Quantum Entanglement Lecture 9 2006-11-27

review – how things change with time

$$\frac{d|\psi\rangle}{dx} = \frac{-i\,H}{\hbar}\,|\psi(0)\rangle$$ ***The Schrodinger equation***

Einstein's photon equation

vector that is the sum of eigenvectors of the Hamiltonian

what is the time derivative of the average of the Hamiltonian itself? zero

Spin in a magnetic field

energy states with 2 electrons

Prof. Leonard Susskind; videos on Stanford on iTunes U
Susskind's Blog: Physics for Everyone

00:30 review – how things change with time

$|\Psi(t) = U(t)\ |\Psi(0)>$

$\Psi(t)$ state at time t

$U(t)$ linear time operator

$\Psi(0)$ state at time 0

U(t) is a lineary time operator describing how states evolve in time space. It is a postulate the U(t) is linear – we cannot prove this, but it appears to be true.

$<\Phi(t)| = < \Phi(0)|U^\dagger(t)$

Same rule but expressed as a complex conjugate of the above; the complex conjugate is the bra vector of the above ket vector – which means we use the dagger of the U matrix. *U† is complex conjugate then transpose.*

Expectation.

The inner product of a vector with itself $<\Psi\ |\ \Psi> = 1$ (if a unit vector). If the inner product of any two vectors does not change in time space this means the length and angle between them does not change. If so, then this relationship is true:

$<\Phi(t)|\Psi(t)> = <\Phi(0)|\Psi(0)>$

substituting: with this expression $= <\varphi(0)|\Psi(0)>$

$<\Phi(t)|\Psi(t)> = <\Phi(0)|U^\dagger(t)\ U(t)|\ \Psi(0)>$ this means that *U†(t)U(t) is a unitary operator.*

10:00 review of ϵ derivation in 6-8

$$U = 1 - \frac{i\,\epsilon\,H}{\hbar}$$

$$U^\dagger = 1 + \frac{i\,\epsilon\,H}{\hbar}$$

there is no physical concept that requires I or $\hbar$ separate from H – just done to make the calculations more transparent.

These values $U\,U^\dagger = 1$, which leads to (see 6-8) $H = H^\dagger$ This condition means that U must be Hermitian, that is, a *Hermitian matrix equals its complex conjuate*

$(1 + I\,\epsilon\,H/\hbar)(\,1 - I\,\epsilon\,H/\hbar)$

$i\epsilon\, H^{\dagger}/\hbar - i\epsilon\, H/\hbar = 0$
$H^{\dagger} = H$

17:00 Hermitian operators are observables – observables are Hermitian operators. This is a fancy way of saying the values observed in a laboratory are real. *So the matrix H must correspond to some quantity that we can measure*.

Not only must H corresond to some observable but that observable must be very general as (1)every system has an "H" and (2)without the "H" the system couldn't change with time.

H is the Hamiltonian, the Hamiltonian means the energy

21:00 review, differential equation and base formula

$$\frac{d}{dt}|\psi\rangle = \frac{|\psi(\varepsilon)\rangle - |\psi(0)\rangle}{\varepsilon}$$

$$\frac{d}{dt}|\Psi> = \frac{|\Psi(\epsilon)>-|\Psi(0)>}{\epsilon}$$

substituting for |Ψ(ϵ)> we get how Ψ changes with the time derivative of it

$$\frac{d|\psi\rangle}{dx} = \frac{-i\,H}{\hbar}\,|\psi(0)\rangle$$ ***The Schrodinger equation***

26:00 Einsteins photon equation:

$E = \hbar\omega$ ω is radians per second so use ℏ which is h/2π

H must have eigenvectors (all hermitians have eigenvectors) so solve to determine the basis of the space state.

$H|\psi_E\rangle = E|\psi_E\rangle$ pick a particular eigenvector ΨE, with eignenvalue E

note that if Ψe is an eigenvector then it will stay an eigenvector if a small amount of time is added – so look for a function f that relates Ψ in time to the above eigenvector

$|\psi(t)\rangle = f(t)|\psi_E\rangle$ need to find function f(t)

$\dot{f}\,|\psi_E\rangle = -\frac{iE}{\hbar} f(t)|\psi_E\rangle$ differentiating w/r time

$\dot{f}(t) = -\frac{iE}{\hbar} f(t)$ remove common term |ψϵ⟩

$f = e^{-i\frac{E}{\hbar}t}$ if the derivative equals function then the function is the exponential

31:00 inserting into Schrodinger

$$|\Psi(t)\rangle = e^{-i\frac{E}{\hbar}t}|\Psi(0)\rangle$$

$$\cos\frac{E}{\hbar}t - i\sin\frac{E}{\hbar}t$$

$|\psi(t) = e^{(-i\epsilon/\hbar)\,t}\,|\psi(0)\rangle$

$|\psi(t)\rangle = (\cos((-i\epsilon/\hbar)\,t\,) - \, i\sin((-i\epsilon/\hbar)\,t\,)\,)|\psi(0)\rangle$

$$f = e^{-i\omega t}$$

where $\omega = E/\hbar$ --- which is the Einstein equation for energy of a photon.

