PN-JUNCTION

1 The PN-junction in Equilibrium

2 The I–V Characteristics of the PN-Junction

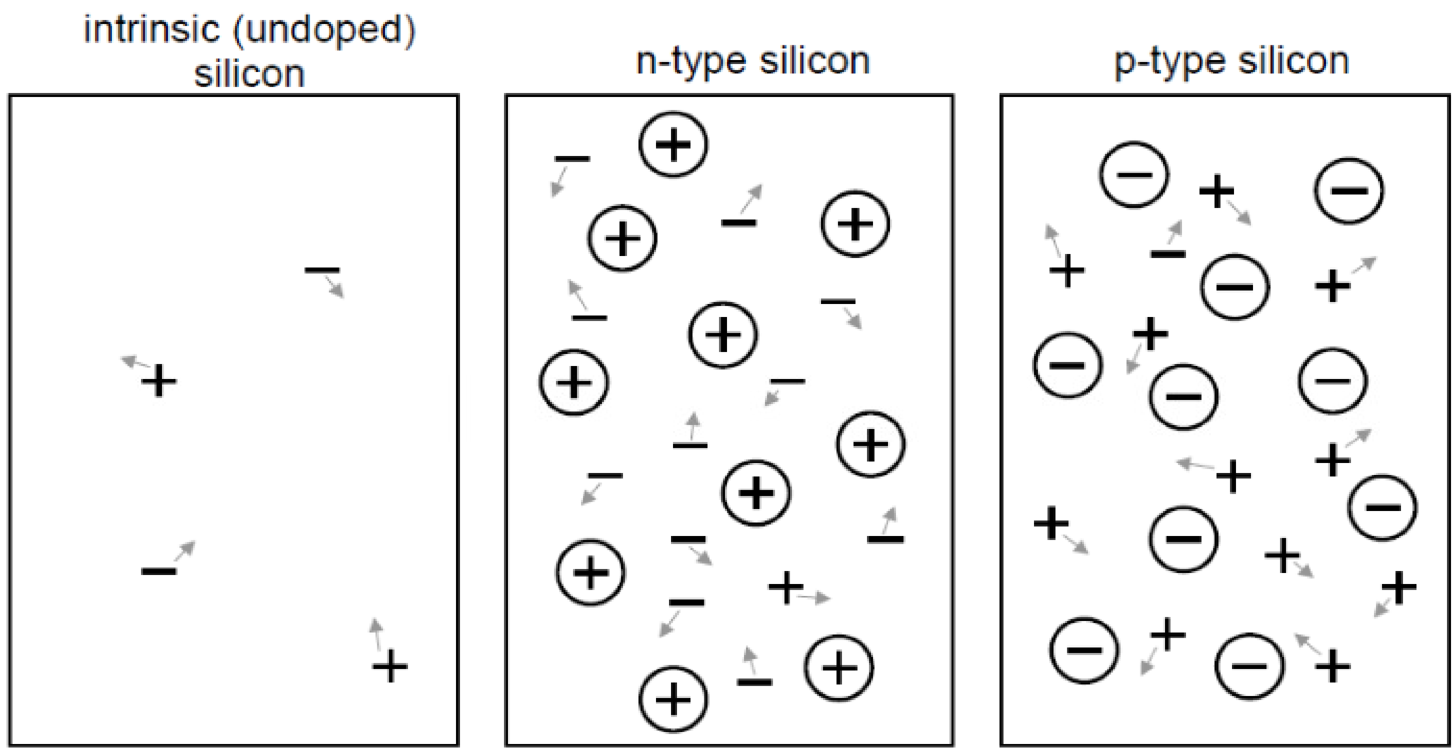
3 Deviations from the Ideal Diode

⊕ donor atom (positive ion)

⊖ acceptor atom (negative ion)

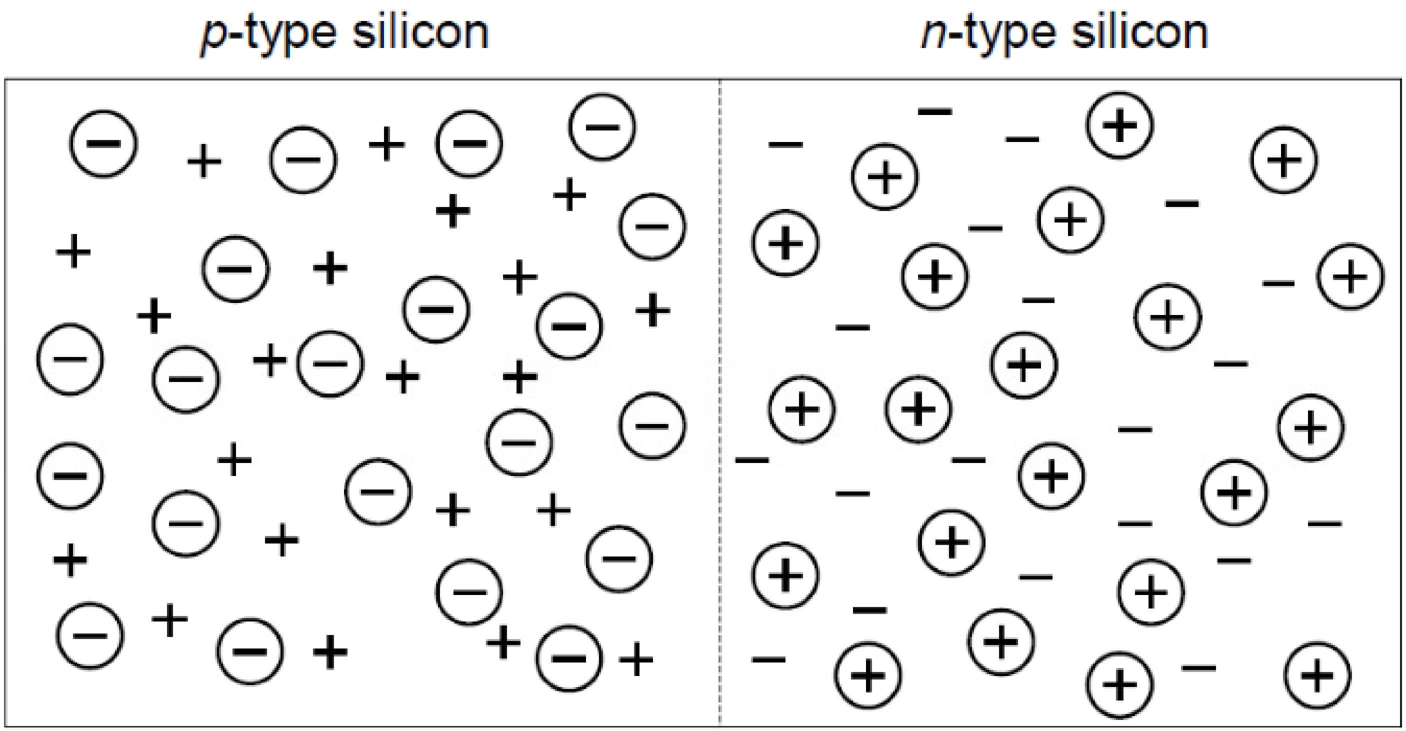
+ hole

– (free) electron

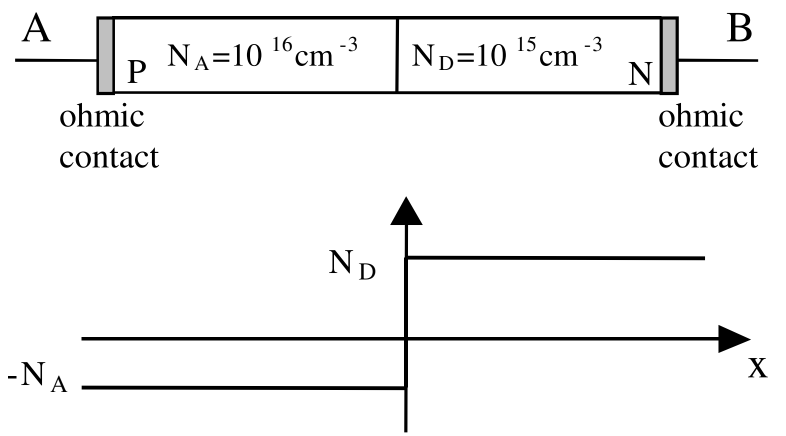


Notice that each piece of silicon is electrostatically **neutral** on the macroscopic level; there are equal numbers of positive and negative charges.

