability

No bridging effect

A selective chemical etch exits Low shear forces

Disadvantage: not a good candidate for polycides

20. $RC = R_s^2 L^2 / t_{ox} = [(1000 \times 10^{-6})/(5000 \times 10^{-8})] * [(1^2 * 3.9 * 8.85 \times 10^{-14})/(10^{-4})]$

➔ RC ~ 70 ns.

21. (a) $R = \rho \dfrac{L}{A} = 2.67 \times 10^{-6} \times \dfrac{1}{0.28 \times 10^{-4} \times 0.3 \times 10^{-4}} = 3.2 \times 10^3 \Omega$

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon TL}{S} = \frac{3.9 \times 8.85 \times 10^{-14} \times 0.3 \times 10^{-4} \times 1 \times 10^4 \times 10^{-6}}{0.36 \times 10^{-4}} = 2.6 \times 10^{-13}\ F$$

$RC = 3.2 \times 10^5 \times 2.9 \times 10^{-15} = 0.93$ ns

(b) $R = \rho \dfrac{L}{A} = 1.7 \times 10^{-6} \times \dfrac{1}{0.28 \times 10^{-4} \times 0.3 \times 10^{-4}} = 2 \times 10^3 \Omega$

$$C = \frac{\varepsilon \mathrm{A}}{d} = \frac{\varepsilon TL}{S} = \frac{2.8 \times 8.85 \times 10^{-14} \times 0.3 \times 10^{-4} \times 1}{0.36 \times 10^{-4}} = 2.1 \times 10^{-13}\ \mathrm{F}$$

$RC = 2 \times 10^3 \times 2.1 \times 10^{-13} = 0.42$ ns

(c) We can decrease the RC delay by 55%. Ratio $= \dfrac{0.42}{093} = 0.45.$

22. (a) The aluminum runner can be considered as two segments connected in series: 20% (or $0.4\ \text{mm}$) of the length is half thickness $(0.5\ \mu\text{m})$ and the remaining $1.6\ \text{mm}$ is full thickness $(1\ \mu\text{m})$. The total resistance is

$$R = \rho\left[\frac{\ell_1}{A_1} + \frac{\ell_2}{A_2}\right] = 3\times10^{-6}\left[\frac{0.16}{10^{-4}\times10^{-4}} + \frac{0.04}{10^{-4}\times(0.5\times10^{-4})}\right]$$
$$= 72\ \Omega.$$

The limiting current I is given by the maximum allowed current density times cross-sectional area of the thinner conductor sections:

$I = 5\times10^{5}\ \text{A/cm}^2\times(10^{-4}\times0.5\times10^{-4}) = 2.5\times10^{-3}\ \text{A} = 2.5\ \text{mA}.$

The voltage drop across the whole conductor is then

$V = RI = 72\Omega\times2.5\times10^{-3}\ \text{A} = 0.18\text{V}.$

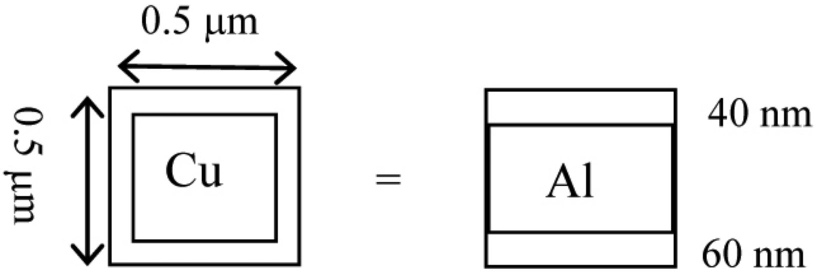
23. Let: $MTF = C\{\exp[E_a/(kT)]\}/J^2$, where C is the proportionality constant.

From data given: $72.2 = C\{\exp[E_a/(25.9\times10^{-3})]\}/10^{6}$, and

$0.00722 = C\{\exp[E_a/(25.9\times10^{-3})]\}/10^{10}.$

Solving yields: $C = 2.78\times10^{-6}\text{h cm}^4/\text{A}^2$ and $E_a = 0.8\ \text{eV}.$

24.



h: height , W : width , t : thickness, assume that the resistivities of the cladding layer and TiN are much larger than $\rho_{A\ell}$ *and* ρ_{Cu}

$$R_{Al} = \rho_{Al} \times \frac{\ell}{h \times W} = 2.7 \frac{\ell}{(0.5-0.1)\times 0.5}$$

$$R_{Cu} = \rho_{Cu} \times \frac{\ell}{h \times W} = 1.7 \frac{\ell}{(0.5-2t)\times(0.5-2t)}$$

When $R_{Al} = R_{Cu}$

Then $\frac{2.7}{0.4\times 0.5} = \frac{1.7}{(0.5-2t)^2}$

$\Rightarrow t = 0.073\ \mu\text{m} = 73\ \text{nm}.$

CHAPTER 13

1. With reference to Fig. 2 for class 100 clean room we have a total of 3500 particles/m^3 with particle sizes ≥ 0.5 μm

$$\frac{21}{100}\times 3500 = 735 \text{ particles/m}^2 \text{ with particle sizes} \geq 1.0\ \mu\text{m}$$

$$\frac{4.5}{100}\times 3500 = 157 \text{ particles/m}^2 \text{ with particle sizes} \geq 2.0\ \mu\text{m}$$

Therefore, (a) $3500 - 735 = 2765$ particles/m^3 between 0.5 and 1 μm

(b) $735 - 157 = 578$ particles/m^3 between 1 and 2 μm

(c) 157 particles/m^3 above 2 μm.

2. $$Y = \prod_{n=1}^{9} e^{-D_1 A}$$

$\text{A} = 50\ \text{mm}^2 = 0.5\ \text{cm}^2$

$$Y = e^{-4(0.1\times 0.5)} \times e^{-4(0.25\times 0.5)} \times e^{-1(1\times 0.5)} = e^{-1.2} = 30.1\%.$$

3. If a reticle contains a defect, that defect will appear in every exposed field on the wafer. If the field contains a single chip and the defect is fatal, then every chip will be inoperative. A mask projected onto the whole wafer at once need not be entirely defect-free. A single defect will affect only one chip in this case.

4. (a) The resolution of a projection system is given by

$$l_m = k_1 \frac{\lambda}{NA} = 0.6 \times \frac{0.193\mu\text{m}}{0.65} = 0.178\ \mu\text{m}$$

$$DOF = k_2 \frac{\lambda}{(NA)^2} = 0.5\left[\frac{0.193\ \mu\text{m}}{(0.65)^2}\right] = 0.228\ \mu\text{m}$$

(b) We can increase NA to improve the resolution. We can adopt resolution enhancement techniques (RET) such as optical proximity correction (OPC) and phase-shifting Masks (PSM). We can also develop new resists that provide lower k_1 and higher k_2 for better resolution and depth of focus.

(c) PSM technique changes k_1 to improve resolution.

5. In practice, reducing the wavelength below about 193 nm, light tends to be absorbed by the lenses that are often used in standard lithographic processes. This can led to

material damage and reductions in intensity. Also, at low wavelengths (e.g., below 157 nm) there are few options for light sources.

6. Using equation (1), minimum linewidth ~ $(\lambda g)^{½}$, the values for the two cases turn out to be: $(430*10)^{½} = 65.5744$, and $(250*40)^{½} = 100$, respectively. Hence, the 1st printer (with 10 μm mask) has a smaller resolution and is better.

7. (a) Using resists with high γ value can result in a more vertical profile but throughput decreases.

   (b) Conventional resists can not be used in deep UV lithography process because these resists have high absorption and require high dose to be exposed in deep UV. This raises the concern of damage to stepper lens, lower exposure speed and reduced throughput.

8. (a) A shaped beam system enables the size and shape of the beam to be varied, thereby minimizing the number of flashes required for exposing a given area to be patterned. Therefore, a shaped beam can save time and increase throughput compared to a Gaussian beam.

