Quantum Entanglement Lecture 6 2006-10-30

review of entangled states, sub-spaces

review of projection operators, probabilities

review: classical probability (Bell's)

the 2 slit experiment, one hole, two hole

Destructive interference of a reording device

Entanglement of the experiment with an apparatus

Prof. Leonard Susskind; videos on [Stanford on iTunes U]

[Susskind's Blog: Physics for Everyone]

01:00 review of entangled states

If you have the 1^{st} electron entangled with 2^{nd} electron.

--- if the 2^{nd} electron is in bathtub the entire state (1^{st} electron plus bathtub electrons becomes entangled;

--- if bathtub evaporates, electrons still remain entangled;

2^{nd} electron moves into bathtub and entangles all electrons

--- two electrons get entangled providing they are close enough together;

--- similarly, two entangled electrons disentangle providing they are close enough to interact with each other.

06:00 projection operators (elementary level)

07:00 dimensionality is maximum number of linearly independent vectors – that is you cannot write any one of them as a sum of any of the others (opposite of linear dependent) --- they do not have to be orthogonal, just non-related

11:00 definition of basis vectors;

given a vector n (|n> (n=1,…d) n-dimensional space) there will always be (a family of) n mutually orthogonal unit vectors.

<m|n> = δmn (Kronecker delta) =1,m=n; =0 m≠n

$$\langle m|n\rangle = \delta_{mn}$$ evaluated *over all m and n*

definition: any vector can be expressed as a sum of orthogonal basis vectors

$$|\Psi\rangle = \Sigma_n \; a_n|n\rangle$$

vector ψ projected onto m (basis vector) is just a_m

$$\langle m|\Psi\rangle = \Sigma \, a_n \langle m|m\rangle = a_m$$

vector ψ can be also be expressed by the inner product of the n basis vectors, times n

$$|\Psi\rangle = \Sigma \, |n\rangle \langle n|\Psi\rangle \qquad \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} \begin{bmatrix} x1 & x2 & x3 \end{bmatrix} \begin{bmatrix} (\Psi_1) \\ (\Psi_2) \\ (\Psi_3) \end{bmatrix}$$

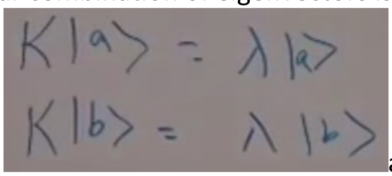
18:00 dyad operator – this is the unit operator; I

$\Sigma \, |n\rangle\langle n|$

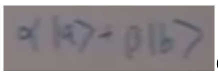
$$\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} \begin{bmatrix} x1 & x2 & x3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

20:00 Sub-spaces
K is a Hermitian operator, operating on two independent vectors a b
if λ is an eigenvalue of a and b, then any linear combination of eigenvectors is also an eigenvector

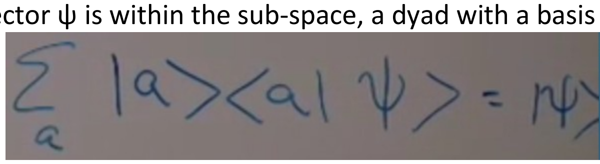
this linear combination of eigenvectors is called a sub space



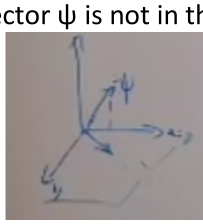
a b have common eigenvalues


combination also an eigenvector

you can form a set of basis vectors for that sub-space. Say a b are the basis vectors of that sub-space; if the vector ψ is within the sub-space, a dyad with a basis vector just returns ψ

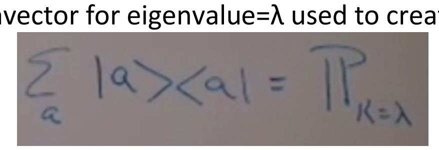


if the vector ψ is not in the sub-space, you get the projection onto the sub-space


dyad operator on ψ gives projection onto the sub-space

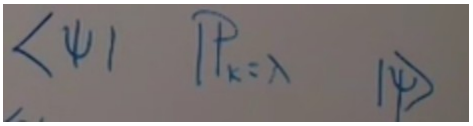
28:00 projection operator. Thus the expression Σ |a><a| is the projection operator where K=λ (i.e. eigenvectors of hermitian operator) thus projection operators are specific, i.e. $\mathbb{P}_{\sigma 3+}$ projects onto z-axis in the positive "up" direction.

a eigenvector for eigenvalue=λ used to create projection onto k=λ sub-space

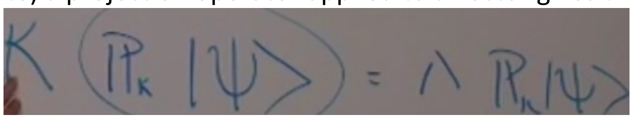


to determine a projection operator for a property you are interested in
---find the subspace the property corresponds to; (eigenvectors, Hermitian)
---find the basis vectors for that subspace
---then construct the projection operator Σ|a><a|