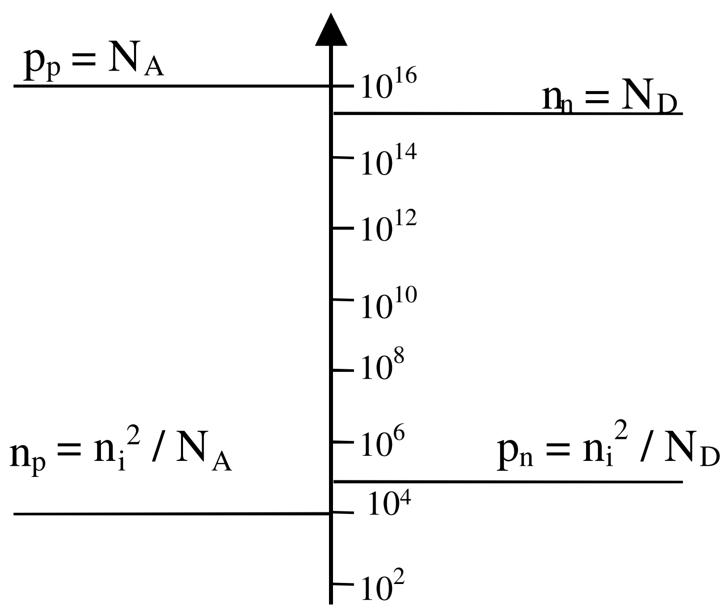
PN-junction



"Contactless" pn-junction



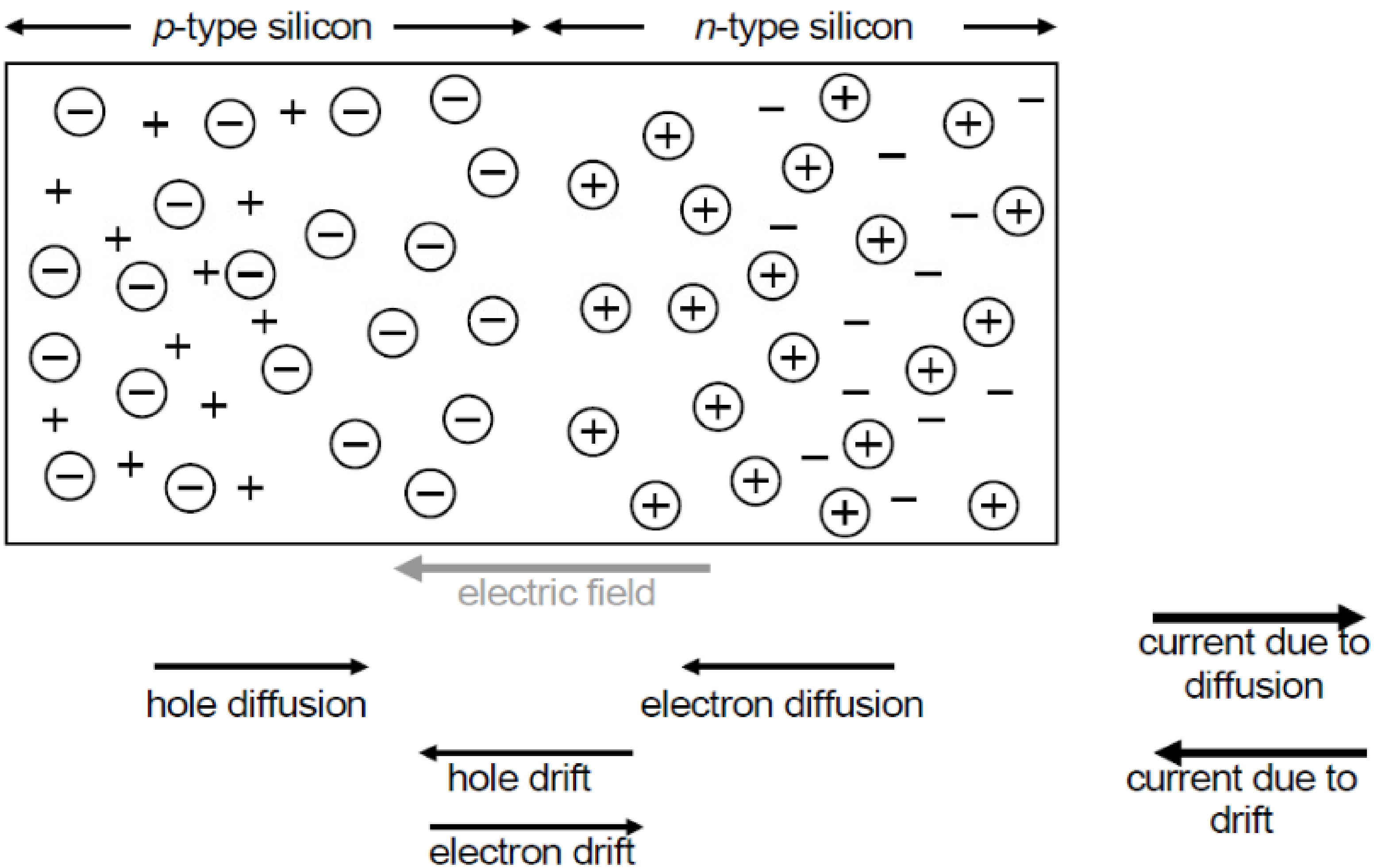
(a) Step junction



(b) "initial" carrier concentrations

$n_i=10^{10}cm^{-3}$

"Before contacting", the two regions are electrically neutral ($\rho = 0$).

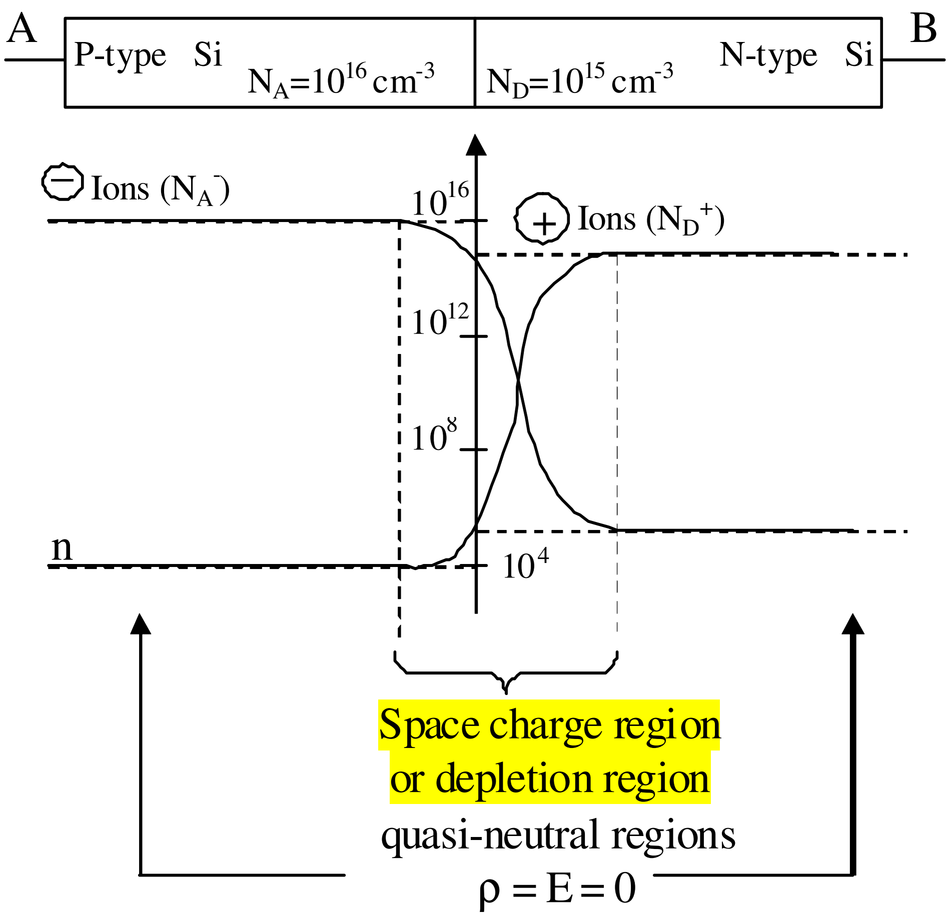


With no external voltage applied to the *p-n* junction, the diffusion and drift currents balance exactly, and there is no net current flow.

http://www.pveducation.org/pvcdrom/pn-junction/formation-pn-junction

pn-junction in thermal equilibrium

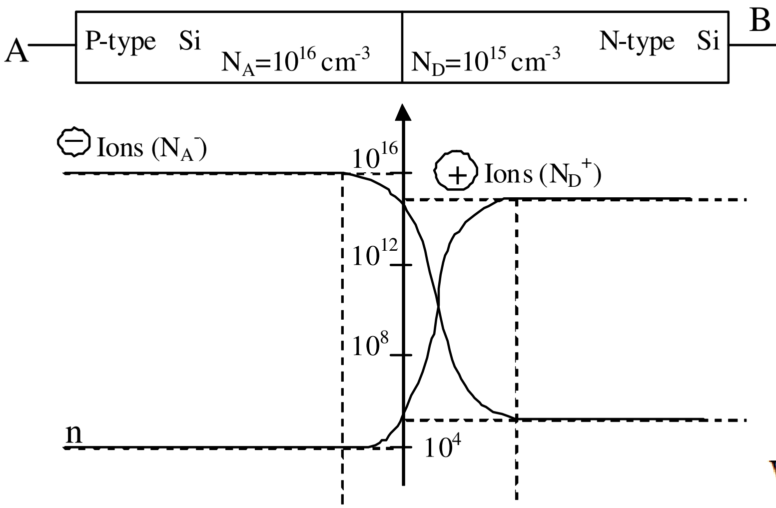
$V_{AB} = 0$

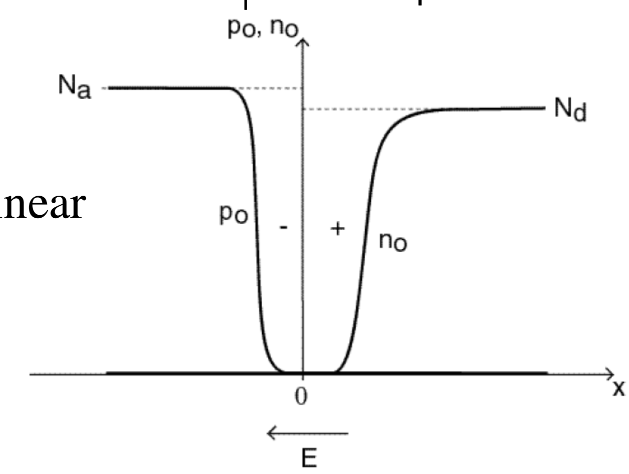


* electrons (holes) diffuse from the n (p) side to the p (n) side, leaving behind N_D^+ (N_A^-) ionized donor (acceptor) atoms and, consequently, a net charge density $\rho \neq 0$, which gives rise to an electric field $\neq 0$

* $pn = n_i^2$ because $V_{AB} = 0$

pn-junction in thermal equilibrium





On a linear scale

We can divide semiconductor into three regions

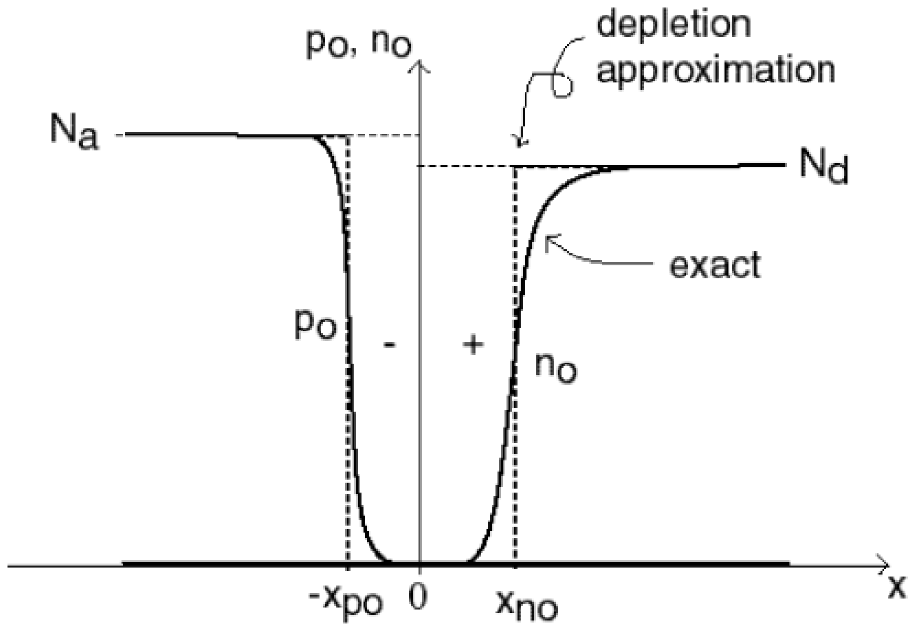
- Two quasi-neutral n- and p-regions (QNR's)
- One space-charge region (SCR)

Now, we want to know $n_o(x)$, $p_o(x)$, $\rho(x)$, $E(x)$ and $\phi(x)$.