   (b) We can make alignment marks on wafers using e-beam and etch the exposed marks. We can then use them to do alignment with e-beam radiation and obtain the signal from these marks for wafer alignment.

   X-ray lithography is a proximity printing lithography. Its accuracy requirement is very high, therefore alignment is difficult.

   (c) X-ray lithography using synchrotron radiation has a high exposure flux so X-ray has better throughput than e-beam.

9. (a) To avoid the mask damage problem associated with shadow printing, projection printing exposure tools have been developed to project an image from the mask. With a 1:1 projection printing system is much more difficult to produce defect-free masks than it is with a 5:1 reduction step-and-repeat system.

   (b) It is not possible. The main reason is that X-rays cannot be focused by an optical lens. When it is through the reticle. So we can not build a step-and-scan X-ray lithography system.

10. As shown in the figure, the profile for each case is a segment of a circle with origin at the initial mask-film edge. As overetching proceeds the radius of curvature increases so that the profile tends to a vertical line.

11. (a) 20 sec

$0.6 \times 20/60 = 0.2$ μm.....(100) plane

$0.6/16 \times 20/60 = 0.0125$ μm.....(110) plane

$0.6/100 \times 20/60 = 0.002$ μm.....(111) plane

$W_b = W_0 - \sqrt{2}l = 1.5 - \sqrt{2} \times 0.2 = 1.22$ μm

(b) 40 sec

$0.6 \times 40/60 = 0.4$ μm.....(100) plane

$0.6/16 \times 40/60 = 0.025$ μm.....(110) plane

$0.6/100 \times 40/60 = 0.004$ μm.....(111) plane

$W_b = W_0 - \sqrt{2}l = 1.5 - \sqrt{2} \times 0.4 = 0.93$ μm

(c) 60 sec

$0.6 \times 1 = 0.6$ μm.....(100) plane

$0.6/16 \times 1 = 0.0375$ μm.....(110) plane

$0.6/100 \times 1 = 0.006$ μm.....(111) plane

$W_b = W_0 - \sqrt{2}l = 1.5 - \sqrt{2} \times 0.6 = 0.65$ μm.

12. Using the data in Prob. 11, the etched pattern profiles on <100>-Si are shown in below.

(a) 20 sec $l = 0.012$ μm, $W_0 = W_b = 1.5$ μm

(b) 40 sec $l = 0.025$ μm, $W_0 = W_b = 1.5$ μm

(c) 60 sec $l = 0.0375$ μm, $W_0 = W_b = 1.5$ μm.

13. (a) The equation states that the activation energy plays a role. Hence, one can expect silicon etching to be an activation-limited process. This, rather than the in-diffusion of the etchant into silicon, would be the critical step.

(b) Assuming the activation energy to play a role, illumination by photons which can deposit energy onto the material, can be expected to have an influence and increase the etch rate.

(c) The high silicon bond density on the {111} surface results in a denser silicon environment and hence leads to a slower rate of etching.

14. $\text{Si Etch Rate (nm/ min)} = 2.86\times10^{-13}\times n_F\times T^{1/2}\times e^{-E_a/RT}$

$$= 2.86\times10^{-13}\times3\times10^{15}\times(298)^{1/2}\times e^{\frac{-2.48\times10^3}{1.987\times298}}$$
$$= 224.7\ \text{nm/min.}$$

15. $\text{SiO}_2\ \text{Etch Rate (nm/min)} = 0.614\times10^{-13}\times3\times10^{15}\times(298^{1/2})\times e^{\frac{-3.76\times10^3}{1.987\times298}} = 5.6\ \text{nm/min}$

$\text{Etch selectivity of SiO}_2\ \text{over Si} = \dfrac{5.6}{224.7} = 0.025$

$\text{Or etch rate (SiO}_2\text{)/etch rate (Si)} = \dfrac{0.614}{2.86}\times e^{(-3.76+2.48)/1.987\times298} = 0.025.$

16. $1\,\text{Pa} = 7.52\ \text{m Torr}$

$\text{PV} = \text{nRT}$

$7.52/760\times10^{-3} = \text{n/V}\times0.082\times273$

$\text{n/V} = 4.42\times10^{-7}\,\text{mole/liter} = 4.42\times10^{-7}\times6.02\times10^{23}/1000 = 2.7\times10^{14}\,\text{cm}^{-3}$

Mean-free-path

$\lambda = 5\times10^{-3}/\ \text{P cm} = 5\times10^{-3}\times1000/7.52 = 0.6649\ \text{cm} = 6649\ \mu\text{m}$

$150\text{Pa} = 1128\ \text{m Torr}$

$\text{PV} = \text{nRT}$

$1128/760\times10^{-3} = \text{n/V}\times0.082\times273$

$\text{n/V} = 6.63\times10^{-5}\,\text{mole/liter} = 6.63\times10^{-5}\times6.02\times10^{23}/1000 = 4\times10^{16}\,\text{cm}^{-3}$

mean-free-path

$\lambda = 5\times10^{-3}/\text{P cm} = 5\times10^{-3}\times1000/1128 = 0.0044\ \text{cm} = 44\ \mu\text{m}.$

17. If the etch rate can be controlled to within 10 %, the polysilicon may be etched 10 % longer or for an equivalent thickness of 40 nm . The selectivity is therefore

$40\ \text{nm}/1\ \text{nm} = 40.$

18. Assuming a 30% overetching, and that the selectivity of Al over the photoresist maintains 3. The minimum photoresist thickness required is

$(1+30\%)\times1\ \mu\text{m}/3 = 0.433\ \mu\text{m} = 433.3\ \text{nm}.$

19. $\omega_e = \dfrac{qB}{m_e}$

$$2\pi \times 2.45 \times 10^{9} = \frac{1.6 \times 10^{-19} \times B}{9.1 \times 10^{-31}}$$

$$\begin{aligned} B &= 8.75 \times 10^{-2}\,(\text{tesla}) \\ &= 875\,(\text{gauss}). \end{aligned}$$

20. Traditional RIE generates low-density plasma (10^{9} cm^{-3}) with high ion energy. ECR and ICP generate high-density plasma (10^{11} to 10^{12} cm^{-3}) with low ion energy. Advantages of ECR and ICP are low etch damage, low microloading, low aspect-ratio dependent etching effect, and simple chemistry. However, ECR and ICP systems are more complicated than traditional RIE systems.

21. The corrosion reaction requires the presence of moisture to proceed. Therefore, the first line of defense in controlling corrosion is controlling humidity. Low humidity is essential, especially if copper containing alloys are being etched. Second is to remove as much chlorine as possible from the wafers before the wafers are exposed to air. Finally, gases such as CF_4 and SF_6 can be used for fluorine/chlorine exchange reactions and polymeric encapsulation. Thus, Al-Cl bonds are replaced by Al-F bonds. Whereas Al-Cl bonds will react with ambient moisture and start the corrosion process , Al-F bonds are very stable and do not react. Furthermore, fluorine will not catalyze any corrosion reactions.

CHAPTER 14

1. $\mathrm{E_a(boron) = 3.46\ eV,\ D_0 = 0.76\ cm^2/sec}$

From Eq. 6,

$$D = D_0 \exp\left(\frac{-E_a}{kT}\right) = 0.76 \exp\left(\frac{-3.46}{8.614\times10^{-5}\times1223}\right) = 4.142\times10^{-15}\ \mathrm{cm^2/s}$$

$$L = \sqrt{Dt} = \sqrt{4.142\times10^{-15}\times1800} = 2.73\times10^{-6}\ \mathrm{cm}$$

From Eq. 9, $C(x) = C_s \mathrm{erfc}\left(\frac{x}{2L}\right) = 1.8\times10^{20} \mathrm{erfc}\left(\frac{x}{5.46\times10^{-6}}\right)$

If $x = 0$, $C(0) = 1.8\times10^{20}$ atoms/cm^3;

$x = 0.05\times10^{-4}$, $\mathrm{C}(5\times10^{-6}) = 3.6\times10^{19}$ atoms/cm^3;

$x = 0.075\times10^{-4}$, $\mathrm{C}(7.5\times10^{-6}) = 9.4\times10^{18}$ atoms/cm^3;

$x = 0.1\times10^{-4}$, $\mathrm{C}(10^{-5}) = 1.8\times10^{18}$ atoms/cm^3;

$x = 0.15\times10^{-4}$, $\mathrm{C}(1.5\times10^{-5}) = 1.8\times10^{16}$ atoms/cm^3.