37:30 ***vector that is the sum of eigenvectors of the Hamiltonian***

the hamiltonian is a hermitian operator which has a complete set of eigenvectors. Since the hermiitian is a linear operator it can be expanded into a set of vectors:

$$\sum_n \alpha_n|\Psi_n\rangle$$

expansion into set of vectors

since the evolution is a linear operator we can figue out what happens to each side and add them up.

$$\sum_n \alpha_n|\Psi_n\rangle \longrightarrow \sum_n \alpha_n e^{-i\frac{E_n t}{\hbar}}|\Psi_n\rangle$$

each one of the eigenvectors is evolving with a differrent frequency. this is the general solution of the schrodinger eq. – the high energy eigenvectors evolve quickly, lower energy, slower.

42:00 other observables than hamiltonian - how do they change with time?

---for now, how does the expectation value change with time?

call the observable A. Now the Ψ s change with time, not the As (the As are a very particular set of observables)

$$\bar{A} = \langle\Psi|A|\Psi\rangle$$

average value of the observable A

now how does this change with time: differentiate with time. only Ψ varies with time so the differential is a product rule:

$$\frac{d\bar{A}}{dt} = \langle\dot{\Psi}|A|\Psi\rangle + \langle\Psi|A|\dot{\Psi}\rangle$$

product rule of differentiation

to resolve – use the schrodinger eq. for $\dot{\Psi}$ = -I H/$\hbar$.

on the left we need the complex conjugate for$\dot{\Psi}^*$ = +I H/$\hbar$ - also because the order of multiplication H A --- A H

$$\langle\Psi|\frac{iH}{\hbar}A|\Psi\rangle + \langle\Psi|A\frac{-iH}{\hbar}|\Psi\rangle$$

<ψ|iH/$\hbar$ A|ψ> <ψ|A iH/$\hbar$ |ψ>

$$\frac{i}{\hbar}\langle\Psi|HA - AH|\Psi\rangle = \dot{\bar{A}}$$

simplyfing:

46:50 *AB-BA is called the commutator of B and A , denoted by, [A,B]*

$$AB - BA = [A,B]$$

48:30 The average of the time derivative of an operator is equal to the average of the commutor with the hamiltonian.

$$\dot{\bar{A}} = \frac{i}{\hbar}\overline{[HA]}$$

Note: [H,A] average is <Ψ |HA-AH| Ψ>.
Thus the commutator is itself an operator so I can ask what is the expectation value of the commutator?

50:00 what is the time derivative of the average of the Hamiltonian itself? zero.
the value [H,H] is always zero. The expectation of the Hamiltonian does not change with time.
(conservation law) every system has a Hamiltonian so every system conserves energy.
--- of course providing that the system does not have a force applied.

53:20 The Hamiltonian is a hermitian operator: so the Hamiltonian is just a diagonal matrix with energy eigenvalues along the diagonal

$$H = \begin{pmatrix} E_1 & 0 & \cdot & - \\ 0 & E_2 & \cdot & \cdot \\ \cdot & & E_3 & \\ \cdot & & & E_4 \end{pmatrix}$$

54:00 U – the linear time evolution matrix defined above for each eigenvalue as: |Ψ(t)> = U(t)|Ψ(0)> is:

$$U = \begin{pmatrix} e^{-i\frac{E_1}{\hbar}t} & 0 & 0 \\ 0 & e^{-i\frac{E_2}{\hbar}t} & \\ & & \ddots \\ 0 & 0 & \end{pmatrix}$$

the cross-diagonal terms being zero means the eigenvectors do not get mixed up with each other. this is usually written as:

$$U = e^{-i\frac{H}{\hbar}t}$$

57:55 *systems which do not interact with each other have Hamiltonians which are the sum of the individual Hamiltonians*.

58:00 Spin in a magnetic field

we have the sigma matricies which represent the spin of the electron and are also proportional to the magnetic moments. and the magnetic field B

$$\sigma_1, \sigma_2, \sigma_3 \qquad \vec{B}$$

the energy of a spin in a magnetic field is given by the dot product of the spin and the magnetic field. (really the dot product of the magnetic moment and the magnetic field)

magnetic moment is spin times a factor µ (needs an electric charge to create a magnetic field so the magnitude of the electric charge is contained in µ) (the /2 is some convention???)

$$\frac{\mu\vec{\sigma}}{2}$$

energy is: which we take to be the hamiltonian.

$$H = \frac{\mu\vec{\sigma}}{2} \cdot \vec{B}$$

taking the direction of B in a preferred direction (3rd direction, along the z-axis)

$$H = \frac{\mu}{2} B \sigma_3$$

62:45 which is the energy of the Hamiltonian of a spin in a magnetic field.

