Quantum Entanglement Lecture 7 2006-11-06

review 2 slit experiment

formal calculation of probability that electron found at m

destroying the interference pattern

shrodinger's cat is not in a superposition of alive and dead

classical entropy

Trace of a matrix

quantum density matrix

Quantum mechanical entropy of a density matrix

Prof. Leonard Susskind; videos on Stanford on iTunes U
Susskind's Blog: Physics for Everyone

©Brian Carpenter, 2009 – Please acknowledge when copying

01:00 review 2 slit experiment.

states change in a linear way

```
                         m
          A              .
 0                       .
          B              .
                         .
```

electron emitted at 0, has equal probability of going through A or B (quantum superposition – add amplitudes)

$$|0\rangle \longrightarrow |A\rangle + |B\rangle$$

$$|A\rangle \longrightarrow \sum_n \psi_m |m\rangle$$ if electron goes through A, the quantum probability of arriving at the m[th] position:

$$|B\rangle \longrightarrow \sum_n \phi_m |m\rangle$$ or, if started at B, then will arrive a position m with probability amplitudes:

$$P^A_m = \psi^*_m \psi_m$$ if slot B is closed, then probability arrive at the m[th] position is $\mathbb{P}^A{}_m$;

$$P^B_m = \phi^*_m \phi_m$$ similarly if slot A closed, $\mathbb{P}^B{}_m$ is $\varphi^*\varphi$

06:50 In classical statistics, if both holes open, the the classical probability of arriving a position m would be the sum of the individual probabilities; Pa + Pb

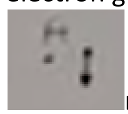
07:00 in quantum probability, going through A or B (both holes open)

$$|A\rangle + |B\rangle \rightarrow \sum_n (\psi_m + \phi_m)|m\rangle$$ A or B is sum of all probabilities at m

the probability is the amplitude squared (ψ + φ)* . (ψ + φ)
which is the sum of the two individual trajectories plus the interference trajectory of going through A (ψ* φ) and going through B (ψ φ*)

$$P_m^{A+B} = \psi_m^* \psi_m + \phi_m^* \phi_m + \psi_m^* \phi_m + \psi_m \phi_m^*$$

If we position at m=0 (equal distance between A B) the ψ φ amplitudes will be equal which gives us 4 four times the probabilty, not twice as in classical.

09:45 you can also, by inserting things or changing wave phase, make ψ = - φ; in which case you will get zero probability (destructive interference)
10:00 what happens if someone records whether electron goes through A or B; i.e. add something that flips, records, …, if electron goes through A  recording device at slot A to determine if electron goes through A

so now we have a different set of conditions. if the electron leaves point 0 with spin down (1)if it goes through A will be flipped up (2)if through B left as down:

$$|0d\rangle \longrightarrow |Au\rangle + |Bd\rangle$$

The two states A and B are correllated and entangled with the spin.
now states are different at position m, depending on {A up} or {B down}

$$|Au\rangle \longrightarrow \sum_n \psi_m |mu\rangle$$
$$|Bd\rangle \longrightarrow \sum_n \phi_m |md\rangle$$

20:00 formal calculation of probability that electron found at m

using probability operator

1) find sub space;
2) find orthogonal basis for that subspace;
3) calculate proability operator as a sum of dyads |i><i|

2 ways electron found at m, spin up or spin down. we need to build the projection operator onto the state where the electron is at point m.

ℙ $\{ |mu\rangle\langle mu| + |md\rangle\langle md| \}$ projection operator at m (|m u><m u| + |m d><m d|)
eq. as above, but now we have to sum both states – not a simple probability as |ket> vectors are different final state is sum of states through A, through B
state vector;

RH $\{ \sum_n \psi_n |nu\rangle + \phi_n |nd\rangle$ Σψ_n|n u> + φn|n d> summed over n
on other side, the conjugate to calculate the probability.