The $x_j = 2\sqrt{Dt}\left(\mathrm{erfc}^{-1}\frac{C_{sub}}{C_s}\right) = 0.15\mu\mathrm{m}$

Total amount of dopant introduced $= \mathrm{Q(t)}$

$$= \frac{2}{\sqrt{\pi}} C_s L = 5.54\times10^{14}\ \mathrm{atoms/cm^2}.$$

2. $$D = D_0 \exp\left(\frac{-E_a}{kT}\right) = 0.76 \exp\left(\frac{-3.46}{8.614\times10^{-5}\times1323}\right) = 4.96\times10^{-14}\mathrm{cm^2/s}$$

From Eq. 15, $C_s = C(0,t) = \frac{S}{\sqrt{\pi Dt}} = 2.342\times10^{19}$ atoms/cm^3

$$C(x) = C_s \mathrm{erfc}\left(\frac{x}{2L}\right) = 2.342\times10^{19} \mathrm{erfc}\left(\frac{x}{2.673\times10^{-5}}\right)$$

If $x = 0$, $\mathrm{C(0)} = 2.342\times10^{19}$ atoms/cm^3; $x = 0.1\times10^{-4}$, $\mathrm{C}(10^{-5}) = 1.41\times10^{19}$ atoms/cm^3;

$x = 0.2\times10^{-4}$, $\mathrm{C}(2\times10^{-5}) = 6.79\times10^{18}$ atoms/cm^3;

$x = 0.3\times10^{-4}$, $\mathrm{C}(3\times10^{-5}) = 2.65\times10^{18}$ atoms/cm^3;

$x = 0.4\times10^{-4}$, $\mathrm{C}(4\times10^{-5}) = 9.37\times10^{17}$ atoms/cm^3;

$x = 0.5\times10^{-4}$, $\mathrm{C}(5\times10^{-5}) = 1.87\times10^{17}$ atoms/cm^3;

$x = 0.6\times10^{-4}$, $\mathrm{C}(6\times10^{-5}) = 3.51\times10^{16}$ atoms/cm^3;

$x = 0.7\times10^{-4}$, $\mathrm{C}(7\times10^{-5}) = 7.03\times10^{15}$ atoms/cm^3;

$x = 0.8\times10^{-4}$, $\mathrm{C}(8\times10^{-5}) = 5.62\times10^{14}$ atoms/cm^3.

The $x_j = \sqrt{4Dt\ln\dfrac{S}{C_B\sqrt{\pi Dt}}} = 0.72\ \mu\mathrm{m}$.

3. From equation (14): $\mathrm{C(x,t)} = [\mathrm{S/(nDt)^{1/2}}]\ \exp[-\mathrm{x^2/(4Dt)}]$, one has for this situation:

   $10^{18} = 10^{15}\ \exp[(4\times10^{-8})/(4^*\mathrm{D}^*3600)]$ è $\mathrm{D} = 4\times10^{-13}$ cm^2/s.

4. $$1\times10^{15} = 1\times10^{18}\ \exp\left(\frac{10^{-8}}{4\times2.3\times10^{-13}t}\right)$$

   $\mathrm{t} = 1573\ \mathrm{s} = 26\ \mathrm{min}$

   For the constant-total-dopant diffusion case, Eq. 15 gives $C_s = \dfrac{S}{\sqrt{\pi Dt}}$

   $S = 1\times10^{18}\sqrt{\pi\times2.3\times10^{-13}\times1573} = 3.4\times10^{13}$ atoms/cm^2.

5. At 1100°C, $\mathrm{n_i} = 6\times10^{18}\ \mathrm{cm^{-3}}$. Therefore, the doping profile for a surface concentration of $4\times10^{18}\ \mathrm{cm^{-3}}$ is given by the "intrinsic" diffusion process:

   $$C(x,t) = C_s\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$

where $\mathrm{C_s = 4\times10^{18}\ cm^{-3}}$, $\mathrm{t = 3\ hr = 10800\ s}$, and $\mathrm{D = 5\times10^{-14}\ cm^2/s}$. The diffusion length is then

$\sqrt{Dt} = 2.32\times10^{-5}\mathrm{cm} = 0.232\,\mu\mathrm{m}$

The distribution of arsenic is $C(x) = 4\times10^{18}\ \mathrm{erfc}\left(\dfrac{x}{4.64\times10^{-5}}\right)$

The junction depth can be obtained as follows

$$10^{15} = 4\times10^{18}\ \mathrm{erfc}\left(\frac{x_j}{4.64\times10^{-5}}\right)$$
$$x_j = 1.2\times10^{-4}\ \mathrm{cm} = 1.2\,\mu\mathrm{m}.$$

6. The process is called the ramping of a diffusion furnace. For the ramp-down situation, the furnace temperature T is given by

$$\mathrm{T = T_0 - rt}$$

where $\mathrm{T_0}$ is the initial temperature and r is the linear ramp rate. The effective Dt product during a ramp-down time of $\mathrm{t_1}$ is given by

$$(Dt)_{eff} = \int_0^{t_1} D(t)dt$$

In a typical diffusion process, ramping is carried out until the diffusivity is negligibly small. Thus the upper limit $\mathrm{t_1}$ can be taken as infinity:

$$\frac{1}{T} = \frac{1}{T_0 - rt} \approx \frac{1}{T_0}\left(1 + \frac{rt}{T_0} + \ldots\right)$$

and

$$D = D_0\ \exp\left({}^{-E_a}\!\big/_{kT}\right) = D_0\ \exp\left[\frac{-E_a}{kT_0}\left(1 + \frac{rt}{T_0} + \ldots\right)\right] = D_0\left(\exp\frac{-E_a}{kT_0}\right)\left(\exp\frac{-rE_a t}{kT_0^2}\ldots\right)$$
$$\approx D(T_0)\exp\frac{-rE_a t}{kT_0^2}$$

where $D(T_0)$ is the diffusion coefficient at T_0. Substituting the above equation into the expression for the effective Dt product gives

$$(Dt)_{eff} \approx \int_0^\infty D(T_0)\exp\frac{-rE_a t}{kT_0^2}dt = D(T_0)\frac{kT_0^{\ 2}}{rE_a}$$

Thus the ramp-down process results in an effective additional time equal to $kT_0^{\ 2}/rE_a$ at the initial diffusion temperature T_0.

For phosphorus diffusion in silicon at 1000°C, we have from Fig. 4:

$$D(T_0) = \mathrm{D(1273\ K)} = 2\times10^{-14}\ \mathrm{cm^2/s}$$

$$r = \frac{1273-773}{20\times60} = 0.417\ K/s$$

$$E_a = 3.66\ \mathrm{eV}$$

Therefore, the effective diffusion time for the ramp-down process is

$$\frac{kT_0^{\ 2}}{rE_a} = \frac{1.38\times10^{-23}(1273)^2}{0.417(3.66\times1.6\times10^{-19})} = 91s \approx 1.5\ \mathrm{min.}$$

7. For low-concentration drive-in diffusion, the diffusion is given by Gaussian distribution. The surface concentration is then

$$C(0,t) = \frac{S}{\sqrt{\pi Dt}} = \frac{S}{\sqrt{\pi D_0 t}}\exp\left(\frac{E_a}{2kT}\right)$$

$$\frac{dC}{dt} = \frac{S}{\sqrt{\pi D_0}}\exp\left(\frac{E_a}{2kT}\right)\left(\frac{-t^{3/2}}{2}\right) = -0.5\times\frac{C}{t}$$

$$\text{or } \frac{dC}{C} = -0.5\times\frac{dt}{t}$$

which means 1% change in diffusion time will induce 0.5% change in surface concentration.

$$\frac{dC}{dT}=\frac{S}{\sqrt{\pi D_0 t}}\exp\left(\frac{E_a}{2kT}\right)\left(\frac{-E_a}{2kT^2}\right)=-C\frac{E_a}{2kT^2}$$

$$\text{or }\frac{dC}{C}=\frac{-E_a}{2kT}\times\frac{dT}{T}=\frac{-3.6\times1.6\times10^{-19}}{2\times1.38\times10^{-23}\times1273}\times\frac{dT}{T}=-16.9\times\frac{dT}{T}$$

which means 1% change in diffusion temperature will cause 16.9% change in surface concentration.