We need to solve Poisson's equation using a simple but powerful approximation

3. The Depletion Approximation

- Assume the QNR's are perfectly **charge neutral**
- Assume the SCR is **depleted** of carriers
  - *depletion region*
- Transition between SCR and QNR's sharp at
  - $-x_{po}$ and x_{no} (**must calculate where to place these**)



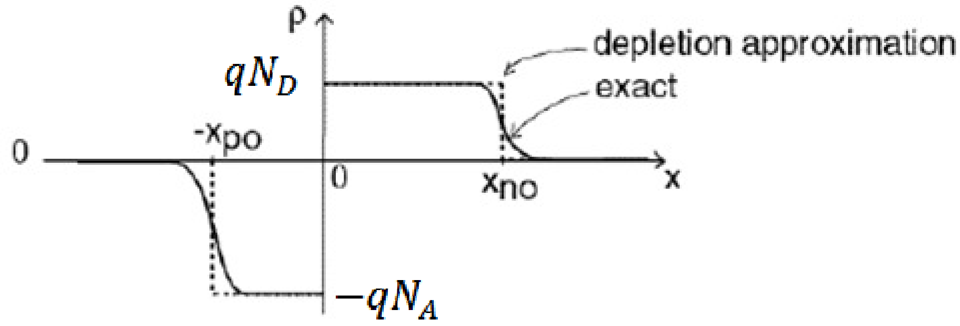
$x < -x_{po}$; $\quad p_o(x) = N_A, \quad n_o(x) = \frac{n_i^2}{N_A}$

$-x_{po} < x < 0$; $\quad p_o(x), \quad n_o(x) \ll N_A$

$0 < x < x_{no}$; $\quad n_o(x), \quad p_o(x) \ll N_D$

$x > x_{no}$; $\quad n_o(x) = N_D, \quad P_o(x) = \frac{n_i^2}{N_D}$

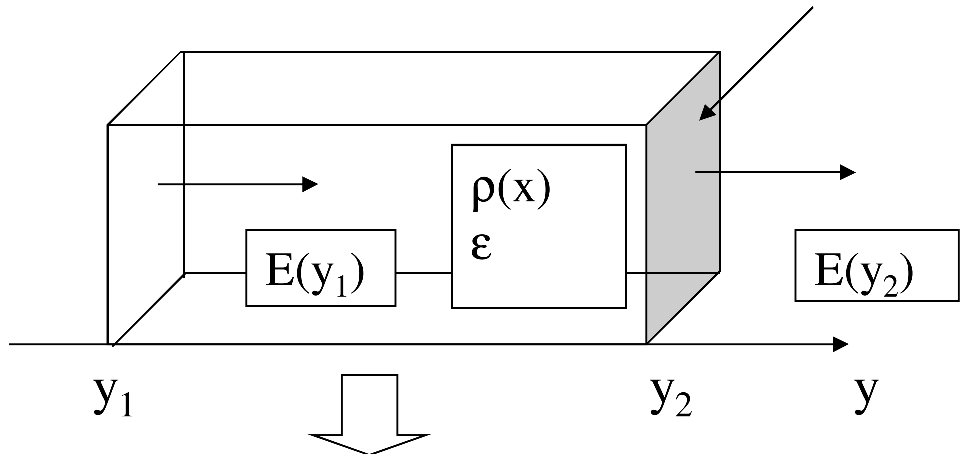
Space Charge Density



$$\begin{aligned} \rho(x) &= 0; && x < -x_{po} \\ &= -qN_A\ ; && -x_{po} < x < 0 \\ &= \ qN_D\ ; && 0 < x < x_{no} \\ &= 0; && x > x_{no} \end{aligned}$$

A review on Poisson's equation

ρ - charge density   ε - permittivity   A - area



$$A\ \varepsilon E(y_2) - A\ \varepsilon E(y_1) = A\int_{y_1}^{y_2} \rho(y)\,dy$$

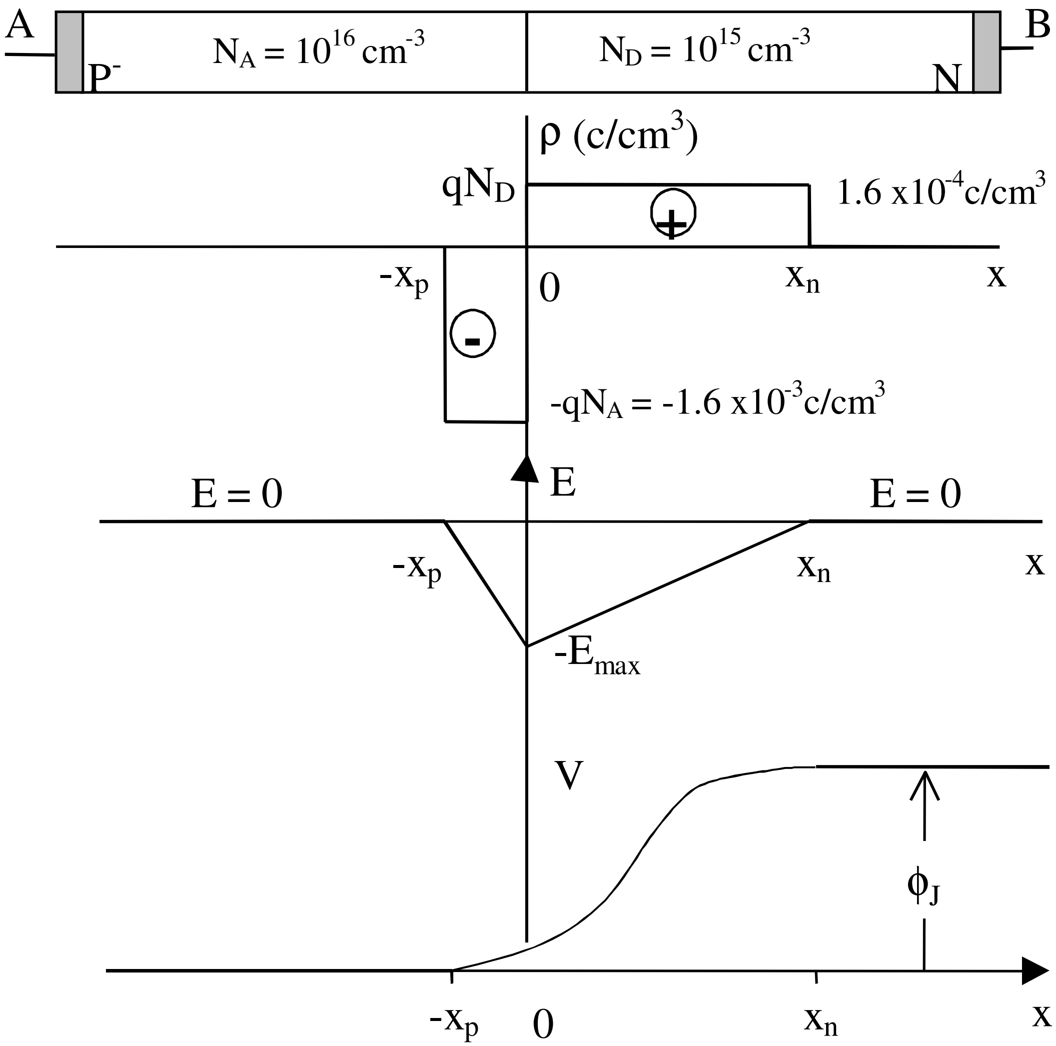
$$\frac{dV}{dy} = -E(y)$$

$$-\frac{d^2V}{dy^2} = \frac{dE}{dy} = \frac{\rho}{\varepsilon}$$

Gauss' law: $\oiint \vec{D}\, d\vec{s} = Q$

In one dimension:
Poisson's equation relates the potential V to the charge density ρ

pn-junction in thermal equilibrium



$$\frac{dE}{dx} = \rho / \varepsilon_{Si}$$

$E = 0 \quad x_n \leq x; \quad x \leq -x_p$

$E = -qN_A / \varepsilon_{Si} \; (x + x_p)$

$E = -qN_D / \varepsilon_{Si} \; (x_n - x)$

Continuity of electrical field

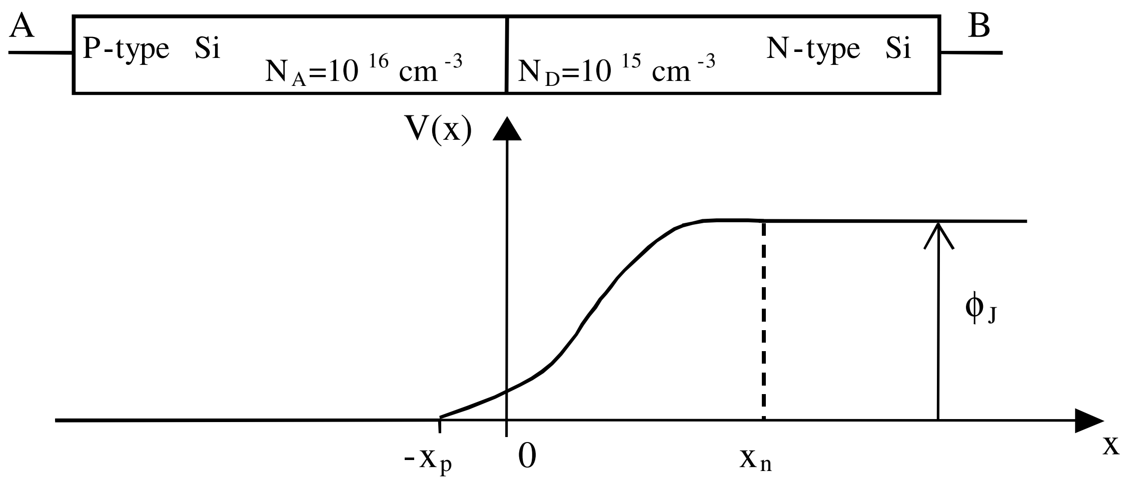
$N_a x_p = N_D x_n$

$$E = -\frac{dV}{dx}$$

$V = 0$ at $x = -x_p$

$$V = \begin{cases} 0 \; ; \text{ for } \; x \leq -x_p \\ \dfrac{qN_A}{2\varepsilon_{Si}}\left(x + x_p\right)^2 \\ \phi_J - \dfrac{qN_D}{2\varepsilon_{Si}}\left(x - x_n\right)^2 \\ \phi_J; \text{ for } \; x_n \leq x \end{cases}$$

The Built-in Voltage



$$J_n = qn\mu_n E + qD_n \frac{dn}{dx} = 0 \rightarrow E = -\frac{D_n}{\mu_n}\frac{d(\ln n)}{dx}$$

Example: $n_i=10^{10}$cm^{-3}

$N_A=10^{16}$cm^{-3} , $N_D=10^{15}$cm^{-3}

$$\frac{D_n}{\mu_n} = \frac{kT}{q} = \phi_t \qquad -\int_a^b E\,dx = V_b - V_a = \phi_t \ln\left(n_b / n_a\right)$$

$$\phi_J = \phi_t \ln\left(\frac{10^{31}}{10^{20}}\right)$$

V = 0 and $n = n_i^2 / N_A$; for $x < -x_p$

$$\phi_J = \phi_t \ln\left(\frac{n(x > x_n)}{n(x < -x_p)}\right)$$

$$\phi_J = \phi_t \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$\boldsymbol{\phi_J = 632 \text{ mV}} \quad (\phi_t = 25\,mV)$$

Depletion-layer width and maximum electric field

$$V_{(x=0)} = \frac{q N_A}{2\varepsilon_{SI}} x_p^2 = \phi_J - \frac{q N_D x_n^2}{2\varepsilon_{Si}}$$

$$N_A x_p = N_D x_n$$

$$X_{do} = x_n + x_p$$

$$X_{do} = \sqrt{\frac{2\varepsilon_{Si}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_J}$$

