Quantum Entanglement Lecture 4 2006-10-16

Review: completion of single bit system
probability of finding an electron in a particular state
calculate eigenvectors of σ.n
notes on preparing and measuring a system (not in video)
simultaneous measurement
entanglement – simple definition
entangled state, prepared together

Prof. Leonard Susskind; videos on [Stanford on iTunes U]
[Susskind's Blog: Physics for Everyone]

05:00 Review: completion of single bit system

an entangled system involves 2 or more systems but we need to

unit vectors shown with ^ on top;

$\hat{n}.\hat{n} = n1^2 + n2^2 + n3^2 = 1$

for any vector n (n1,n2,n3) the dot product with the sigma matrices:

$\sigma.\hat{n} = \sigma1*n1 + \sigma2*n2 + \sigma3*n3$

$\sigma.\hat{n} = \begin{bmatrix} 0 & n1 \\ n1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -in2 \\ in2 & 0 \end{bmatrix} + \begin{bmatrix} n3 & 0 \\ 0 & -n3 \end{bmatrix} = \begin{bmatrix} n3 & n1 - in2 \\ n1 + in2 & -n3 \end{bmatrix}$

definitions or notations: in vector n1 n2 n3, let

$n1 - i\,n2 = n_-$

$n1 + i\,n2 = n_+$

$(n-)(n+) = n1^2 + n2^2 + n3^2 - n3^2 = 1 - n3^2$

$\sigma.\hat{n} = \begin{bmatrix} n3 & n_- \\ n_+ & -n3 \end{bmatrix}$

for sigmas (for i ne j), reversing order of matrix multiplication reverses the sign:

$\sigma i\,.\,\sigma j = -\,\sigma j\,.\,\sigma i$

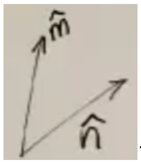
measurable component checked by placing magnet in direction of n̂ and check to see if it emits a photon. If it doesn't emit a photon then the electron is aligned with n.

dot product of sigma matrices with a unit vector is one (amplitude)

$(\sigma.\hat{n})^2 = 1$

σ.n eigenvalues are +1,-1,

12:00 probability of finding an electron in a particular state



force the component of spin along n̂ axis by placing in a large magnetic field; then what is the probability I will find the measure along m̂ axis to be +1?
(note: later uses n1 for n, n2 for m)

Notation: eigenvector of σ.n with eigen vector of +1 is: | σ.n = 1>

if you apply eigenvector against above, you get +1 times the same vector
σ.n | σ.n = 1> = +1| σ.n = 1>
σ.m | σ.m = 1> = +1| σ.m = 1>
this is the eigenvector of the state when, if measured, σ.n is definitely equal to +1

1—first find eigen vectors along n1
2-then dot product n2 and n1

probability if σ.n=1 (spin along n vector) what is the probability of spin=1 along m axis
< σ.m=1 | σ.n=1> (amplitude, probability is the square)

21:00 calculate eigenvectors of σ.n, i.e. solve:

$$\begin{bmatrix} n3 & \mathrm{n}_- \\ \mathrm{n}_+ & -n3 \end{bmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}\alpha \\ \beta\end{pmatrix}$$

step 1. set α=1, find z

$$\begin{bmatrix} n3 & \mathrm{n}_- \\ \mathrm{n}_+ & -n3 \end{bmatrix}\begin{pmatrix}1 \\ z\end{pmatrix} = \begin{pmatrix}1 \\ z\end{pmatrix}$$

n3 + zn- = 1
z = 1-n3 / n-

25:00 $\begin{pmatrix} n_3 & n_- \\ n_+ & -n_3 \end{pmatrix}\begin{pmatrix}1 \\ z\end{pmatrix} = \begin{pmatrix}1 \\ z\end{pmatrix} = \begin{pmatrix}1 \\ \frac{1-n_3}{n_-}\end{pmatrix}$ eigenvector for a.m

so eigen vector is $\begin{pmatrix}1 \\ \frac{1-n3}{\mathrm{n}_-}\end{pmatrix}$; forming the dot product. note we use n+ on the ro0w vector conjugate:

$$\begin{pmatrix}1 & \frac{1-n3}{\mathrm{n}_+}\end{pmatrix}\begin{pmatrix}1 \\ \frac{1-n3}{\mathrm{n}_-}\end{pmatrix} = 1 + \left(\frac{(1-n3)^2}{\mathrm{n}_+\mathrm{n}_-}\right) = \frac{\mathrm{n}_+\mathrm{n}_- + 1 - 2\mathrm{n}3 + \mathrm{n}3^2}{\mathrm{n}_+\mathrm{n}_-}$$

$$= \frac{1-\mathrm{n}3^2 + 1 - 2\mathrm{n}3 + \mathrm{n}3^2}{1-\mathrm{n}3^2} = \frac{2-2n3}{(1+n3)(1-n3)} = \frac{2}{1+n3}$$

now must normalize by setting dot product to 1; multiply by $\sqrt{\frac{1+n3}{2}}$

normalized eigenvector for +1 is: $\sqrt{\frac{1+n3}{2}}\begin{pmatrix}1 \\ \frac{1-n3}{\mathrm{n}_-}\end{pmatrix}$

30:18 $\sqrt{\frac{1+n_3}{2}}\begin{pmatrix}1\\ \frac{1-n_3}{2}\end{pmatrix}$ normalized eigenvector

multiply by matrix to check $M|a\rangle = \lambda\ |a\rangle$

$$\begin{bmatrix} n3 & \mathrm{n}_- \\ \mathrm{n}_+ & -n3 \end{bmatrix}\begin{pmatrix} 1 \\ \frac{1-n3}{\mathrm{n}_-} \end{pmatrix} =$$

calculating the top element

$n3 + n_- * (1\text{-}n3)/\ n_- = n3 + 1 - n3 = 1$

calculating the bottom element

$n_+ \text{-}n3 * (1\text{-}n3)/\ n_- = (n_+ n_- - n3 + n3*n3)/\ n_- = (1\text{-}n3*n3 - n3 + n3*n3)/\ n_- = (1-n3)\ /\ n_-$

so the eigenvector for eigenvalue +1 is valid (need scale factor)

$$\begin{bmatrix} n3 & \mathrm{n}_- \\ \mathrm{n}_+ & -n3 \end{bmatrix}\begin{pmatrix} 1 \\ \frac{1-n3}{\mathrm{n}_-} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1-n3}{\mathrm{n}_-} \end{pmatrix}$$

31:50 we calculate σ .m the same way, replacing n with m;

to calculate <σ.m|σ.n>

$$\left(\sqrt{\frac{1+m_3}{2}} \quad \frac{1-m_3}{m_+}\sqrt{\frac{1+m_3}{2}}\right)\begin{pmatrix}\sqrt{\frac{1+n_3}{2}} \\ \frac{1-n_3}{n_-}\sqrt{\frac{1+n_3}{2}}\end{pmatrix}$$

hard to read – formatted below

$$\langle\sigma.m|\sigma.n\rangle = \left(\sqrt{\frac{1+m3}{2}} \quad \sqrt{\frac{1+m3}{2}}\,(\frac{1-m3}{\mathrm{m}_+})\right)\begin{pmatrix}\sqrt{\frac{1+n3}{2}} \\ \sqrt{\frac{1+n3}{2}}\,(\frac{1-n3}{\mathrm{n}_-})\end{pmatrix}$$

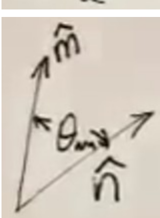
to get probability: square this eq. by multiplying with complex conjugate (takes about 15-30 minutes)

Probability of finding the state along the m-vector of a system that was prepared in the n-vector state:

$P = |\langle\sigma.m|\sigma.n\rangle|^2$

34:50

$\frac{1+n_1m_1+n_2m_2+n_3m_3}{2}$ Probability or (1+ m.n)/2 $\frac{1+n.m}{2}$



n.m is the cosine of the angle between vector n and vector m

$\frac{1+n.m}{2} = \frac{1+\cos\theta_{nm}}{2}$ probability expressed in angle θ between m & n

notes on preparing and measuring a system (not in video)

1- you first prepare the system by placing a magnet in the $\hat{n}$ direction;
2- you then move the magnet to the $\hat{m}$ direction;
3- the electron may precess to the $\hat{m}$ direction, emitting a photon of energy;
4- if a photon is emitted then the electron spin is observed in the $\hat{m}$ direction;
5- once measured – the system stay in that state until a new measurement or change.

you have to repeat the experiment many times to get the probability ending up with something like “483 out of 1000” to get the experimental probability – but it should eventually match the calculated probability