8. At 900°C, $n_i = 2\times10^{18}\ \text{cm}^{-3}$. For a surface concentration of $4\times10^{18}\ \text{cm}^{-3}$, given by the "extrinsic" diffusion process

$$D = D_0 e^{\frac{-E_a}{kT}}\times\frac{n}{n_i}=45.8e^{\frac{-4.05\times1.6\times10^{-19}}{1.38\times10^{-23}\times1173}}\times\frac{4\times10^{18}}{2\times10^{18}}=3.77\times10^{-16}\ \text{cm}^2/\text{s}$$

$$x_j = 1.6\sqrt{Dt} = 1.6\sqrt{3.77\times10^{-16}\times10800} = 3.23\times10^{-6}\ \text{cm} = 32.3\ \text{nm}.$$

9. The Gaussian profile goes in deeper and has a longer tail. By contrast, the high-density profile has much higher values for low depths. Since the total number of atoms is constant, the area under both profiles is the same.

10. For impurity in the oxidation process of silicon,

$$\text{segregation coefficient} = \frac{\text{equilibrium concentration of impurity in silicon}}{\text{equilibrium concentration of impurity in SiO}_2}.$$

11. $\kappa = \dfrac{3\times10^{11}}{5\times10^{13}} = \dfrac{3}{500} = 0.006.$

12. The ion dose per unit area is

$$\frac{N}{A}=\frac{\frac{It}{q}}{A}=\frac{\frac{10\times10^{-6}\times5\times60}{1.6\times10^{-19}}}{\pi\times\left(\frac{10}{2}\right)^2}=2.38\times10^{12}\ \text{ions/cm}^2$$

From Eq. 25 and Example 3, the peak ion concentration is at $x = R_p$. Figure. 17 indicates the σ_p is $20\ \text{nm}$.

Therefore, the ion concentration is

$$\frac{S}{\sigma_p\sqrt{2\pi}} = \frac{2.38\times10^{12}}{20\times10^{-7}\sqrt{2\pi}} = 4.74\times10^{17}\,\text{cm}^{-3}.$$

13. From Fig. 17, the $R_p = 230\ \text{nm}$, and $\sigma_p = 62\ \text{nm}$. The peak concentration is

$$\frac{S}{\sigma_p\sqrt{2\pi}} = \frac{2\times10^{15}}{62\times10^{-7}\sqrt{2\pi}} = 1.29\times10^{20}\,\text{cm}^{-3}$$

From Eq. 25,

$$10^{15} = 1.29\times10^{20}\ \exp\left[\frac{-(x_j - R_p)^2}{2\sigma_p^{\ 2}}\right]$$

$$x_j = 0.53\ \mu\text{m}.$$

14. $-0.5 = -1.1 + qF_B/C_\text{i}$

$$C_{ox} = \frac{\varepsilon_s}{d} = \frac{3.9\times8.85\times10^{-14}}{10^{-6}} = 3.45\times10^{-7}$$

$$F_B = \frac{3.45\times10^{-7}}{1.6\times10^{-19}}\times0.6 = 1.3\times10^{12}\ \text{cm}^{-2}$$

$$\frac{10^{-5}}{\pi(10.16)^2}t = 1.3\times10^{12}\times1.6\times10^{-19}$$

The implant time $t = 6.7\ \text{s}$.

15. Using equation (28):

$$\text{Energy lost} = [4(11\text{m}_\text{p})(28\text{m}_\text{p})]/[(11\text{m}_\text{p} + 28\text{m}_\text{p})^2](100\ \text{keV}) \sim 81\ \text{keV}.$$

16. Current = 0.5 mA ➔ boron charge deposited in 2 minutes $= 0.5\times10^{-3} * 2 * 60$ C. The number of ions $= (0.5\times10^{-3*}2^*60)/(1.6\times10^{-19}) = 3.75\times10^{17}$.

Hence, flux $=$ ions/area/time $= 3.75\times10^{17}/(\Pi * 10^2)/120 = 9.9471\times10^{12}$.

The dose $S = \text{ions/area} = 3.75\times10^{17} / (\Pi * 10^{2}) = 1.1936\times10^{15}$.

$N(x = 0.2\ \mu m) = \{S/[(2n)^{½}\sigma_p]\}\ \exp[-\{(X - R_p)^2\}/(2\sigma_p{}^2)]$ works out to:

$\{1.1936\times10^{15} /[(2n)^{½} * 7\times10^{-6}]\}\ \exp[-\{(2\times10^{-5} - 3.1\times10^{-5})^2\}/\{2*(7\times10^{-6})^2\}] = 1.979\times10^{19}/\text{cc}$.

17. Dose per unit area $= \dfrac{Q}{q} = \dfrac{C_0 \Delta V_T}{q} = \dfrac{3.9\times8.85\times10^{-14}\times1}{250\times10^{-8}\times1.6\times10^{-19}} = 8.6\times10^{11}\ \text{cm}^{-2}$

From Fig. 17 and Example 3, the peak concentration occurs at $140\ \text{nm}$ from the surface. Also, it is at $(140 - 25) = 115\ \text{nm}$ from the Si-SiO_2 interface.

18. The higher the temperature, the faster defects anneal out. Also, the solubility of electrically active dopant atoms increases with temperature.

19. The projected range is $150\ \text{nm}$ (see Fig. 17). The average nuclear energy loss over the range is $60\ \text{eV/nm}$ (Fig. 16).

$60\times0.25 = 15\ \text{eV}$ (energy loss of boron ion per each lattice plane)

the damage volume = $V_D = \pi(2.5\ \text{nm})^2(150\ \text{nm}) = 3\times10^{-18}\ \text{cm}^3$

total damage layer $= 150/0.25 = 600$

displaced atom for one layer $= 15/15 = 1$

damage density $= 600/V_D = 2\times10^{20}\ \text{cm}^{-3}$

$2\times10^{20}/5.02\times10^{22} = 0.4\%$.

20. Fraction= ${}_0\!\int^t[S/\{(2\pi)^{½}\sigma_p\}]\exp[-\{(x-R_p)/(\sigma_p\sqrt{2})\}^2]/{}_0\!\int^\infty[S/\{(2\pi)^{½}\sigma_p\}]$
$\exp[-\{(x-R_p)/(\sigma_p\sqrt{2})\}^2]$
Manipulating the above:
Fraction = $\{\text{erf}[R_p/(\sigma_p\sqrt{2})] - \text{erf}[(R_p-t)/(\sigma_p\sqrt{2})]\} / \{1+\text{erf}[(R_p)/(\sigma_p\sqrt{2})]\}$.
Putting in the values, one gets:
Fraction = $[\text{erf}(1.767767) - \text{erf}(1.502602)]/[1+\text{erf}(1.767767)] = 0.010648$.