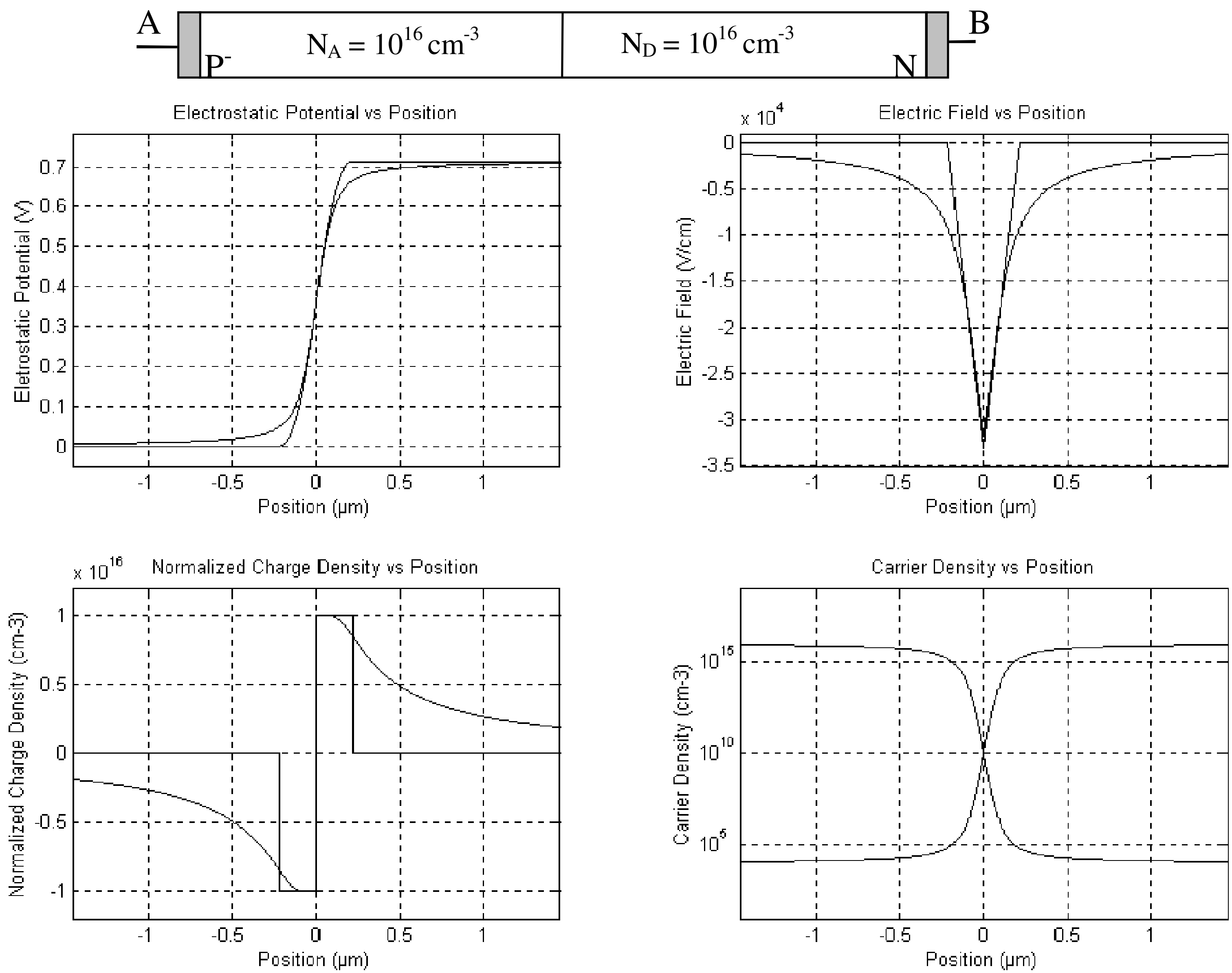
equilibrium

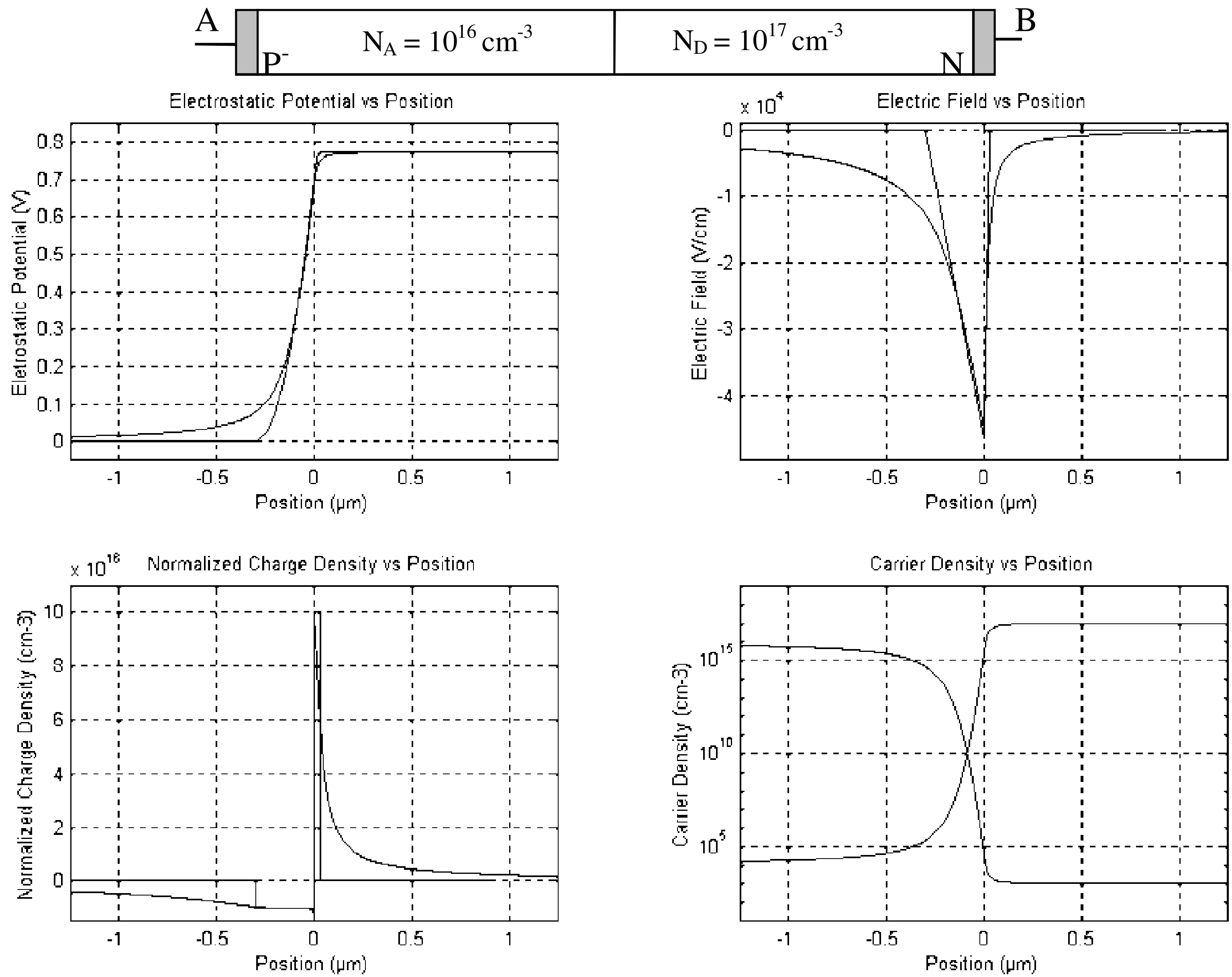
$$x_n = \frac{X_{do}}{1 + \frac{N_D}{N_A}} \qquad x_p = \frac{X_{do}}{1 + \frac{N_A}{N_D}}$$

$\varepsilon_{Si} = 1.04 \times 10^{-12}$ F/cm
$N_A = 10^{16} \text{cm}^{-3}$
$N_D = 10^{15} \text{cm}^{-3}$

$X_{do} = 0.951\ \mu\text{m}$
$x_n = 0.864\ \mu\text{m}$
$x_p = 0.0864\ \mu\text{m}$

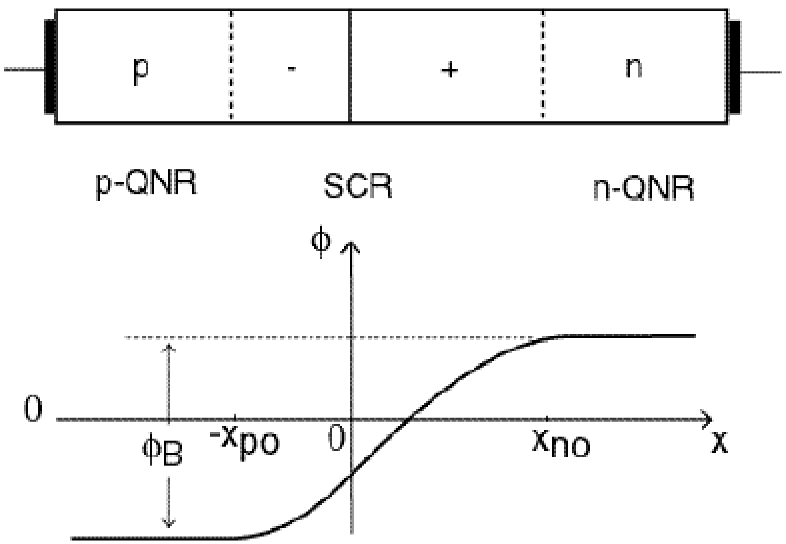
$$E_{max} = \frac{2\phi_J}{X_{do}} \qquad E_{max} = \frac{2 \times 0.632\ \text{V}}{0.951\ \mu\text{m}} \qquad E_{max} = 13.3\ \text{kV/cm}$$


A
B
P-
N
N_A = 10^16 cm^-3
N_D = 10^16 cm^-3
Electrostatic Potential vs Position
Eletrostatic Potential (V)
Position (μm)
Electric Field vs Position
x 10^4
Electric Field (V/cm)
Position (μm)
Normalized Charge Density vs Position
x 10^16
Normalized Charge Density (cm-3)
Position (μm)
Carrier Density vs Position
Carrier Density (cm-3)
Position (μm)


A
B
$N_A = 10^{16}$ cm^{-3}
$N_D = 10^{17}$ cm^{-3}
P$^-$
N
Electrostatic Potential vs Position
Eletrostatic Potential (V)
Position (μm)
Electric Field vs Position
x 10^4
Electric Field (V/cm)
Position (μm)
Normalized Charge Density vs Position
x 10^{16}
Normalized Charge Density (cm-3)
Position (μm)
Carrier Density vs Position
Carrier Density (cm-3)
Position (μm)

4. Contact Potential

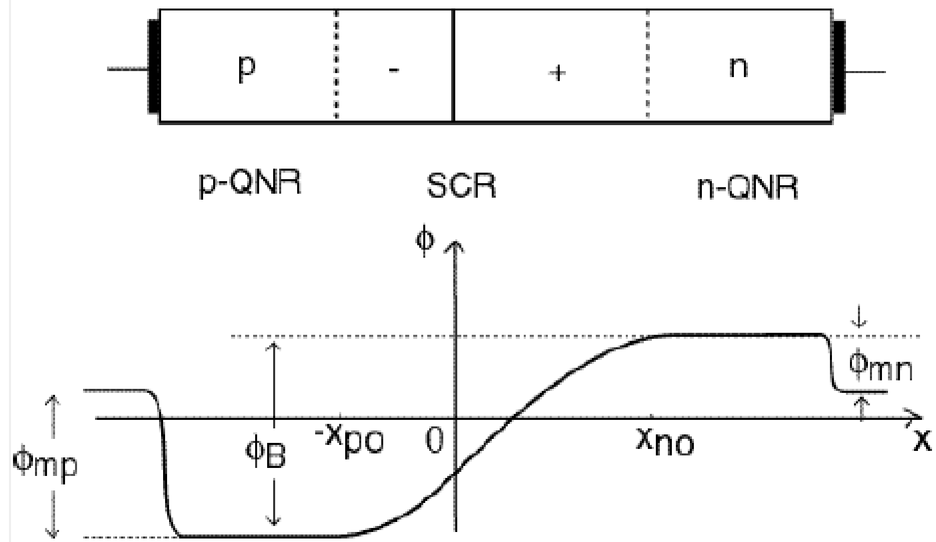
Potential distribution in thermal equilibrium so far:



Question 1: *If I apply a voltmeter across the pn junction diode, do I measure ϕ_B?*