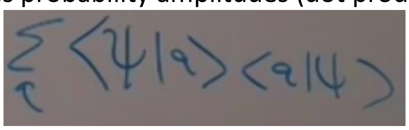
31:00 the probability postulate is calculated using projection operators.
--- given an arbitrary state ψ, which may or may not be in the subspace, the probability has the property of K=λ is the expection value of:

probability of ψ at basis state K defined by λ
or, the sum of all ways you can find that property. (projection onto basis vectors)
Note: Σ means sum over all vectors (-not a particular vector such as a or b)

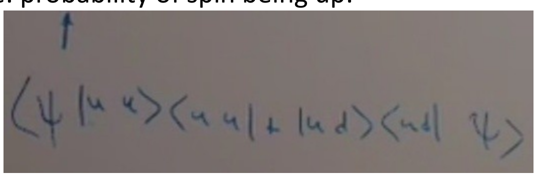
(side note) a projection operator applied to a vector gives the eigenvector in that space



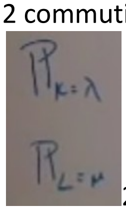
this gives probability amplitudes (dot products) for each basis vector, then summed

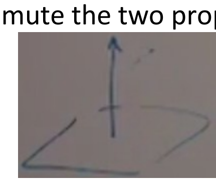


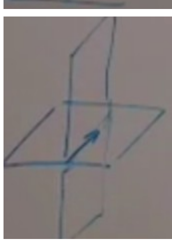
example: probability of spin being up:

probability 1st state is u u plus probability of u d

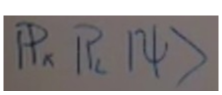
38:00 2 commuting vector operators.

2 vector operators corresponding to K=λ and L=μ
to commute the two properties (K=λ and L=μ) must have vectors in common.

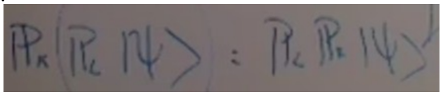
nothing in common, K,L orthogonal

share a common subspace

you need to determine: (1)which basis vectors do they have in common and (2)common projection operator

 the product of ℙk ℙl will have the common properties:

proof:

 If ℙk (ℙl|ψ>) = ℙl(ℙk|ψ> then the operators commute;

the ℙk|ψ> is the probability of ψ have property k;

the ℙl|ψ> is the probability of ψ have property l

ℙk and ℙl commute so the terms are equal.

probability ψ has joint property k and l

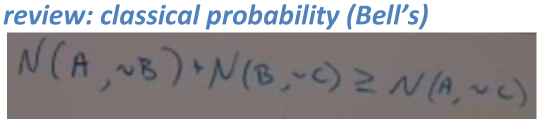
 < ψ|ℙk ℙl|ψ >

45:00 note:

having both properties (and) take the product

having one, other, or both (or) take the sum

50:00 review: classical probability (Bell's)

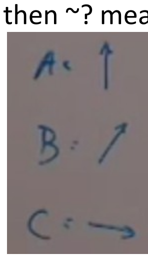


statement in classical probability

determine equivalent quatum state probability statement

take A=90° spin up; B=spin 45° ; C=0° horizontal:

then ~? means electron pointing in the opposite direction; (+180°)

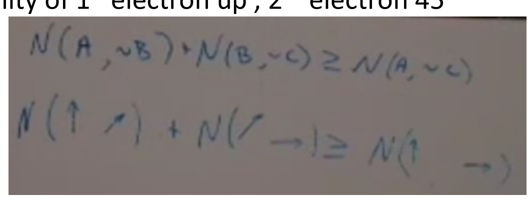


~A=270° ; ~B=225°; ~C=180°

in an entangled state, the property of 1 is the ~ property of 2 (A1 = ~A2).

56:00 we can then write the above N(A,~B), for 2 electrons, as N(A,B)

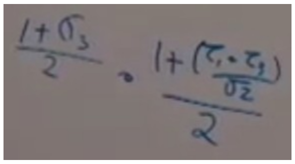
probability of 1st electron up , 2nd electron 45°

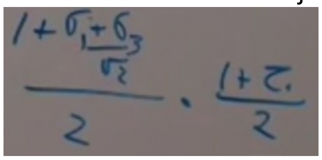


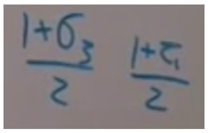
prob. shown in terms of (1st 2nd) "up" angle of electron spin

(1+σ3)/2 is projection operator for 1st spin up along z-axis;

((1+(τ1 + τ3)√2)/2 is projection operator for 2nd spin 45°

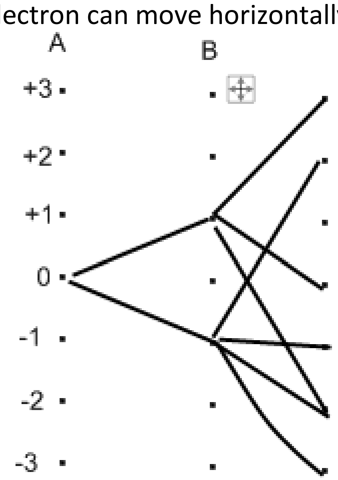
 joint probability is the product, corresponds to A⋂B in set theory

 similarly the second term operators:

 and the third term

73:00 the 2 slit experiment.

simple: electron starts out a 0 and can only go into a finite number of positions (+3 to -3)
assume electron can move horizontally.