21. $\Delta V_t = 1\text{V} = \dfrac{Q_1}{C_{ox}}$

where Q_1 is the additional charge added just below the oxide-semiconductor surface by ion implantation. C_{OX} is a parallel-plate capacitance per unit area given by $C_{ox} = \frac{\varepsilon_s}{d}$

(d is the oxide thickness, ε_s is the permittivity of the semiconductor)

$$Q_1 = \Delta V_1 C_{ox} = \frac{1\text{V} \times 3.9 \times 8.85 \times 10^{-14} \text{F/cm}}{0.4 \times 10^{-6} \text{ cm}} = 8.63 \times 10^{-7} \frac{\text{C}}{\text{cm}^2}$$

$$\frac{8.63 \times 10^{-7}}{1.6 \times 10^{-19}} = 5.4 \times 10^{12} \text{ions/cm}^2$$

$$\text{Total implant dose} = \frac{5.4 \times 10^{12}}{45\%} = 1.2 \times 10^{13} \text{ions/cm}^2.$$

22. The discussion should mention much of Section 13.6. Diffusion from a surface film avoids problems of channeling. Tilted beams cannot be used because of shadowing problems. If low energy implantation is used, perhaps with preamorphization by silicon, then to keep the junctions shallow, RTA is also necessary.

23. From Eq. 35

$$\frac{S_d}{S} = \frac{1}{2} \text{erfc}\left(\frac{0.4 - 0.6}{0.2\sqrt{2}} \right) = 0.84$$

The effectiveness of the photoresist mask is only 16%.

$$\frac{S_d}{S} = \frac{1}{2} \text{erfc}\left(\frac{1 - 0.6}{0.2\sqrt{2}} \right) = 0.023$$

The effectiveness of the photoresist mask is 97.7%.

24. $T = \frac{1}{2\sqrt{\pi}} \frac{e^{-u^2}}{u} = 10^{-5}$

$\therefore u = 3.02$

$d = R_p + 4.27\sigma_p = 0.53 + 4.27 \times 0.093 = 0.927$ µm.

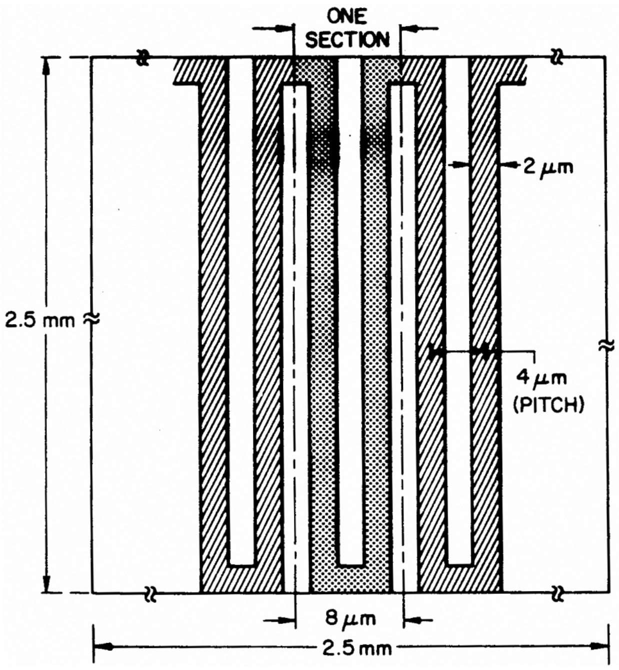
CHAPTER 15

1. Each U-shape section (refer to the figure) has an area of $2500\ \mu\text{m}\times 8\ \mu\text{m} = 2\times 10^4\ \mu\text{m}^2$. Therefore, there are $(2500)^2/2\times 10^4 = 312.5$ U-shaped section. Each section contains 2 long lines with 1248 squares each, 4 corner squares, 1 bottom square, and 2 half squares at the top. Therefore the resistance for each section is

$$1\ \text{k}\Omega/\geqslant(1248\times 2+4\times 0.65+2)=2500.6\quad \text{k}\Omega$$

The maximum resistance is then

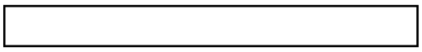
$$312.5\times 2500.6 = 7.81\times 10^8\ \Omega = 781\ \text{M}\Omega$$



2. If the space between lines is $2\ \mu\text{m}$, then there is $4\ \mu\text{m}$ for each turn (i.e., $2\times\text{n}$, for one turn). Assume there are n turns, from Eq.6, $L \approx \mu_0 n^2 r \approx 1.2\times 10^{-6} n^2 r$, where r can be replaced by $2\times\text{n}$. Then, we can obtain that n is 13.

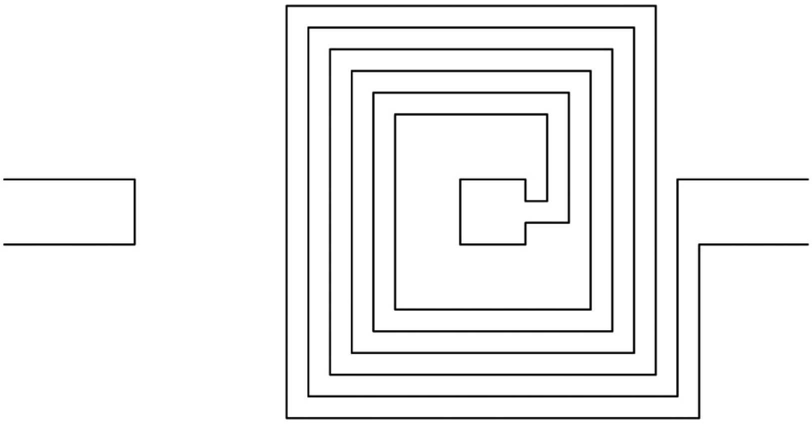
3. (a) Metal 1, (b) contact hole, (c) Metal 2.

(a) Metal 1,

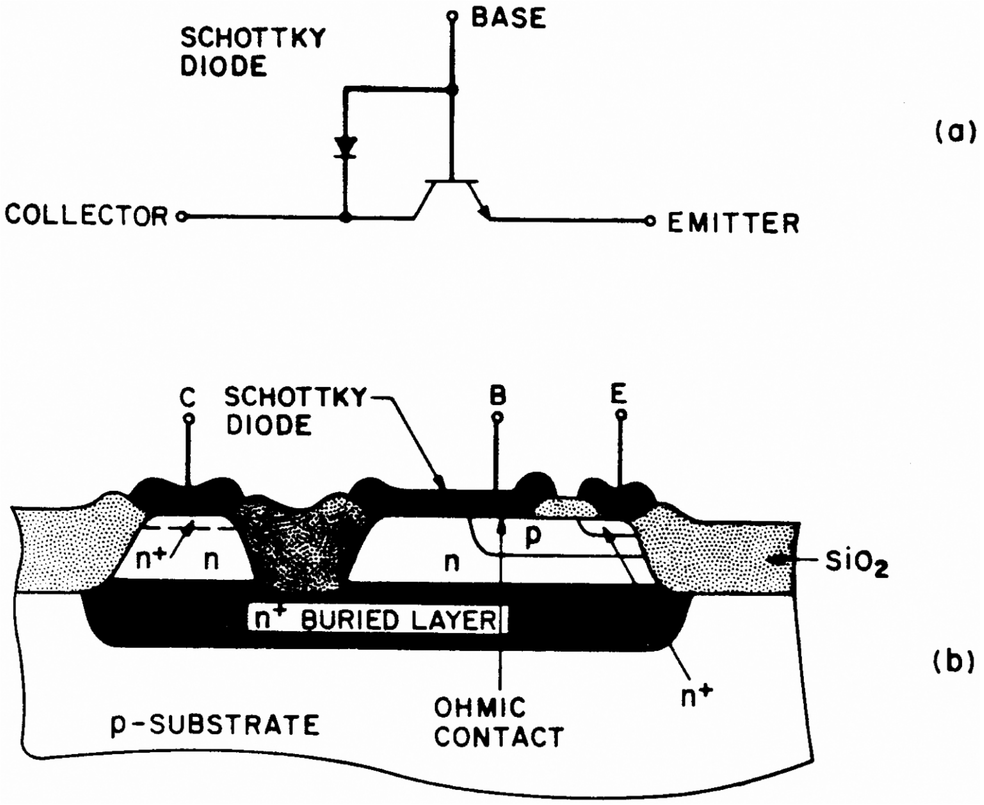


(b) contact hole,

(c) Metal 2.



