

1. Consider an NPN silicon bipolar junction transistor with $N_E = 10^{18} \text{ cm}^{-3}$, $N_B = 10^{17} \text{ cm}^{-3}$, $N_C = 10^{15} \text{ cm}^{-3}$, $w_E = 1 \mu\text{m}$, $w_B = 0.5 \mu\text{m}$ and $w_C = 4 \mu\text{m}$. The minority carrier lifetimes equal $1 \mu\text{s}$ throughout the transistor. $V_{BE} = 0.5 \text{ V}$ and $V_{BC} = 0 \text{ V}$. Ignore the recombination in the depletion region. The base-emitter junction area equals 10^{-4} cm^2 .

- a) Calculate the quasi-neutral region widths in all three regions.

The quasi-neutral region widths are obtained by first calculating the individual depletion layer widths:

$$x_{n,BE} = \sqrt{\frac{2\epsilon_s(\phi_{i,BE} - V_{BE})}{q} \frac{N_B}{N_E} \left(\frac{1}{N_B + N_E} \right)} = 6.68 \text{ nm}$$

$$x_{p,BE} = \sqrt{\frac{2\epsilon_s(\phi_{i,BE} - V_{BE})}{q} \frac{N_E}{N_B} \left(\frac{1}{N_B + N_E} \right)} = 66.8 \text{ nm}$$

$$x_{p,BC} = \sqrt{\frac{2\epsilon_s(\phi_{i,BC} - V_{BC})}{q} \frac{N_C}{N_B} \left(\frac{1}{N_B + N_C} \right)} = 9.51 \text{ nm}$$

$$x_{n,BC} = \sqrt{\frac{2\epsilon_s(\phi_{i,BC} - V_{BC})}{q} \frac{N_B}{N_C} \left(\frac{1}{N_B + N_C} \right)} = 951 \text{ nm}$$

with

$$\phi_{i,BE} = V_t \ln \frac{N_E N_B}{n_i^2} = 0.873 \text{ V}$$

$$\phi_{i,BC} = V_t \ln \frac{N_C N_B}{n_i^2} = 0.695 \text{ V}$$

So that the quasi-neutral region widths equal:

$$w'_E = w_E - x_{n,BE} = 0.993 \mu\text{m}$$

$$w'_B = w_B - x_{p,BE} - x_{p,BC} = 0.425 \mu\text{m}$$

$$w'_C = w_C - x_{n,BC} = 3.05 \mu\text{m}$$

- b) Calculate the emitter efficiency, the base transport factor and the current gain, using the ideal device model.

The emitter efficiency, the base transport factor and the transport factor are given by:

$$\gamma_E = \frac{1}{1 + \frac{D_{p,E} N_B w_B}{D_{n,B} N_E w_E}} = 0.99092$$

$$\alpha_T = 1 - \frac{w_B^2}{2L_{n,B}^2} = 0.999902$$

$$\alpha = \gamma_E \alpha_T = 0.9908$$

The current gain then equals

$$\beta = \frac{I_C}{I_B} = \frac{\alpha}{1 - \alpha} = 110$$

- c) Calculate the base-emitter and base-collector junction capacitances.

$$C_{j,BE} = \frac{\epsilon_s A}{x_{n,BE} + x_{p,BE}} = 14.3 \text{ pF}$$

$$C_{j,BC} = \frac{\epsilon_s A}{x_{n,BC} + x_{p,BC}} = 1.1 \text{ pF}$$

- d) Calculate the majority carrier charge and the excess minority carrier charge in the quasi-neutral region in the base.

$$Q_{p,B} = q A N_B w_B = 0.18 \text{ aC}$$

$$\Delta Q_{n,B} = q A n_i^2 \frac{w_B^2}{2 N_B} = 67.9 \text{ pC}$$

- e) Calculate the base transit time, t_r .

$$t_r = \frac{w_B^2}{2 D_n} = 48.8 \text{ ps}$$

- f) Calculate the ideality factor, n , of the collector current, I_C , and calculate the Early voltage, V_A .

$$n = 1 + V_t \frac{Q_{p,B}}{C_{j,BE}} = 1.0055$$

$$V_A = \frac{Q_{p,B}}{C_{j,BC}} = 61.9$$

- Derive an expression for the base transit time without assuming the quasi-neutral base region to be either much longer or much shorter than the diffusion length. Write your answer as a function of the quasi-neutral base width, w_B , and the electron diffusion constant and diffusion length in the base, $D_{n,B}$ and $L_{n,B}$.

Start from the time independent diffusion equation and apply both boundary conditions to find the electron density in the base. Calculate the diffusion current at both ends of the quasi-neutral region. Find the base transport from the ratio of both currents. Use hyperbolic functions, $\cosh(x)$ and $\sinh(x)$, to simplify the derivation.

The diffusion equation in the base is a second order differential equation with constant coefficients. The solution is obtained by applying the boundary conditions on both sides of the quasi-neutral to the general solution.

$$0 = D_n \frac{d^2 n_p}{dx^2} - \frac{n_p - n_{p0}}{\tau_n}$$

$$n_p(x') = n_{p0} + A \cosh \frac{x'}{L_n} + B \sinh \frac{x'}{L_n}$$

$$n_p(x' = 0) = n_{p0} \exp \frac{V_{BE}}{V_t} \text{ so that } A = n_{p0} (\exp \frac{V_{BE}}{V_t} - 1)$$

$$n_p(x' = w_B') = n_{p0} \text{ so that } B = -A \coth \frac{w_B'}{L_n}$$

$$n_p(x') = n_{p0} + n_{p0} (\exp \frac{V_{BE}}{V_t} - 1) (\cosh \frac{x'}{L_n} - \coth \frac{w_B'}{L_n} \sinh \frac{x'}{L_n})$$

The current density is obtained by calculating the diffusion current. This current is then evaluated at both ends of the quasi-neutral region.

$$J_n(x') = qD_n \frac{dn_p(x')}{dx} = qD_n n_{p0} (\exp \frac{V_{BE}}{V_t} - 1) (\sinh \frac{x'}{L_n} - \coth \frac{w_B'}{L_n} \cosh \frac{x'}{L_n})$$

$$J_n(x' = 0) = J_s \coth \frac{w_B'}{L_n}$$

$$J_n(x' = w_B') = J_s (\sinh \frac{w_B'}{L_n} - \coth \frac{w_B'}{L_n} \cosh \frac{w_B'}{L_n})$$

Where the current J_s is given by:

$$J_s = qD_n n_{p0} (\exp \frac{V_{BE}}{V_t} - 1)$$

The transport factor equals the ratio of the electron current density emerging from the base region at $x' = w_p'$ to the current density going in to the base at $x' = 0$.

$$\alpha_T = \frac{J_n(x' = w_B')}{J_n(x' = 0)} = \frac{\sinh^2 \frac{w_B'}{L_n} - \cosh^2 \frac{w_B'}{L_n}}{\cosh \frac{w_B'}{L_n}} = \frac{1}{\cosh \frac{w_B'}{L_n}}$$

This result can be checked for $w_B' \ll L_n$, since it reduces to the base transport factor as obtained using the “short” diode approximation and the Taylor-series expansion for the hyperbolic cosine:

$$\alpha_T = \frac{1}{\cosh \frac{w_B'}{L_n}} \cong \frac{1}{1 + \frac{1}{2} \left(\frac{w_B'}{L_n} \right)^2} \cong 1 - \frac{1}{2} \left(\frac{w_B'}{L_n} \right)^2$$