LH $\psi_n^* \langle n\, u| + \phi_n^* \langle n\, d| \{$ $\psi_n^*\langle n\ u| + \varphi_n^*\langle n\ d|$

$\{\psi_n^*\langle n\ u| + \varphi_n^*\langle n\ d|\}$ **$|m\ u\rangle\langle m\ u| + |m\ d\rangle\langle m\ d|$** $\{\Sigma\psi_n|n\ u\rangle + \varphi n|n\ d\rangle\}$

22:00 destroying the interference pattern

24:00 Calculating – you only get a contribution when up=up (down=down) and when the state vector position "n" is equal to "m" of our test position. when equal, the probability is 1 (because cross interference terms vanish)

$\psi_m^* \psi_m + \phi_m^* \phi_m$ the probability is square of amplitude; same as classical
so- if you do a measurement the same as classical probability.
The measurement destroys the interference pattern (entangled state)

30:00 another way to destroy the interference pattern is by the electron emitting a photon as it goes through A or B. For a slow electron this probability is low (~ 1 in 100) but if you increase the energy of an electron the probability is higher and would be enough to destroy the interfernce pattern.

32:00 this also means you have to use a large number of electrons in the experiment to overcome this "accelerating electron emitting a photon effect"

34:00 if you collect statistics from only when the photon was emitted then you would find that the interfeerence pattern was worse. Some problem though because a photon emitted from A may not be completely orthhogonal to a photon emitted from B so you may still have some interference from the photons.

35:00 another effect is the interference destruction from the atmosphere. The atmosphere acts like a continual measurement thus constantly destroying the interference patterns.

37:00 collapse of the wave packet means you get rid of the additive expresssion and thus elimitate cross terms:

$\sum_n (\psi_m + \phi_m)|m\rangle$ additive function of interference $(\Sigma(\psi_m+\varphi_m)|m\rangle)$

$+ \psi_m^* \phi_m + \psi_m \phi_m^*$ cross terms attributed to additive packet $(+ \psi_m^*\varphi_m + \psi_m\varphi_m^*)$

41:20 shrodinger's cat is not in a superposition of alive and dead.
The cat is entangled with another system. The composite system is in a superposition of |live unfired> |dead fired>
l,d=cat alive,dead. u,f=gn unfired,fired

$|l\,u\rangle \longrightarrow \alpha|d\,u\rangle + \beta|d\,f\rangle$ |l u> → α|d u> + β|d f>

49:00 if you add further measurement devices. schrodinger looks into box thus entangling the cat, gun, and him – and then someone watches schrodinger, etc..
--- because of linearity of actions all come out to be the same thing

50:00 measurement differences classical <-> quantum

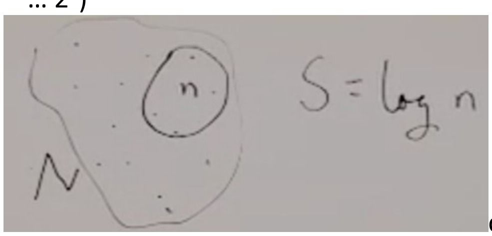
- a classical experiment can always be measured without disturbing the system.
- in a quantum experiment the measurement is not done until entanglement is established;
- and it is that entanglement that disturbs the system;
- Also destroys any previous entanglement.

60:00 entropy
64:00 how do you determine degree of entanglement between two systems?
this measurement is call ***entanglement entropy***.

66:00 classical entropy, probability just uses boolean set theory.
entropy is a state of the system together with your state of knowledge about the system.
the less you know, the more its entropy. The more you know, the less its entropy.