4. The circuit diagram and device cross-section of a clamped transistor are shown in (a) and (b), respectively.



5. (a) The undoped polysilicon is used for isolation.

   (b) The polysilicon 1 is used as a solid-phase diffusion source to form the extrinsic base region and the base electrode.

   (c) The polysilicon 2 is used as a solid-phase diffusion source to form the emitter region and the emitter electrode.

6. (a) For 30 keV boron, $R_p = 100$ nm and $\Delta R_p = 34$ nm. Assuming that R_p and ΔR_p for boron are the same in Si and SiO2 the peak concentration is given by

$$\frac{S}{\sqrt{2\pi}\Delta R_p} = \frac{8\times10^{11}}{\sqrt{2\pi}(34\times10^{-7})} = 9.4\times10^{16}\ \text{cm}^{-3}$$

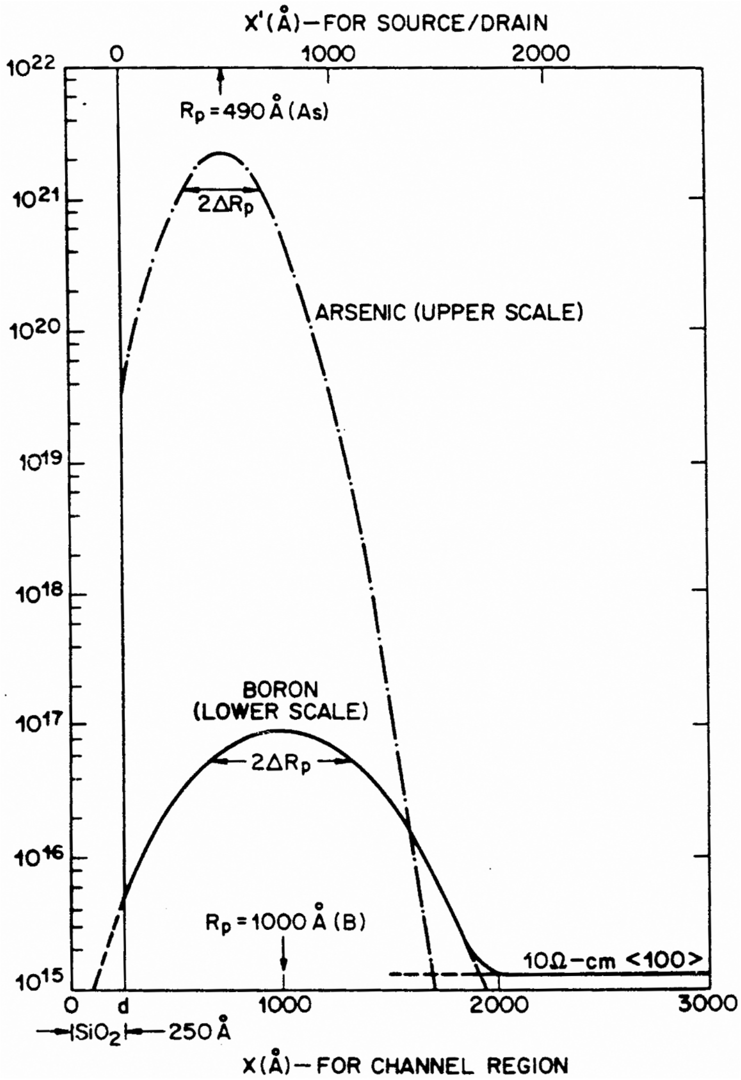
   The amount of boron ions in the silicon is

$$\begin{aligned}\frac{Q}{q} &= \int_d^\infty \frac{S}{\sqrt{2\pi}\Delta R_p}\exp\left[-\frac{(x-R_p)^2}{2\Delta R_p^2}\right]dx\\ &= \frac{S}{s}\left[2-\mathrm{erfc}\left(\frac{R_p-d}{\sqrt{2}\Delta R_p}\right)\right]\\ &= \frac{8\times10^{11}}{2}\left[2-\mathrm{erfc}\left(\frac{750}{\sqrt{2}\times340}\right)\right]\\ &= 7.88\times10^{11}\mathrm{cm}^{-2}\end{aligned}$$

Assume that the implanted boron ions form a negative sheet charge near the Si-SiO$_2$ interface, then

$$\Delta V_T = q\left(\frac{Q}{q}\right)/C_{ox} = \frac{1.6\times10^{-19}\times(7.88\times10^{11})}{3.9\times8.85\times10^{-14}/(25\times10^{-7})} = 0.91\ \mathrm{V}$$

(b) For 80 keV arsenic implantation, $R_p = 49\ \mathrm{nm}$ and $\Delta R_p = 18\ \mathrm{nm}$. The peak arsenic concentration is $\frac{S}{\sqrt{2\pi}\Delta R_p} = \frac{10^{16}}{\sqrt{\pi}\times(18\times10^{-7})} = 2.21\times10^{21}\ \mathrm{cm}^{-3}$.



7. (a) Because (100)-oriented silicon has lower (~ one tenth) interface-trapped charge and a lower fixed oxide charge.

   (b) If the field oxide is too thin, it may not provide a large enough threshold voltage for adequate isolation between neighboring MOSFETs.

   (c) The typical sheet resistance of heavily doped polysilicon gate is 20 to $30\ \Omega/\square$, which is adequate for MOSFETs with gate lengths larger than $3\ \mu m$. For shorter gates, the sheet resistance of polysilicon is too high and will cause large RC delays. We can use refractory metals (e.g., Mo) or silicides as the gate material to reduce the sheet resistance to about $1\ \Omega/\square$.

(d) A self-aligned gate can be obtained by first defining the MOS gate structure, then using the gate electrode as a mask for the source/drain implantation. The self-aligned gate can minimize parasitic capacitance caused by the source/drain regions extending underneath the gate electrode (due to diffusion or misalignment).

(e) P-glass can be used for insulation between conducting layers, for diffusion and ion implantation masks, and for passivation to protect devices from impurities, moisture, and scratches.

8. Charge stored in each cell $= \mathrm{CV} = 50\times10^{-15}\times5 = 2.5\times10^{-13}$ C.

So, number of electrons $= 2.5\times10^{-13}/1.6\times10^{-19} = 1.5625\times10^{6}$.

Leakage current $= dQ/dt = 2.5\times10^{-13}/4\times10^{-3} = 0.625\times10^{-10}$ A $= 62.5$ pA.

9. The lower insulator has a dielectric constant $\varepsilon_1/\varepsilon_0 = 4$ and a thickness $d_1 = 10$ nm The upper insulator has a dielectric constant $\varepsilon_2/\varepsilon_0 = 10$ and a thickness $d_2 = 100$ nm. Upon application of a positive voltage V_G to the external gate, electric field E_1 and E_2 are established in the d_1 and d_2 respectively. We have, from Gauss' law, that

$\varepsilon_1 E_1 = \varepsilon_2 E_2 + Q$ and $V_G = E_1 d_1 + E_2 d_2$ where Q is the stored charge on the floating gate. From these above two equations, we obtain

$$E_1 = \frac{V_G}{d_1 + d_2(\varepsilon_1/\varepsilon_2)} + \frac{Q}{\varepsilon_1 + \varepsilon_2(d_1/d_2)}$$

$$J = \sigma E_1 = 10^{-7}\left\{\frac{10\times10^{7}}{10+100\left(\frac{4}{10}\right)} + \frac{Q}{\left[4+10\left(\frac{10}{100}\right)\right]\times8.85\times10^{-14}}\right\} = 0.2 - 2.26\times10^{5}\,|Q|$$

(a) If the stored charge does not reduce E_1 by a significant amount (i.e., $0.2 \gg 2.26\times10^{5}\,|Q|$, we can write