Question 2: *If I short terminals of pn junction diode, does current flow on the outside circuit?*

We are missing *contact potential* at the metal-semiconductor contacts:



Metal-semiconductor contacts: junction of dissimilar materials

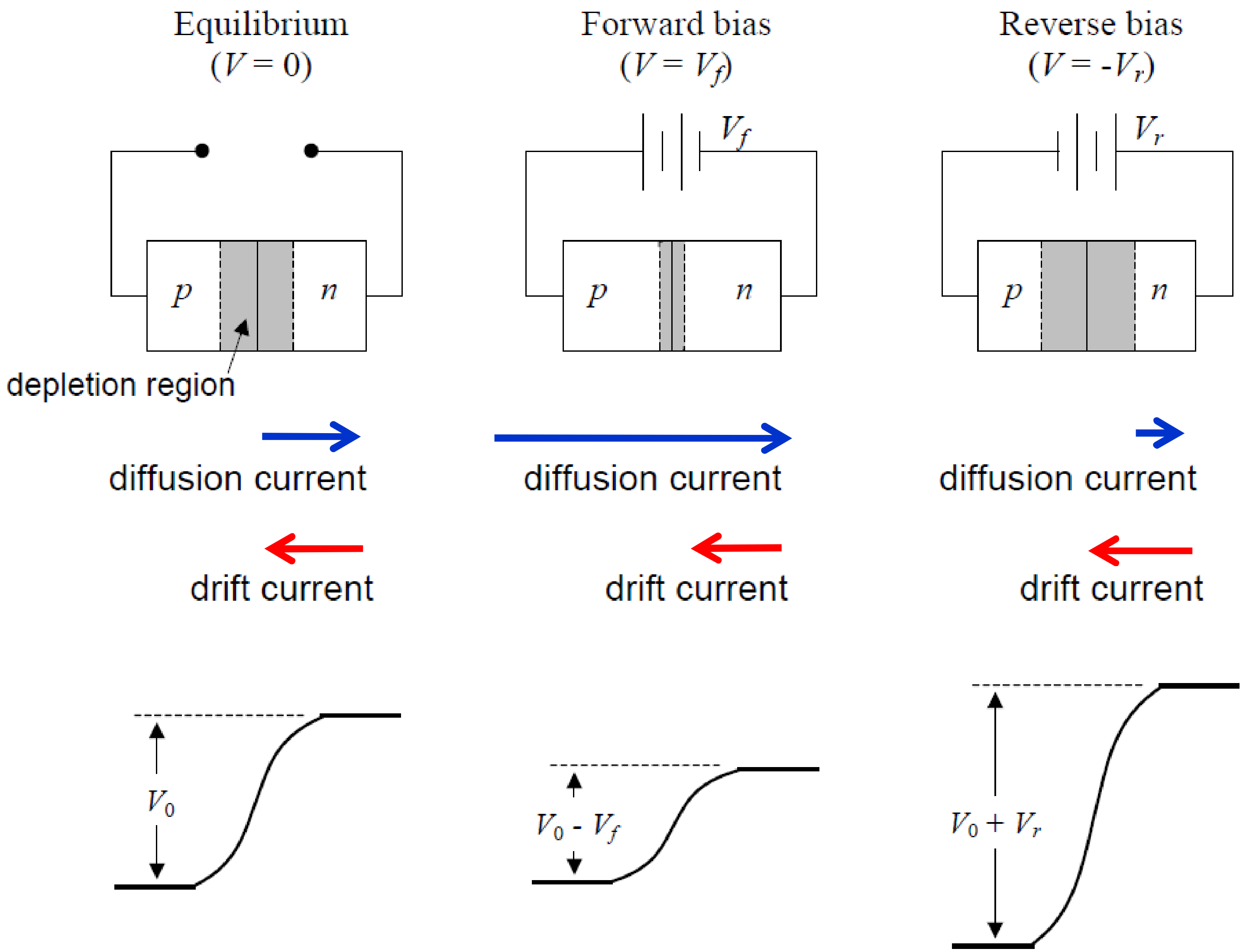
⇒ built-in potentials at contacts ϕ_{mn} and ϕ_{mp}.

Potential difference across structure must be zero

⇒ Cannot measure ϕ_B.

$$\phi_B = |\phi_{mn}| + |\phi_{mp}|$$

Biased pn-junction

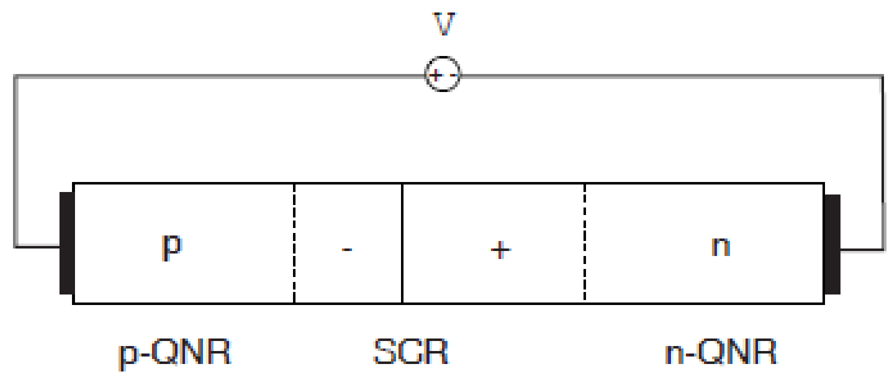


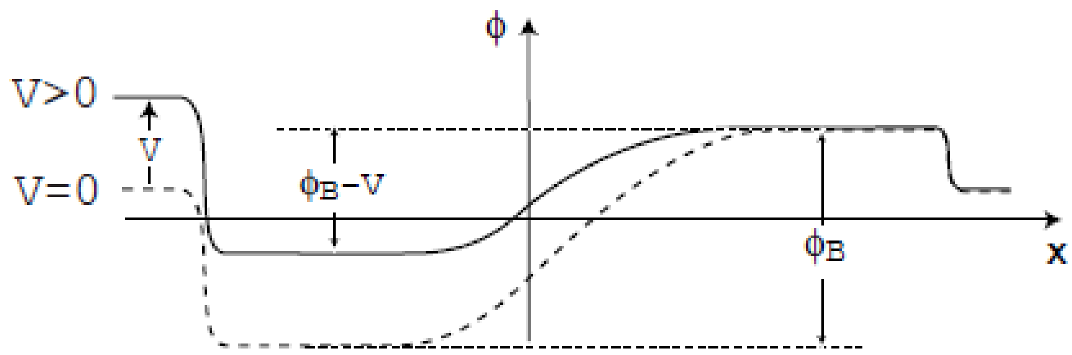
Biased pn-junction

- Voltage drop at the ohmic contacts remain the same;
- Voltage drops across the quasi-neutral regions is zero (not valid for high currents);

→ **All applied voltage drops across the space charge region**

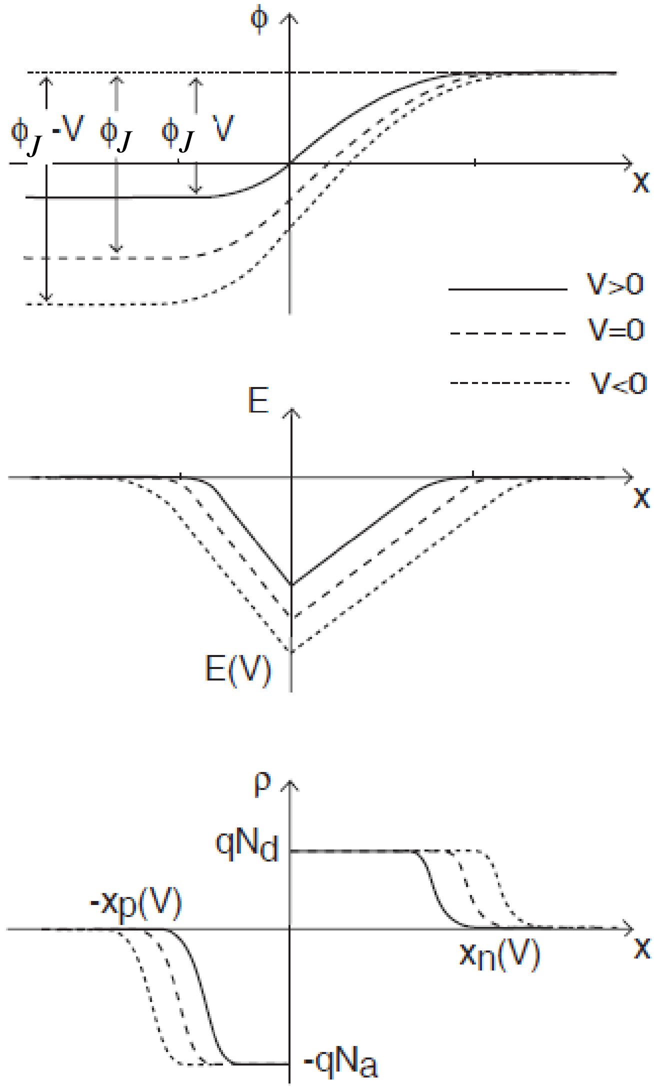
→**Electrostatics of the SCR under bias is unchanged from thermal equilibrium**





Biased pn-junction

Electrostatics of the SCR under bias is unchanged from thermal equilibrium



Depletion approximation

$$x_n(V) = \sqrt{\frac{2\epsilon_s(\phi_J - V)N_a}{q(N_a+N_d)N_d}} \qquad x_n(V) = x_{no}\sqrt{1-\frac{V}{\phi_J}}$$

$$x_p(V) = \sqrt{\frac{2\epsilon_s(\phi_J - V)N_d}{q(N_a+N_d)N_a}} \qquad x_p(V) = x_{po}\sqrt{1-\frac{V}{\phi_J}}$$

$$x_d(V) = \sqrt{\frac{2\epsilon_s(\phi_J - V)(N_a+N_d)}{qN_aN_d}} \qquad x_d(V) = x_{do}\sqrt{1-\frac{V}{\phi_J}}$$

$$|E|(V) = \sqrt{\frac{2q(\phi_J - V)N_aN_d}{\epsilon_s(N_a+N_d)}} \qquad |E|(V) = |E_o|\sqrt{1-\frac{V}{\phi_J}}$$