63:00 Time dependence of the averages of the 3 components of the spin.

this is the analog of the classical question of determine what the spin does (rotate, …)

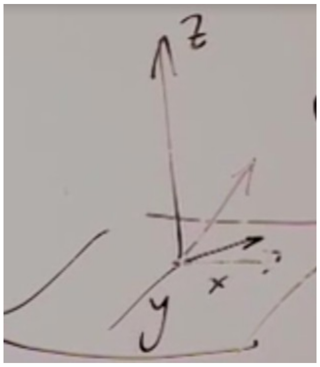
note the average bar is not explicitly written down;

$$\dot{\sigma}_3 = \frac{i}{\hbar}[H\, \sigma_3] = 0$$

but, from above, H is σ3 so the commutator is zero.

so this means the projection onto the z-axis doesn't change – and the angle relative to the z-axis does not change. so whatever happens the spin must move as a regular cone around the z-axis. also the motion of the x-y spin rotates

71:00 Precession around the z-axis. can use any axis. means the component of the spin around a magnetic field does not change with time. (gyroscope)



rotation in x-y plane, precession z-axis (right hand rule?)

now along the x-axis 1st direction. Note: the $\sigma 1$ dot is an expectation value – should be $\sigma 1$ with a dot and bar overtop

$$\dot{\sigma}_1 = \frac{i}{\hbar}\frac{\mu}{2}B\left[\sigma_3\ \sigma_1\right]$$

matrix calculations to show the commutator $[\sigma 3\ \sigma 1] = \sigma 3\sigma 1 - \sigma 1\sigma 3 = i\,\sigma 2 - (-i\,\sigma 2) = 2i\sigma 2$

$$\dot{\sigma}_1 = \frac{i}{\hbar}\frac{\mu}{2}B\,2i\sigma_2$$

$$\dot{\sigma}_1 = -\frac{\mu B}{\hbar}\sigma_2$$

$$\dot{\sigma}_2 = +\frac{\mu B}{\hbar}\sigma_1$$

this is just a rotation in x-y, chose the values

$$\sigma_2 = \sin\frac{\mu B}{\hbar}t$$

$$\sigma_1 = \cos\frac{\mu B}{\hbar}t$$

then the derivatives give the above equations.

80:00 energy states with 2 electrons, singlet, triplet;

the states are combined you have three different energy levels:

u u gives E1
d d gives E1
the singlet state: ud-du gives E2
the triplet state: ud+du gives E1 again.
this is why it is called the triplet state.

(triplet enery occurs when the 2 magnets are aligned – singlet energy is < triplet energy)

$$\sigma \cdot \tau$$

$$\sigma_1 \tau_1 + \sigma_2 \tau_2 + \sigma_3 \tau_3$$

$$\uparrow\uparrow = E_1 \qquad ud - du \longrightarrow E_2$$

$$\downarrow\downarrow = E_1 \qquad ud + du \longrightarrow E_1$$

82:00 lets start a system as u d, eigenvectors are:
|u u>, |d d> and (|u d> ± |d u>)/√2 (√2 to normalize unit vector $\sqrt{1^2 + 1^2}$)

we can then write |u d> as: (Note: the RHS reduces to |u d>)

$$|ud\rangle = \left[\frac{|ud\rangle + |du\rangle}{\sqrt{2}\sqrt{2}}\right] + \left[\frac{|ud\rangle - |du\rangle}{\sqrt{2}\sqrt{2}}\right]$$

$$\frac{|t\rangle}{\sqrt{2}} + \frac{|s\rangle}{\sqrt{2}}$$ names for easier writing. t=teresa=triplet; s=seymor=singlet

$$e^{-i\frac{E_t t}{\hbar}} \frac{|t\rangle}{\sqrt{2}} + \frac{|s\rangle}{\sqrt{2}} e^{-i\frac{E_s t}{\hbar}}$$ now evolve with time.

where the Et and Es are the energy phases E1, E2 above.

88:00 we can substitute the entangled state into eq. above to get separate energy states;

$$\frac{e^{-i\frac{E_t t}{\hbar}}}{2}\left(|ud\rangle + |du\rangle\right) + \frac{e^{-i\frac{E_s t}{\hbar}}}{2}\left(|ud\rangle - |du\rangle\right)$$

89:00 separating the |u d> and the |d u> states:

$$\frac{1}{2}\left(e^{-i\frac{E_T t}{\hbar}} + e^{-i\frac{E_s t}{\hbar}}\right)|ud\rangle$$

$$\frac{1}{2}\left(e^{-i\frac{E_T t}{\hbar}} - e^{-i\frac{E_S t}{\hbar}}\right)|du\rangle$$

91:00 initially, at time t=0, the |d u> is zero. But after a little bit of time the |u d> component diminishes and the |d u> component increases – becoming a mixed state. and the oscillates between |du> and |ud>

$$\cos\left(\frac{E_t - E_s}{2\hbar}\right)t$$ substituting cos for expression 1;
this means the average energy (the frequency) only depends upon the difference of energy of σ τ in an entangled state.