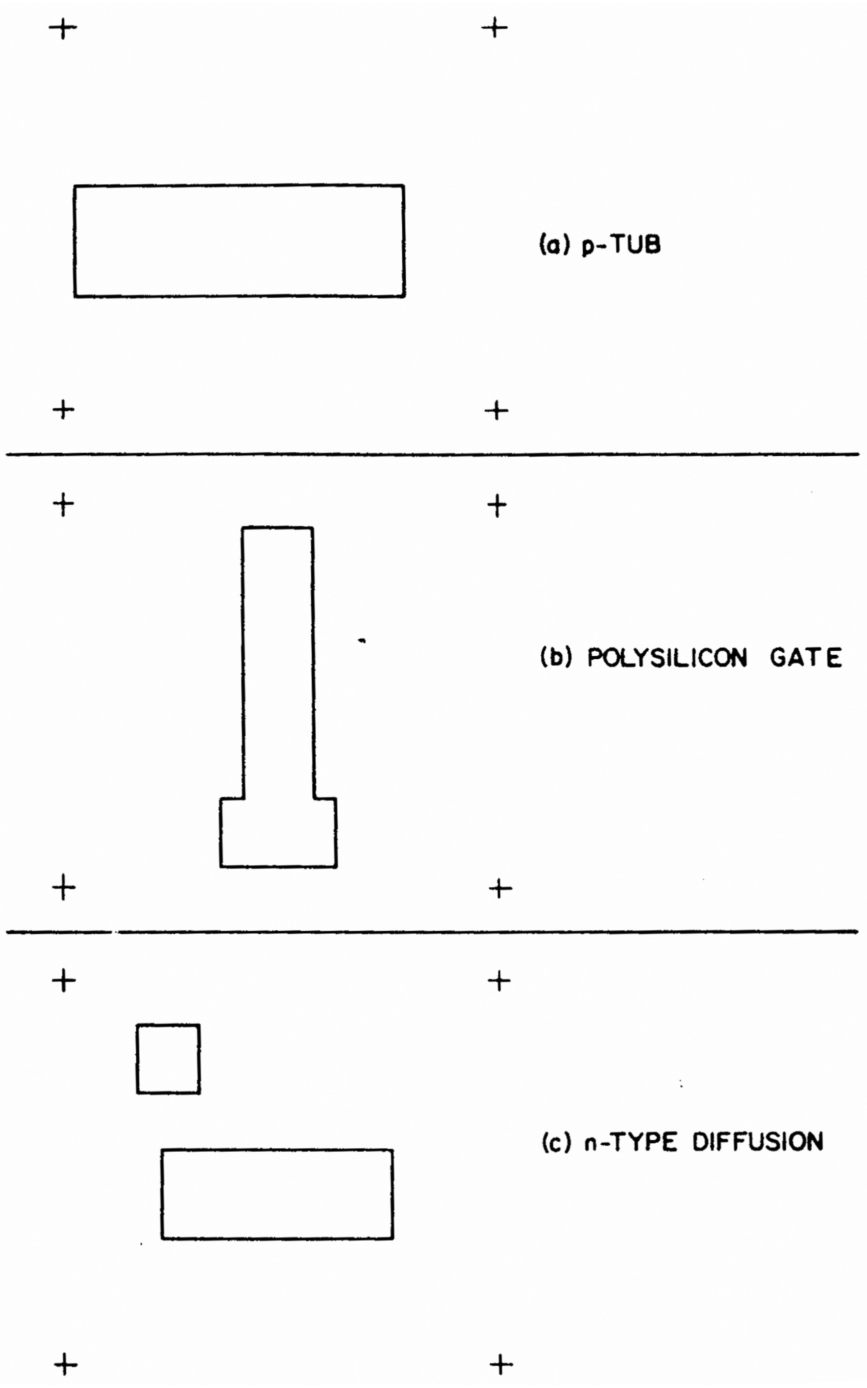
$$Q = \int_0^t \sigma E_1 dt' \approx 0.2\Delta t = 0.2 \times (0.25 \times 10^{-6}) = 5 \times 10^{-8} \text{ C}$$

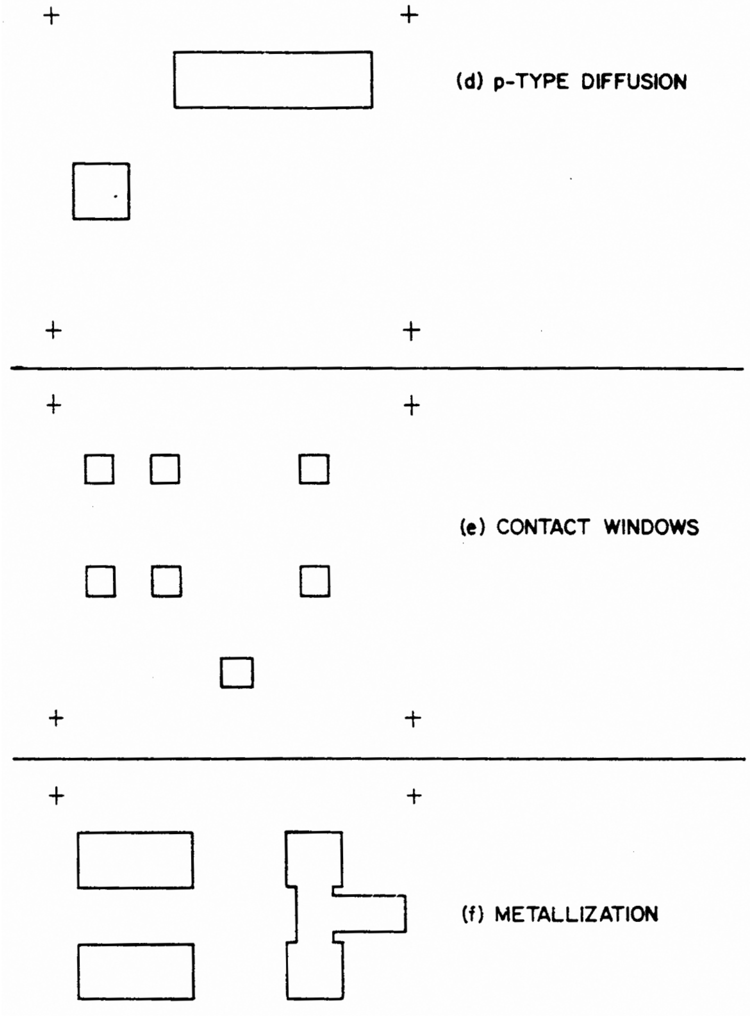
$$\Delta V_T = \frac{Q}{C_2} = \frac{5 \times 10^{-8}}{(10 \times 8.85 \times 10^{-14}) / (100 \times 10^{-7})} = 0.565 \text{ V}$$

(b) when $t \to \infty$, $J \to 0$ we have $|Q| \to 0.2/2.26 \times 10^5 \cong 8.84 \times 10^{-7}$ C.

Then $\Delta V_T = \dfrac{Q}{C_2} = \dfrac{8.84 \times 10^{-7}}{(10 \times 8.85 \times 10^{-14})/10^{-5}} = 9.98$ V.