PN-Junction in Thermal Equilibrium

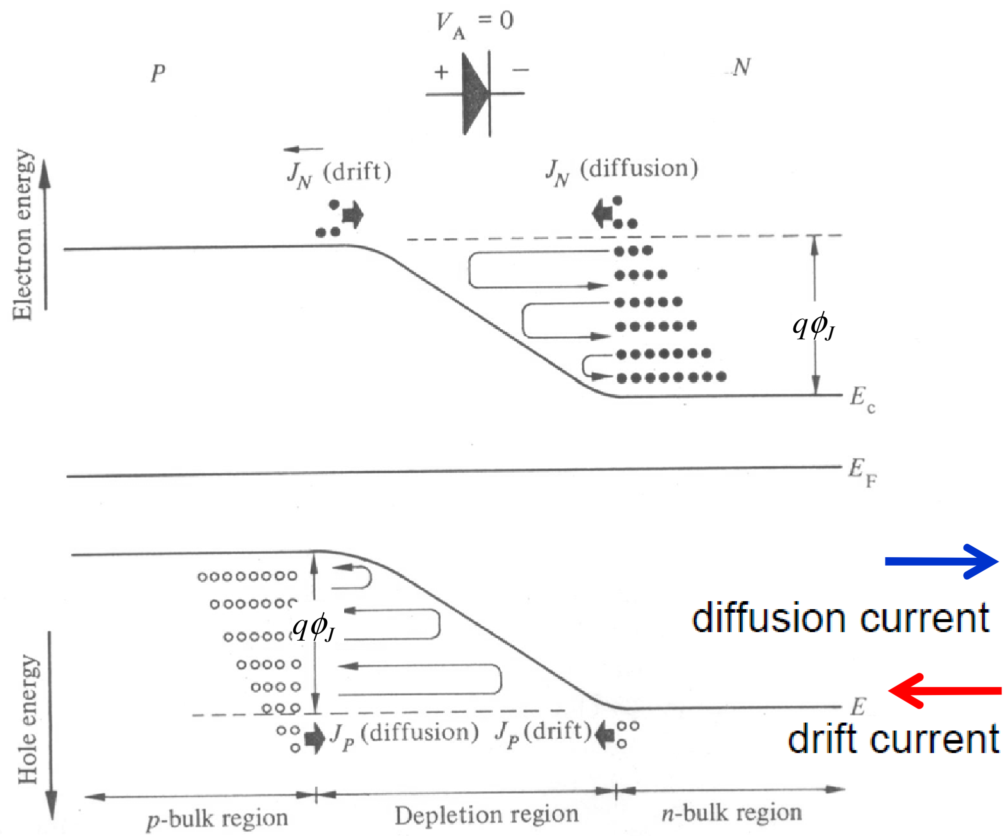


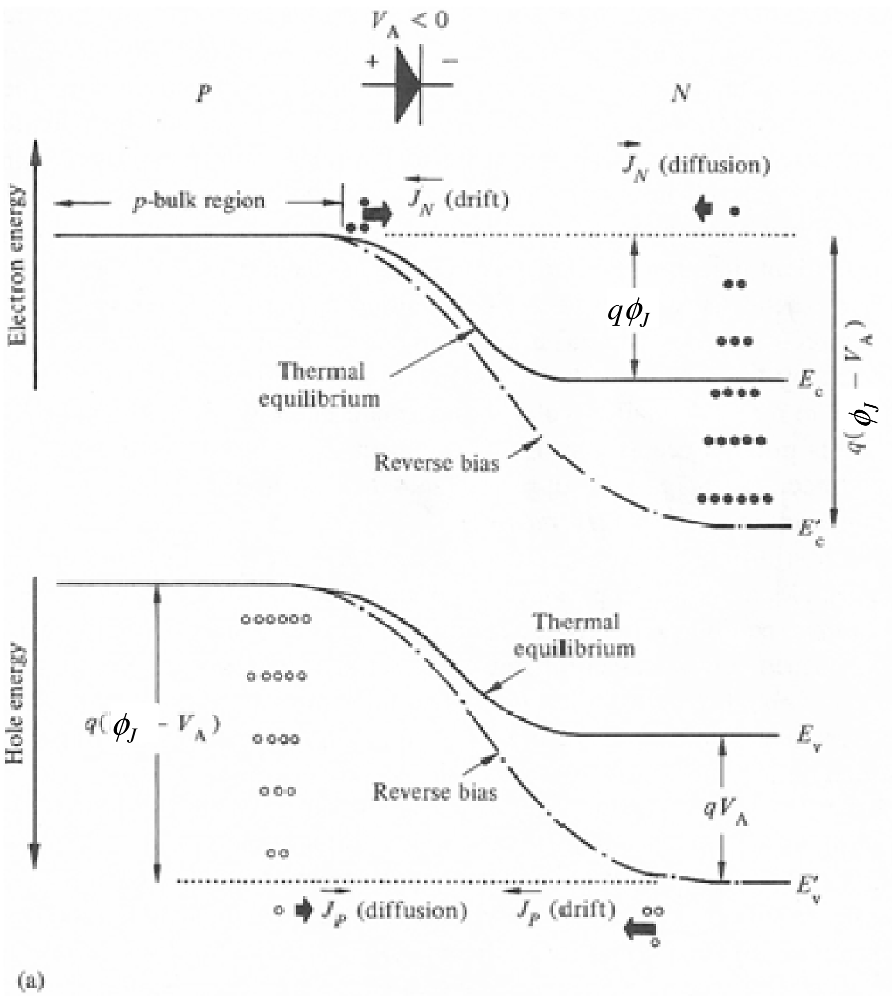
Fig. 3.1 Thermal equilibrium: energy band diagram and carrier flux.

$$J_{n\ diff.} + J_{n\ drift} = 0$$

$$J_{p\ diff.} + J_{p\ drift} = 0$$

Thermal equilibrium: energy band diagram and carrier flux

PN-Junction Under Reverse Bias



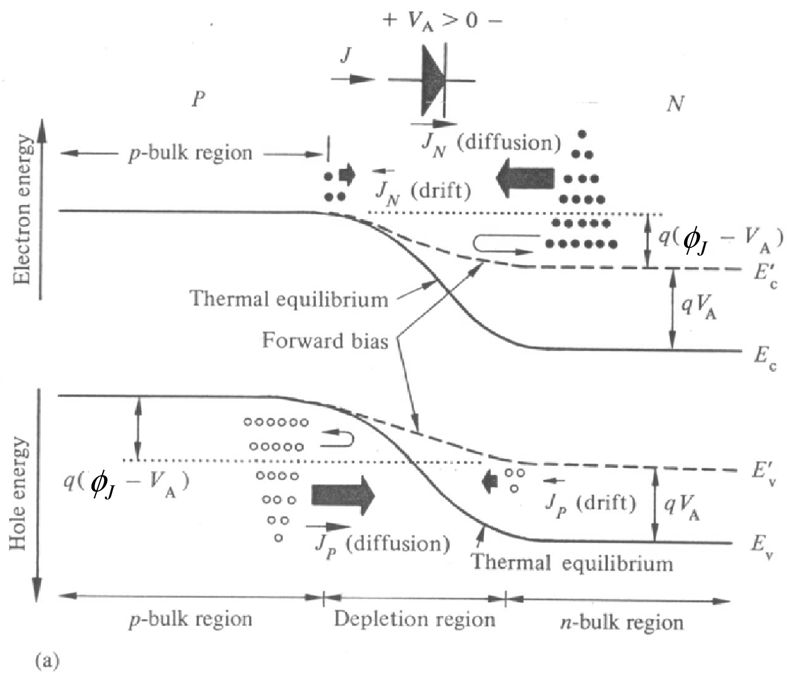
Energy band diagram for reverse bias (– · – · –)
and at thermal equilibrium (______)

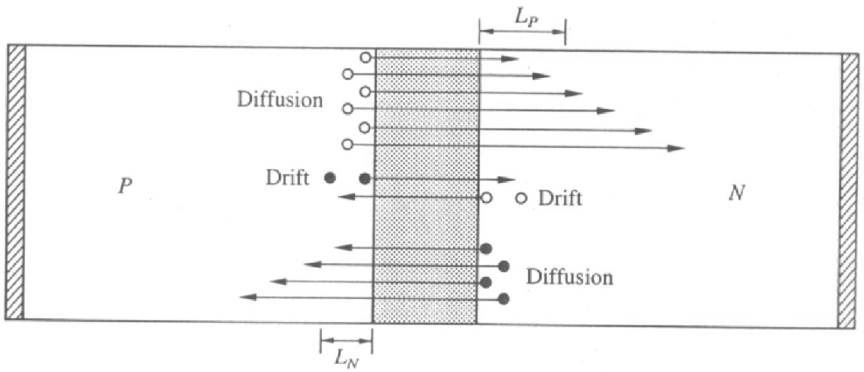
$$J_n = J_{n\ diff.} + J_{n\ drift} \approx J_{n\ drift}$$
$$J_p = J_{p\ diff.} + J_{p\ drift} \approx J_{p\ drift}$$

→ diffusion current

← drift current

PN-Junction Under Forward Bias





Energy band diagram for forward bias (– · – · –) and at thermal equilibrium (______)

$$J_n = J_n\,diff. + J_n\,drift \approx J_n\,diff.$$
$$J_p = J_p\,diff. + J_p\,drift \approx J_p\,diff.$$

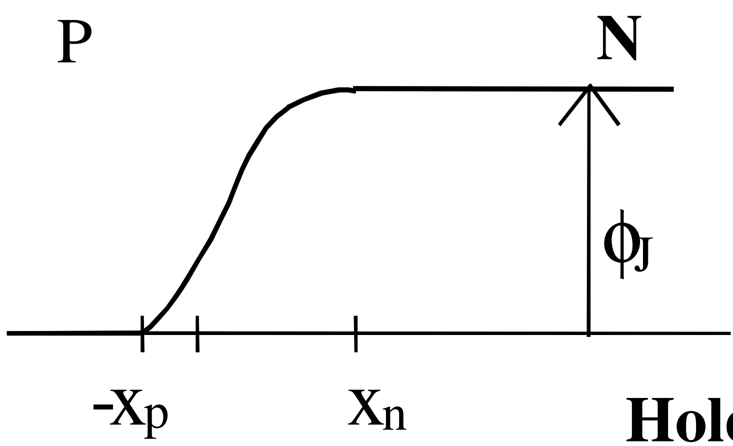
$$J = J_n + J_p$$

diffusion current

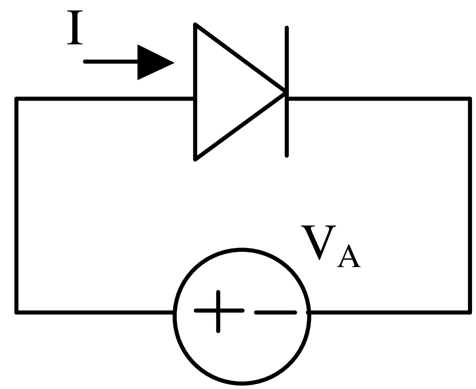
drift current

$V_A > 0$

The voltage drop across the (quasi)-neutral regions is ≈ zero for low-level injection



Equilibrium
(V_A=0)



Hole current

(same reasoning for electrons)

A few holes on the P-side approach the barrier with enough energy to carry over it and reach the N-side, where they recombine

I_{UP} is balanced by a continual generation of pairs by thermal fluctuations near the junction on the N-side and some of the holes produced fall down the energy gradient into the P-side giving a current I_{DO}

Boltzmann

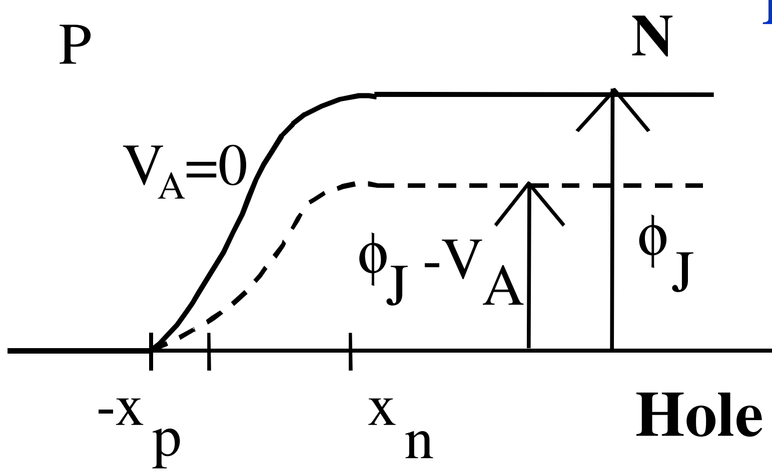
I_{UP} – current uphill $\alpha \ N_A \exp(-\phi_J / \phi_t)$

I_{DO} – current downhill α p (N-side)

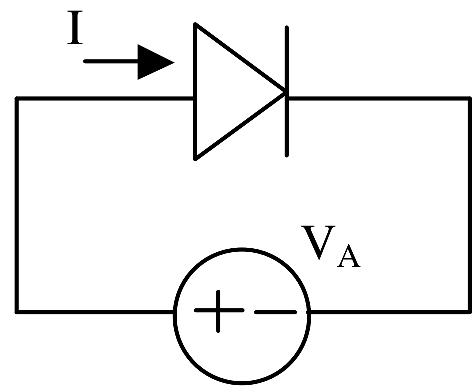
Equilibrium: $I_{UP} = I_{DO} = I_S \ \alpha \ N_A \exp(-\phi_J / \phi_t)$

First order model: I_{DO} is independent of V_A – the rate of thermal generation of pairs will not change for $V_A \neq 0$ since it depends only on local properties of the crystal near the junction.