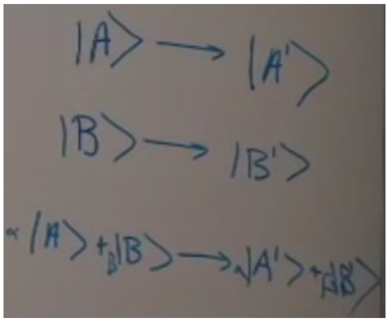
electron starts at 0; all slots at B blocked except for +1, -1 then calculate probability that the electron will arrive at a particular slot.

Assumption: the quantum evolution of a system is linear. That means if a system starts in a state |A> and evolves to a state |A'> ; and |B> to |B'> then if we start the system in a state A+B it will evolve to a state of A' plus B'

|A> → |A'>

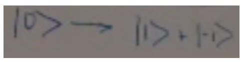
|B> → |B'>

then |A> + |B> → |A'> + |B'>

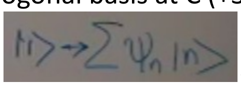


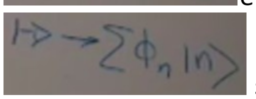
can use arbitrary coefficients: $\alpha|A> + \beta|B> \rightarrow \alpha|A'> + \beta|B'>$

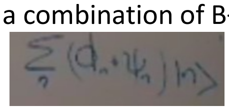
80:00 starting the electron at zero and forced through +1 or -1 – equal likelyhood of being at either slot:



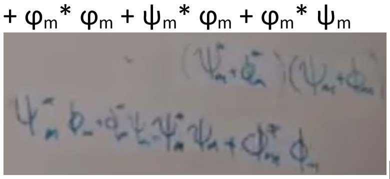
now what happens if an electron starts at B +1;
we will make an arbitrary assumption that electron arrives at C as $\Sigma\, \psi_n|n>$. That is some combination of the orthogonal basis at C (+3, +2, +1 … -3) – assume they do not get mixed up.


electron starts at B+1; arrives at C positions with various probabilities

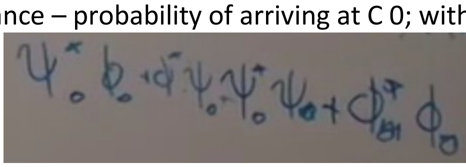

similarly, if started at B -1; then some combination of $\Sigma\, \varphi_n$
start at a combination of B+1; B-1; - will be some combination of the two


$\Sigma\, (\varphi_n + \psi_n)\, |n\rangle$ 2 electrons is some combination of above

probability that the electron will arrive at position n is the square of the 2 probabilities:
$(\psi_m^* + \varphi_m^*)(\psi_m + \varphi_m)$
$\psi_m^*\, \psi_m + \varphi_m^*\, \varphi_m + \psi_m^*\, \varphi_m + \varphi_m^*\, \psi_m$


pretty bad image --- camera guy not focused

for instance – probability of arriving at C 0; with both electrons:


2 holes open; 4 times the probability of C+0
Having both holes open (+1 and -1) has 4 times the probability of only one (+1 or -1) of arriving at a particular point.

92:00 Destructive interference of a reording device

If you insert a device to change the sign, say before B -1; then;
--- the probability if one one route open stays the same;
--- the probability of arriving at a point, with both routes open, is zero.

basically this means if you insert a device to measure which hole (B+1, B-1) the electron goes through you destroy or change the experiment.

98:00 entanglement of the experiment with an apparatus which measures it.
we have an apparatus which, if the electron goes through B+1, flips the spin to up.
no detector needed at B-1.


2 entangled electrons. “up” at slit B+1 forces “down” at slit b-1

103:00 say you start at A+0 with spin down;
---B+1 open, flips, state |B+1 d>
---B-1 open, state |B-1 d>
--- both holes open; combination |+1 u> + |-1 d>

105:00 now what happens to the probability?
if nothing happens to the spin, conditions stay the same:

$$|1u\rangle \rightarrow \sum_n \psi_n |nu\rangle$$ only B+1 is open, no change

$$|-1d\rangle \rightarrow \sum \Phi_n |nd\rangle$$ only B-1 is open, no change

both routes are open – entangled state with the measuring apparatus.

$$\sum_n \psi_n |nu\rangle + \phi_n |nd\rangle$$ entangled, states are summed

what is the probability that we have arrived at the origin, C+0??
turns out the probability is twice – a classical probability.

110:00 results of different slit experiments;

a) both holes open, probability is 4 times (not twice)
b) both holes, but modified before reaching B, probability is zero (destructive)
c) both holes, but measure (and change) after B, probability is twice.