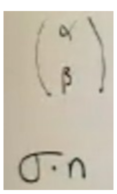
40:00 various discussions about a single bit electron state

Notes:
1 – probability only depends on the angle – you can rotate the whole experiment with no change;
2 - rotational invariance;
3 - symmetric in m & n – depends only on the angle between them;
4 - if m & n are in the same direction, the probability is one (cos 0 = 1);
5 - if m & n are in the opposite direction, the probability is zero (cos 180 = -1);

55:00 simultaneous measurement

if two systems A, B have the same eigenvectors (eigenvalues may be different)
then you can measure them simultaneously – otherwise not

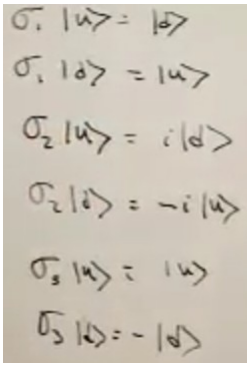
$A|\alpha\rangle = \lambda|\alpha\rangle$
$B|\alpha\rangle = \mu|\alpha\rangle$
then AB = BA (they commute)
[A,B] is math notation for commutation AB-BA=0

58:00 

for any vector there is always some direction where the spin is definitely plus one.
that is always some eigenvector of σ.n (some vector direction n) where the state is 1 is always true:

$\sigma.n|a\rangle = \lambda\ |a\rangle$ shows connection between electron spin state and direction in space.

68:00 entanglement – simple definition



action on sigma matrices: u=up {1 0}, d=down {0 1}

an example:

σ3|d⟩ = -|d⟩ $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{matrix} 0 \\ d \end{matrix} = \begin{matrix} 0 \\ -d \end{matrix}$

pair of electrons:
|u u> electron 1,up electron 2,up
|d u>
|u d>
|d d>

72:00 two electrons, two sets of observables – introduce labels to keep track of the states:
first electron spin labelled σ sigma;
second electron spin labelled τ tau

sigma acts on 1st electron, 2nd electron doesn't change
σ1|u d> = |d d>
σ1|d d> = |u d>
tau acts on 2nd electron, 1st doesn't change
τ2|u d> = -i|u u>

80:00 two independent variables are needed to specify the state of one electron; (up, down |u d>)
for two electrons:
this is a 4 dimensional vector space: |u u> |u d> |d u> |d d>
you can multiply each of the above by a complex number to get the most general state:
you get 8 complex variables (removing 2 for |u u> |d d>) you end up with 6 possible states.
--- seems two electron state more complicated than just "two separate electrons" ‼‼!

84:00 two electrons prepared separately
a1|u> + b1|d> => state of electron 1
a2|u> + b2|d> => state of electron 2

joint product state:
(a1|u> + b1|d>)*(a2|u> + b2|d>)
a1a2|u u> + a1b2|u d> + b1a2|d u> + b1b2|d d>
can always find direction corresponding to state 1,2
can always find direction(s) corresponding to product state.

88:00 entangled state, prepared together

$\frac{1}{\sqrt{2}}\{|u\,d\rangle + |d\,u\rangle\}$

(|u d> + |d u>) / √2

if 1st electron up, 2nd down, etc…

That is: if you measure the state of one electron you know something about the state of the other electron.

90:00 all sigma expectation values are 0 (probabilities are $\frac{1}{2}$ $\frac{1}{2}$ along any direction)

you calculate the expectation (average value) by <a|M|a> - which is the expectation of the observable M being in the state a

92:00 $\{\langle u\,d| + \langle d\,u|\}\ \sigma_1\ \{|u\,d\rangle + |d\,u\rangle\}$ σ1 between <bra| and |ket> states

sigma 1 state bra ket

{<u d| + <d u|} σ1 {|u d> + |d u>}

applying σ1 (check table above)

{<u d| + <d u|} {|d d> + |u u>}

multplying you always get zero as {<u d|}*{|d d>} are orthogonal

(they all are)

σ 1,2,3 expectation values are all zero

σ3

{<u d| + <d u|} σ3 {|u d> + |d u>}

applying σ1

{<u d| + <d u|} {|u d> - |d u>}

product: 1 – 1 = 0

101:00 general discussion

to get into an entangloed state just bring the electrons close enough so there ± spins interact – after a while they will be in an entangled state. (a photon may or may not be emitted)