68:00 say we have a finite system of N states; we know the system is in one of the (small) n states;
--- the smaller the area of little n; the more defined is the state of the system .

n is a measure of our degree of ignorance about the system.
--- if little n is one state, then the entropy is 1
--- if little n = big N, then we know nothing and the entropy is maximum;
69:00 S is a measure of entropy. use log because total number of states is exponential (state1 * state2 * state3 * … 2^3)



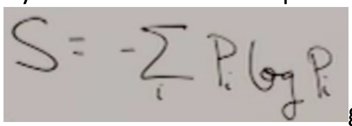
classical: use set theory to measure entropy

72:00 probability of being in state i is 1/n if i is in the subset n. otherwise the probability is zero (n being the subset where we know the system is in).

$$P(i) = \frac{1}{n} \quad \text{if } i \in n$$
$$P(i) = 0 \quad \text{if } i \notin n$$

probability of i being in n

74:00 formula for general probability distribution. Probability of the i^{th} state times the logarithm of the probability of the i^{th} state. note probability is < 1 so log is negative.



general probability distributation

75:00 probability of a state inside n (contribution of any state within subset "n")

$\mathbb{P}$ of 1 state inside n: $S = -\left(\frac{1}{n}\right)\log\left(\frac{1}{n}\right) = \left(\frac{1}{n}\right)\log(n)\ as\ -\log\left(\frac{1}{n}\right) = \log(n)$

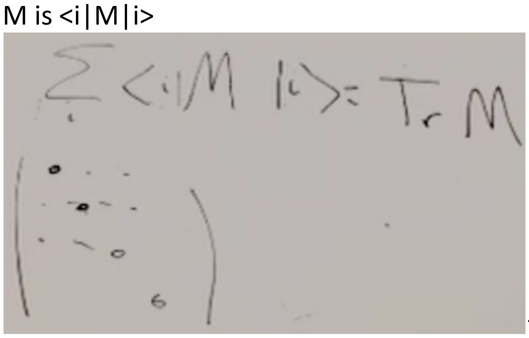
$\mathbb{P}$ of all n states in n: $S = n\left(\frac{1}{n}\right)\log(n) = \log(n)$

In the case of a probability distribution of either zero or one, the probability is normal classical probability

78:00 the difference between maximum entropy and actual entropy is called information.

80:00 definition: Trace of a matrix

let M be any matrix;
I is any diagonal element
trace of M is <i|M|i>



trace is sum of diagonal elements

the trace of M is independent of the basis vectors. all basis vectors give same trace.

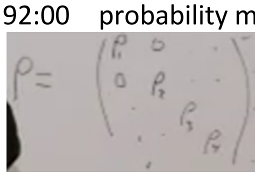
83:00 if M is a diagonal matrix (say a Hermitian) then the diagonal elements are the eigenvalues. *The sum of the eigenvalues equals the trace of the matrix*.

87:00 quantum density matrix.

you use this when you do not know what state a system is in but you do know the probability of being in one state or the other.

90:00 someone has prepared the system along one of the basis vectors I with a probability of P_i . ρi

92:00 probability matrix – trace is(Σ P I) = 1



trace of $\mathbb{P}$ matrix = 1 (i.e. sum of probabilities = 1)

say we have an observable F (is also a Hermitian operator) then the average of F is just that expectation value of the state < I | F | I >

93:50 Definition the average of F is the trace times the product of F and rho.

$\bar{F} = T_r F\rho$ average of vector F =Tr (Fρ)

$\sum_i \langle i|F\rho|i\rangle$ take the basis, take the expectation and sum over i

$I = \sum |j\rangle\langle j|$ the unit vector is formed from the DYAD of the basis unit vectors.

inserting between F and P (because a unit vector can be inserted anywhere)

$\sum_{i,j} \langle i|F|j\rangle\langle j|\rho|i\rangle$ ---only valid if I = j

but <j|P|i> is just Pi (diagonal) and <i|F|i> is Fi – expectation value. which summed gives the Tr F P (trace being sum of values)

$$\langle i|F|i\rangle \quad \rho_{ii}$$

which is the quantum mechanical version of the classic probability:

$\sum F_i P_i$ classical probability function x probability of the function

another analogy: classical probabilities sum to one; density matrix trace sums to one.

99:00 Quantum mechanical entropy of a density matrix

- if any of the (diagonal) ρ is equal to 1, then the entropy is zero.
- if all equal then the probability is the log of the number of states

entropy is the trace Tr times the density matrix ρ times the log density matrix ρ

$$S = -T_r \rho \log \rho$$