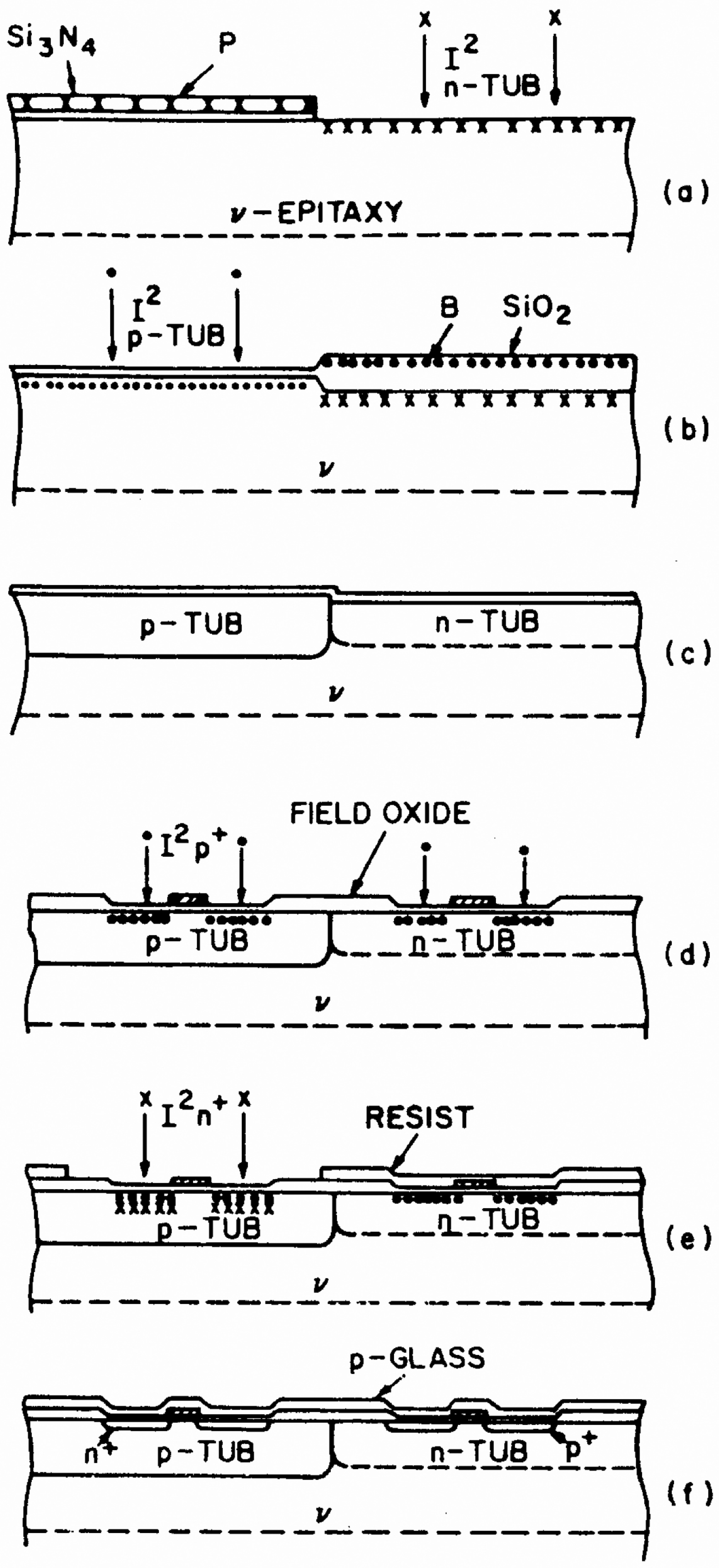
10.





11. The charge will shift the threshold voltage by an amount ΔV_T. This will be given by: $\Delta V_T = -\Delta Q / \varepsilon$ where "*d*" is the thickness of the top oxide layer. This shift will affect the $I_D - V_G$ characteristic. This will also lead to a shift in the channel conductance g_D given by: $g_D = I_D / V_D = (Z/L)\mu C_{ox}(V_G - V_T)$ è $\Delta g_D = (Z/L)\mu C_{ox}\Delta Q/\varepsilon$. Thus, the $g_D - V_D$ plot will shift to the right by ΔV_T.

12.


Si$_3$N$_4$
P
I^2
n-TUB
ν-EPITAXY
(a)
I^2
p-TUB
B
SiO$_2$
ν
(b)
p-TUB
n-TUB
ν
(c)
FIELD OXIDE
I^{2}p$^+$
p-TUB
n-TUB
ν
(d)
I^{2}n$^+$
RESIST
p-TUB
n-TUB
ν
(e)
p-GLASS
n$^+$
p-TUB
n-TUB
p$^+$
ν
(f)

13. To solve the short-channel effect of devices.

14. The device performance will be degraded from the boron penetration. There are methods to reduce this effect: (1) using rapid thermal annealing to reduce the time at high temperatures, consequently reduces the diffusion of boron, (2) using nitrided oxide to suppress the boron penetration, since boron can easily combine with nitrogen and becomes less mobile, (3) making a multi-layer of polysilicon to trap the boron atoms at the interface of each layer.

15. This structure: (a) overcomes short channel effects of typical MOSFETs while still making transistors scalable. (b) The structure allows a small-length transistor that can thus have a large intrinsic gain compared to its bulk counterpart. (c) The heavily-doped poly-Si film that wraps around the fin greatly reduces the source/drain series resistance and provides a convenient means for local interconnects and for making connections to the metal. (d) The channel width is roughly twice the fin height plus the fin width. This increases the device current-carrying capacity. (e) Much Lower off-state current compared to bulk counterpart.

16. Total capacitance of the stacked gate structure is:

$$C = \frac{\varepsilon_1}{d_1} \times \frac{\varepsilon_2}{d_2} \Big/ \left(\frac{\varepsilon_1}{d_1} \times \frac{\varepsilon_2}{d_2} \right) = \frac{7}{0.5} \times \frac{25}{10} \Big/ \left(\frac{7}{0.5} + \frac{25}{10} \right) = 2.12$$

$$\frac{3.9}{d} = 2.12$$

$$\therefore d = \frac{3.9}{2.12} = 1.84 \text{ nm.}$$

17. Disadvantages of LOCOS: (1) high temperature and long oxidation time cause V_T shift, (2) bird’s beak, (3) not a planar surface, (4) exhibits oxide thinning effect. Advantages of shallow trench isolation: (1) planar surface, (2) no high temperature processing and long oxidation time, (3) no oxide thinning effect, (4) no bird’s beak.

18. For isolation between the metal and the substrate.

19. GaAs lacks of high-quality insulating film.

20. (a) $$RC = \left(\rho \frac{L}{A}\right)\left(\varepsilon_{ox} \frac{A}{d}\right) = \left(10^{-5} \times \frac{1}{1 \times 0.5 \times 10^{-8}}\right)\left[3.9 \times 8.85 \times 10^{-14} \times \frac{1 \times (1 \times 10^{-4})}{0.5 \times 10^{-4}}\right]$$

$$= 2000 \times (69.03 \times 10^{-14}) = 1.38 \times 10^{-9} s = 1.38 \text{ ns}.$$

(b) For a polysilicon runner

$$RC = \left(R_{square} \frac{L}{W}\right)\left(\varepsilon_{ox} \frac{A}{d}\right)$$

$$= 30\left(\frac{1}{10^{-4}}\right)(69.03 \times 10^{-14}) = 2.07 \times 10^{-7} \text{ s}$$

$$= 207 \text{ ns}$$

Therefore the polysilicon runner's RC time constant is 150 times larger than the aluminum runner.

21. SOI technology helps in the continued miniaturization of microelectronic devices. Some of the benefits of SOI technology relative to conventional silicon processing include:

(a) Lower parasitic capacitance due to isolation from the bulk silicon. This improves power consumption at matched performance.

(b) Higher resistance to latch-up due to complete isolation of the n- and p-well structures.

(c) SOI overcomes the need for complex well structures and isolation processes.

(d) There is no risk of nonuniform inter-diffusion of silicon and Al in the contact regions because of oxide isolation at the bottom of the junction. Hence, the contact barrier is not necessary.

(e) Since modern bulk silicon device needs high doping at the drain and substrate to eliminate short-channel effects and punch-through, this high doping results in high capacitance when the junction is reversed bias. However, with SOI, the maximum capacitance between the junction and substrate is the capacitance of the buried insulator. The related dielectric constant is three times smaller than that of silicon.

(f) Thus SOI offers higher chip performance per watt, lower DC power consumption, and better scalability at smaller geometries over traditional process technologies.

22. When we combine the logic circuits and memory on the chip, we need multiple supply voltage. For reliability issue, different oxide thicknesses are needed for different supply voltages.

23. (a) $1/C_{\text{total}} = 1/C_{\text{Ta}_2\text{O}_5} + 1/C_{\text{nitride}}$

Hence $EOT/3.9 = 75/25 + 10/7 = 17.3$ Å

(b) $\text{EOT} = 16.7$ Å.