M. Born, Atomic Physics, Dover, p. 305
R. Feynman et al., The Feynman Lectures on Physics, Addison Wesley, vol. 3, p. 14.8.



Forward bias
(V_A>0)



Hole current (same reasoning for electrons)

Boltzmann

I_{UP} – current uphill α $N_A \exp(V_A - \phi_J)/\ \phi_t$)

$I_{DO} = I_S$

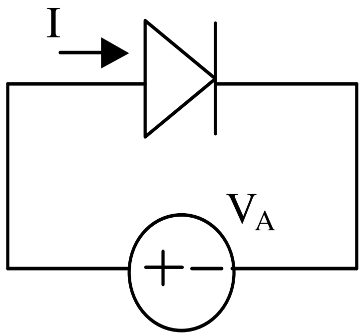
$I_{UP} = I_S \exp\ (V_A\ /\ \phi_t)$

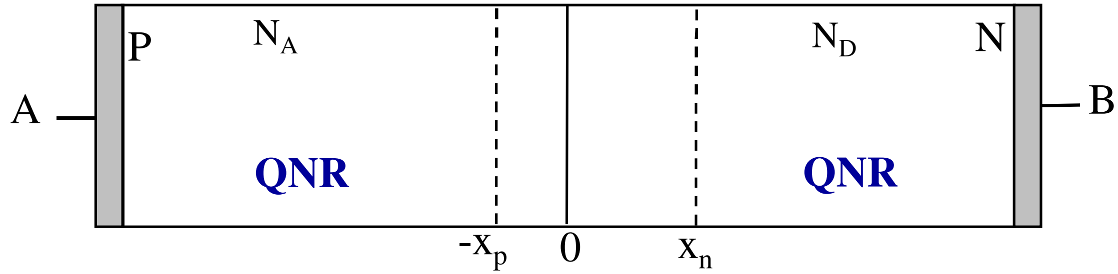
$$I_D = I_{UP} - I_{DO} = I_S\ [\exp\ (V_A\ /\ \phi_t)\ -1]$$

First order model is independent of V_A – the rate of thermal generation of pairs will not change for $V_A \neq 0$ since it depends only on local properties of the crystal near the junction.

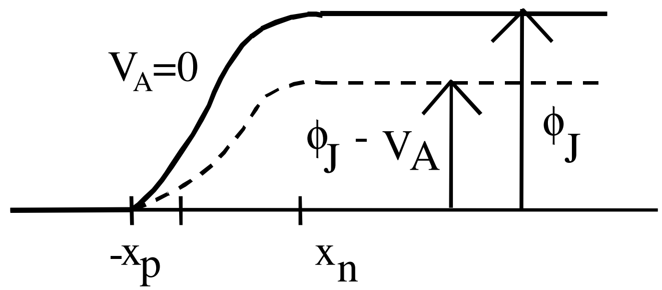
M. Born, Atomic Physics, Dover, p. 305
R. Feynman et al., The Feynman Lectures on Physics, Addison Wesley, vol. 3, p. 14.8.

Development of analytical dc model (I-V characteristics) of the diode



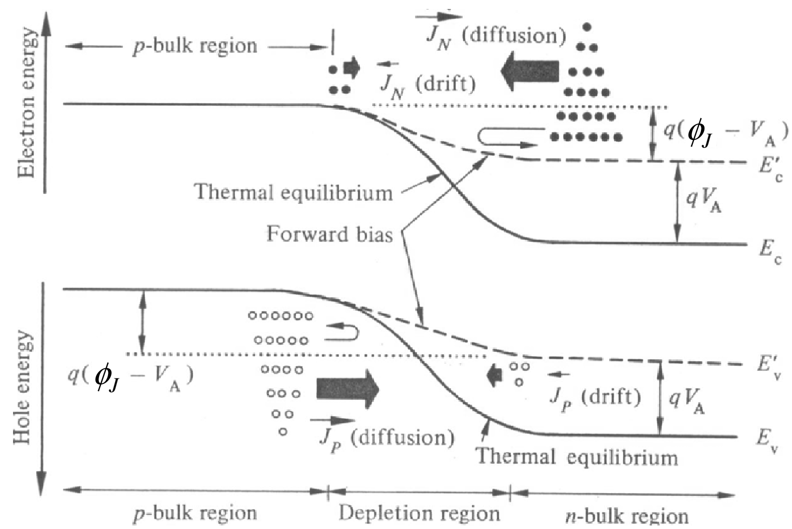


At the edges of the depletion region, $-x_p$ and x_n, equilibrium conditions do not prevail so we must use the "**law of the junction**".

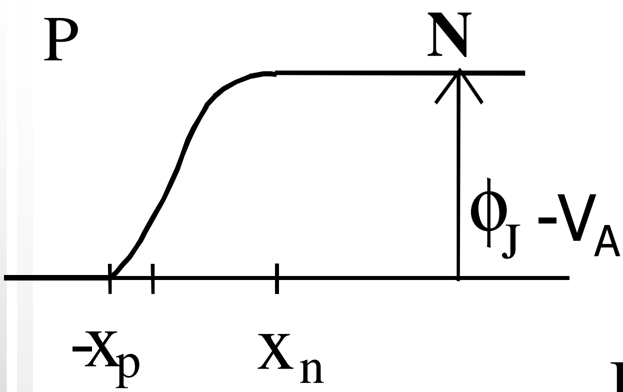


$$\frac{n(-x_p)}{n(x_n)} = \exp\frac{q\left[\phi(-x_p)-\phi(x_n)\right]}{kT} = \exp\left[\frac{V_A-\phi_J}{\phi_t}\right]$$

$$\frac{p(-x_p)}{p(x_n)} = \exp-\frac{q\left[\phi(-x_p)-\phi(x_n)\right]}{kT} = \exp\left[\frac{-(V_A-\phi_J)}{\phi_t}\right]$$



Law of the junction



$$\frac{n(-x_p)}{n(x_n)} = \exp\left[\frac{V_A - \phi_J}{\phi_t}\right] \qquad \textbf{(I)}$$

$$\frac{p(-x_p)}{p(x_n)} = \exp\left[\frac{-(V_A - \phi_J)}{\phi_t}\right] \qquad \textbf{(II)}$$

Boundary conditions:

1. Neutrality at $-x_p$ and x_n (limits of QNR)

$\rightarrow p(-x_p)=n(-x_p)+N_A \qquad n(x_n) = N_D+p(x_n)$ **(III);** ➡ $n(x_n) \cong N_D$; $p(-x_p) \cong N_A$

2. Low-level injection $p(-x_p) >> n(-x_p)$ & $n(x_n) >> p(x_n)$ **(IV)**

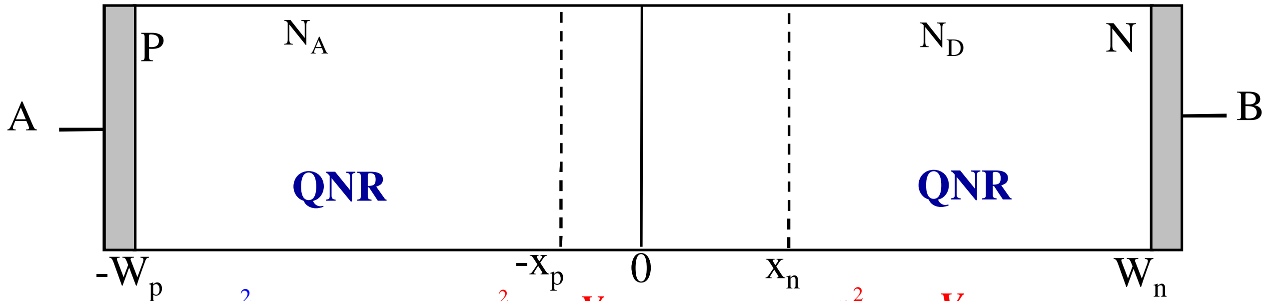
3. Recalling that $\exp\frac{\phi_J}{\phi_t} = \frac{N_A N_D}{n_i^2}$ we find that

Law of the junction ➡

$$n(-x_p) = \frac{n_i^2}{N_A}\exp\frac{V_A}{\phi_t}$$

$$p(x_n) = \frac{n_i^2}{N_D}\exp\frac{V_A}{\phi_t}$$

Boundary conditions for minority carriers



$$n(-W_p) = \frac{n_i^2}{N_A} \qquad n(-x_p) = \frac{n_i^2}{N_A}\exp\frac{V_A}{\phi_t} \qquad p(x_n) = \frac{n_i^2}{N_D}\exp\frac{V_A}{\phi_t} \qquad p(W_n) = \frac{n_i^2}{N_D}$$

Law of the junction

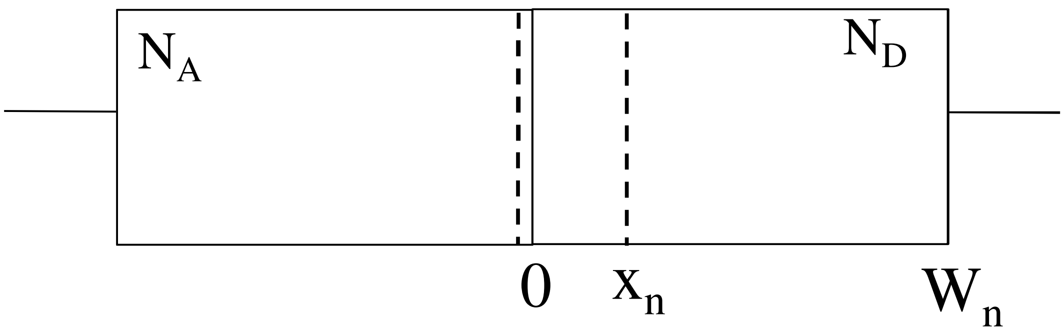
Ohmic contacts: Thermal equilibrium

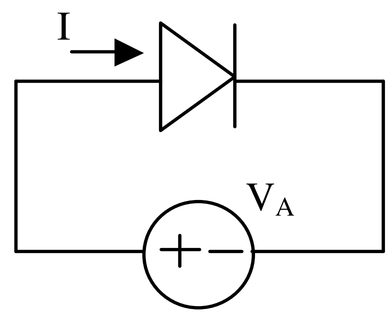
$$n(x_n) \cong N_D;$$

$$p(-x_p) \cong N_A$$

Calculate the current-voltage characteristic of a "short" P+N junction diode

1. Holes are the main carriers;
2. Recombination is negligible in the N region;
3. Diffusion current is dominant





$$p(x_n) = \frac{n_i^2}{N_D} \exp \frac{V_A}{\phi_t}$$

Law of the junction

$$p(W_n) = \frac{n_i^2}{N_D}$$

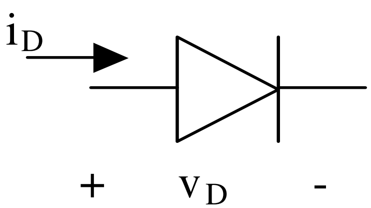
Ohmic contact

$$J_p = -qD_p \frac{dp}{dx} = -qD_p \frac{p(W_n) - p(x_n)}{W_n - x_n}$$

$$I_p = AJ_p = qAD_p \frac{n_i^2}{(W_n - x_n) N_D} \left[\exp \frac{V_A}{\phi_t} - 1 \right]$$

