Quantum Entanglement Lecture 02 notes 2006-10-02

measuring electron spin:

<bra & ket> Dirac notation ;

probability of a vector

Hermitian matrix (a matrix of observables)

Prof. Leonard Susskind; videos on Stanford on iTunes U

Susskind's Blog: Physics for Everyone

01:00 measuring electron spin:

N (magnet, north pole)
e (electron)
S

The simplest experiment involves an electron in a magnetic field (no electric field). The e+ direction will orientate towards north. If the electron is not correctly aligned the electron precesses until aligned. simple experiment,one electron, no electric field, etc. is indeterminate. Experiments involving averages of many electrons over many experiments;
The electron emits a photon during this precessesion process;
photons are emitted in discrete nλ units (frequency, energy dependent);
The interval for photon emission is indeterminate, in general, the stronger the magnetic the shorter the interval – but the interval is still uncertain;
If a photon is emitted then the electron spin direction has changed. If no emission implies the electron spin orientation was originally e+

31:00 <bra & ket> Dirac notation ;

vectors are associated with the state of a system. A vector is associated with a vector space, which is the mathematical space where "systems" reside. Dirac notation is a shorthand method for such representations, others being summation, integral, matrix, linear algebra, etc.

|ket⟩ the "ket-vector" is the vector in vector space;
⟨bra| the "bra-vector" is the representation of a |ket> in adjoint vector space.

The vectors have complex elements. The ket-vector is a column vector. The bra-vector is formed from the ket-vector by taking the complex conjugate and changing to a row vector.

(note: for examples we use the 3-vectors ψ = {1+2i 3 4-i}; φ = {1+3i 2i 3})

|ket⟩ ψ |ψ⟩ is |1+2i 3 4-i⟩ linear form: $\begin{bmatrix} 1+2\mathrm{i} \\ 3 \\ 4-\mathrm{i} \end{bmatrix}$

⟨bra| ψ* ⟨ψ| is ⟨1-2i 3 4+i| linear form: $[1-2\mathrm{i} \quad 3 \quad 4+\mathrm{i}]$

⟨a|b⟩ is the dot product, a scaler value,.

the dot product of a vector with itself is always a positive real number, equal to the square of the vector length;

$\langle\psi|\psi\rangle = \begin{bmatrix} 1-2i & 3 & 4+i \end{bmatrix} \begin{bmatrix} 1+2i \\ 3 \\ 4-i \end{bmatrix}$ = (1-2i)(1+2i) + (3)(3) + (4+i)(4-i) = 31

the dot product of a vector with another is usually a complex scaler;

$\langle\varphi\ |\psi\rangle = \begin{bmatrix} 1+3i & 2i & 3 \end{bmatrix} \begin{bmatrix} 1+2i \\ 3 \\ 4-i \end{bmatrix}$ = (1+3i)(1-2i) + (2i)(3) + (3)(4+i) = 19 + 4i

51:00 probability of a vector is the square of the amplitude, the dot product, or ⟨ a|a ⟩ , or Σ aa*
if a vector a has 2 states, up-designated as +; down-designated as –; then

<a+|a+> probability of a in the up state;
<a-|a-> probability of a in the down state;

the probabilty of a is the sum of states

<a+|a+> + <a-|a>

82:00 observables are real values, i.e. measured in a laboratory
a Matrix is mathematical representation of observables

100: Hermitian matrix (a matrix of observables)

- A Hermitian equals its transposed complex conjugate Mij = Mji*
- M = M† (the † dagger symbol means transpose and complex conjugate)
- The diagonal is real, Mii = Mii* only real numbers equal their complex conjugate

M = $\begin{vmatrix} 3 & 2+i \\ 2-i & 2 \end{vmatrix}$ Hermitian, diagonal has real elements; top-bottom are complex conjugates

M† = $\begin{vmatrix} 3 & 2-i \\ 2+i & 2 \end{vmatrix}^{*} = \begin{vmatrix} 3 & 2+i \\ 2-i & 2 \end{vmatrix}$ = M

97: Shear Matrix – Unit matrix with one off-diagonal a value. (side note)

translation with a shear matrix shifts one axis: $\begin{bmatrix} 1 & 0 & \gamma \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+\gamma z \\ y \\ z \end{bmatrix}$