

The currents in the branches of the graph are as follows:

$$\begin{bmatrix} y_1 & 0 & 0 & 0 \\ 0 & y_2 & 0 & 0 \\ 0 & 0 & y_3 & 0 \\ ay_1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

Doing it the standard way you pre- and post- multiply by the branch incident matrix to yield Kirchoff current node equations.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_1 & 0 & 0 & 0 \\ 0 & y_2 & 0 & 0 \\ 0 & 0 & y_3 & 0 \\ ay_1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} y_1 & 0 & 0 & 0 \\ -y_1 + ay_1 & y_2 & y_3 & 0 \\ -ay_1 & 0 & -y_3 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} y_1 & -y_1 & 0 \\ -y_1 + ay_1 & y_1 + y_2 + y_3 - ay_1 & -y_2 \\ -ay_1 & -y_3 + ay_1 & y_3 \end{bmatrix}$$

We need a diagonal matrix to use the topological method. Let's use the unused column to hold the extra terms from the current controlled current generator. This yields a diagonal matrix admittance matrix. Note the slide of the terms in the incident matrix to the left in the lowest row in the post multiply. This puts the column 4 terms back where they belong in the final resulting matrix.

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_1 & 0 & 0 & 0 \\ 0 & y_2 & 0 & 0 \\ 0 & 0 & y_3 & 0 \\ 0 & 0 & 0 & ay_1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} y_1 & 0 & 0 & 0 \\ -y_1 & y_2 & y_3 & ay_1 \\ 0 & 0 & -y_3 & -ay_1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} y_1 & -y_1 & 0 \\ -y_1 + ay_1 & y_1 + y_2 + y_3 - ay_1 & -y_2 \\ -ay_1 & -y_3 + ay_1 & y_3 \end{bmatrix}$$

NOTE: Post multiply incident matrix $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$ is modified to $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ to put the loaned terms back to where they belong.

For clarities sake the passive entries are zeroed and the active portion shown below. This is how the auxiliary column terms are put back where they belong.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & ay_1 \\ 0 & 0 & 0 & -ay_1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ ay_1 & -ay_1 & 0 \\ -ay_1 & ay_1 & 0 \end{bmatrix}$$

This can be viewed as a superposition of matrix results. The results below

$$= \begin{bmatrix} y_1 & -y_1 & 0 \\ -y_1 & y_1 + y_2 + y_3 & -y_3 \\ 0 & -y_3 & y_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ ay_1 & -ay_1 & 0 \\ -ay_1 & ay_1 & 0 \end{bmatrix}$$

.....Passive Terms..... +.....Active Terms.

The 2x2 second matrix has to be added into the big matrix in the correct spot. Sliding the incident entries to the location corresponding to the control current returns these terms to the correct location within the passive matrix. The correct location is the lower left hand corner of both matrices coincident. This yields the correct result corresponding to nondiagonal method. In the second case the empty column is used to hold the “carry term” from the ay_1 term that was originally held in the extra terms in the first column. So there is the passive network part of the matrix in the first 3 columns and the active part in the column 4.

If we left the original post multiply incident matrix the same the result would have been

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & ay_1 \\ 0 & 0 & 0 & -ay_1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & ay_1 & -ay_1 \\ 0 & -ay_1 & ay_1 \end{bmatrix} \text{ corresponding to:}$$

$$\begin{bmatrix} y_1 & -y_1 & 0 \\ -y_1 & y_1 + y_2 + y_3 + ay_1 & -y_3 - ay_1 \\ 0 & -y_3 - ay_1 & y_3 + ay_1 \end{bmatrix}$$

Which is incorrect with the active terms to the right one position of where they should be.