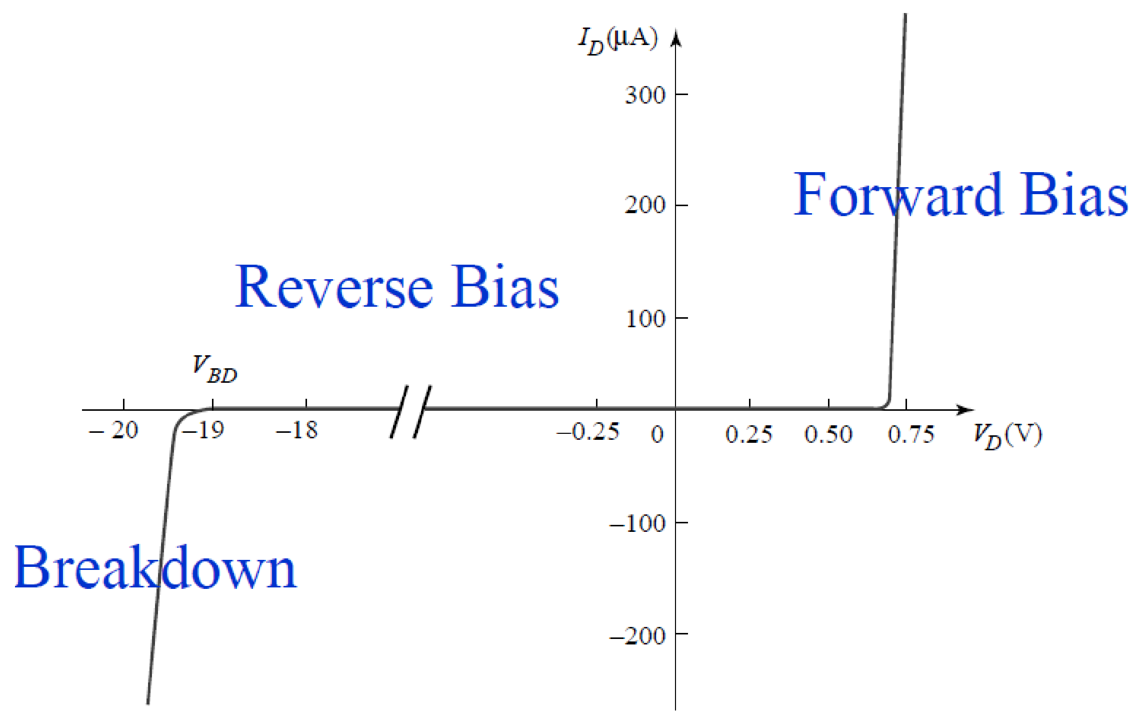
$$I \cong I_p = I_S \left[e^{\frac{V_A}{\phi_t}} - 1 \right];$$

$$I_S \cong qAD_p \frac{n_i^2}{W_n N_D}$$

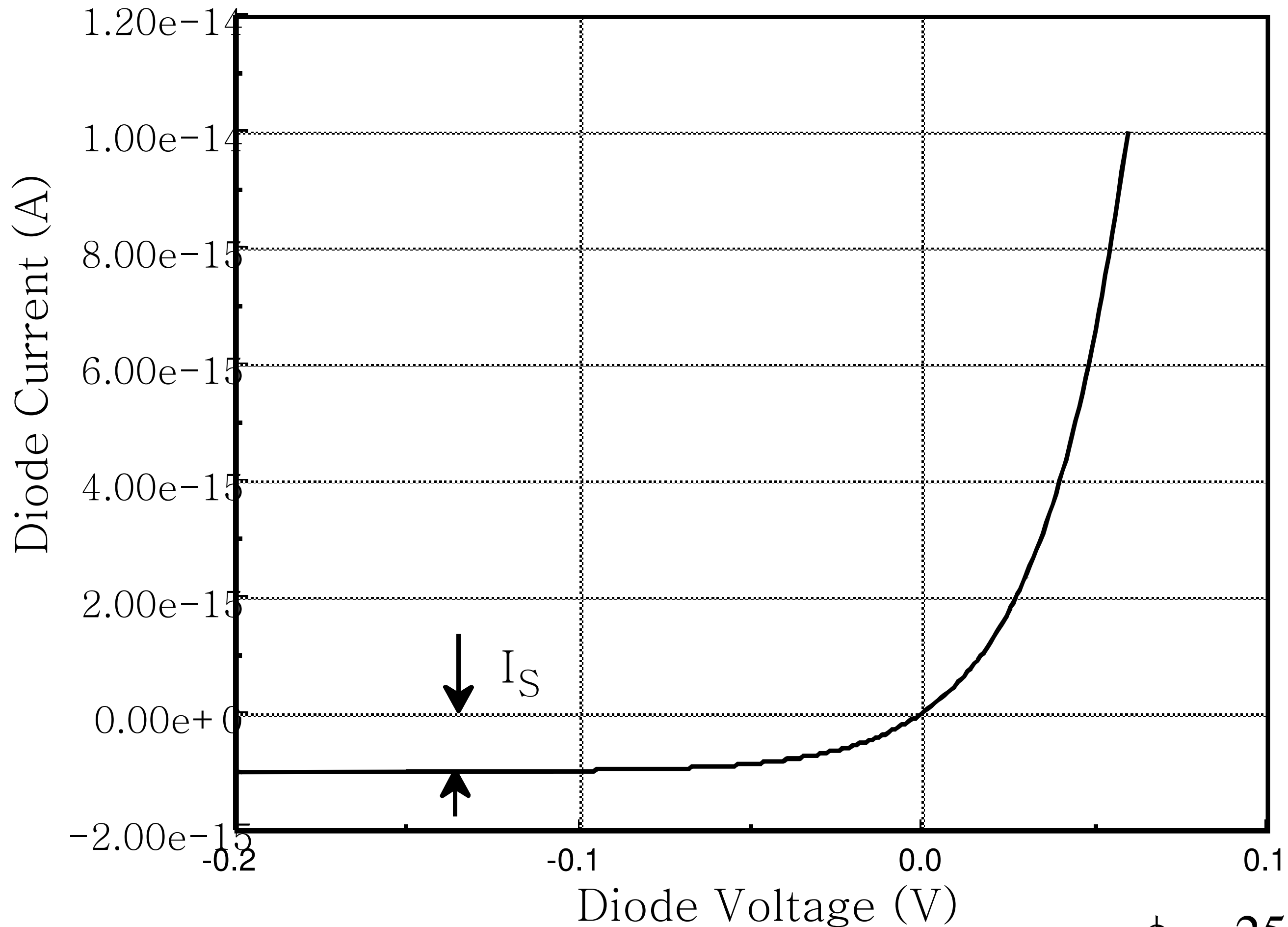


$$i_D = I_S\left[\exp\left(\frac{v_D}{n\phi_t}\right)-1\right] \qquad \phi_t = kT/q$$

I-V Characteristics

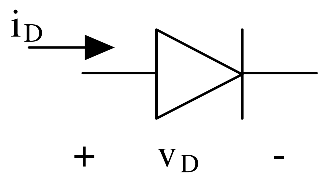


(a)

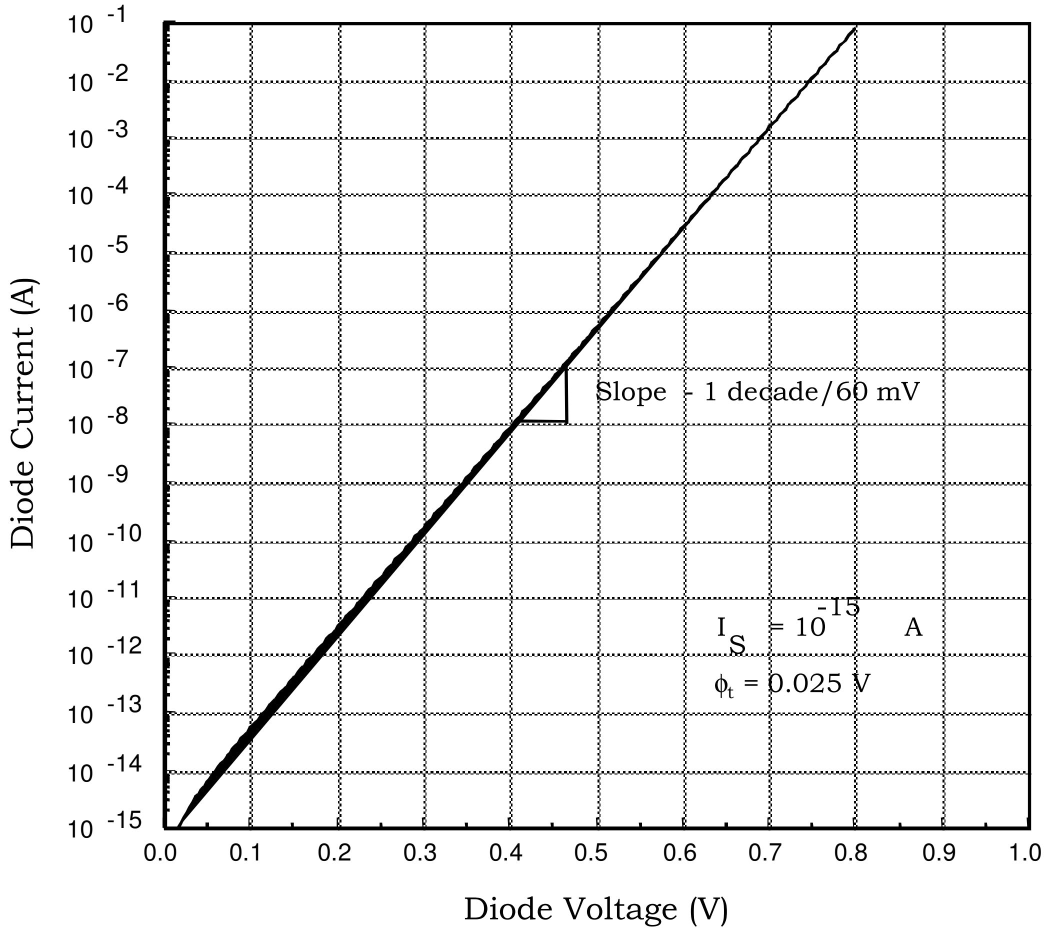


$\phi_t = 25$ mV

Diode behavior near the origin with $I_S=10^{-15}$ A and n=1



$$i_D = I_S\left[\exp\left(\frac{v_D}{n\phi_t}\right) - 1\right]$$



Diode i -v characteristic on semilog scale

I_S : saturation current
n : ideality factor (1 to 2)
ϕ_t : kT/q
k : 1.38 x10^{-23} J/K
Typical values of I_S:
10^{-18} A $\le I_S \le 10^{-9}$ A

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