

Depletion Approximation

Using

$$\psi = \frac{1}{q} (E_f - E_i)$$

We write

$$n = n_i e^{\psi/V_i} \quad \text{and} \quad p = n_i e^{-\psi/V_i}$$

The Poisson equation

$$\frac{d^2 \psi}{dx^2} = -\frac{q}{\epsilon} (p - n + N_d^+ - N_a^-)$$

Or using the voltage concentration relation

$$\frac{d^2 \psi}{dx^2} = \frac{q}{\epsilon} \left[n_i (e^{\psi/V_i} - e^{-\psi/V_i}) + N_d^+ - N_a^- \right]$$

Or else

$$\frac{d^2 \psi}{dx^2} = \frac{q}{\epsilon} \left[2n_i \cdot \sinh(\psi/V_i) + N_d^+ - N_a^- \right]$$

Not solvable analytically.

Typical approximations:

- a) depletion approximation - $n=p=0$ in depletion region
- b) analysis of solution around $-x_p$ and x_n

Example of analysis around x_n where $p=0$ and $N_a^-=0$, thus

$$\frac{d^2 \psi}{dx^2} = -\frac{q}{\epsilon} (-n + N_d^+)$$

For $x \gg x_n$ $\psi = \psi_n = \text{const}$ and $n = n_i e^{\psi_n/V_i} = N_d^+$.

Around $x \approx x_n$ field differs slightly from the constant (by the function ξ) and thus we write

$$\psi = \psi_n - \xi .$$

Because $\psi_n = \text{const}$ we can write

$$\frac{d^2 \xi}{dx^2} = \frac{q}{\epsilon} (-n + N_d^+) .$$

Using $n = n_i e^{\psi/V_t}$ we write

$$n = n_i e^{(\psi_n - \xi)/V_t} = \underbrace{n_i e^{\psi_n/V_t}}_{N_d^+} e^{-\xi/V_t}$$

Therefore, we can write

$$\frac{d^2 \xi}{dx^2} = \frac{q}{\epsilon} (N_d^+ - N_d^+ e^{-\xi/V_t})$$

Because ξ is small we can use the series expansion

$$e^{-\xi/V_t} = 1 - (\xi/V_t) + \frac{1}{2} (\xi/V_t)^2 - \frac{1}{3!} (\xi/V_t)^3 + \dots$$

Which yields

$$\frac{d^2 \xi}{dx^2} = \frac{qN_d^+}{\epsilon V_t} \xi .$$

Changing the variable $y = x - x_n$ we write

$$\frac{d^2 \xi}{dy^2} = \frac{qN_d^+}{\epsilon V_t} \xi .$$

This equation can be solved analytically yielding

$$y = A e^{-y/L_D} + B e^{y/L_D} \quad \text{where} \quad L_D = \sqrt{\frac{\epsilon V_t}{qN_d^+}} \quad \text{is the Debye length.}$$

This result indicate that the increment of potential ξ changes exponentially with the departure from x_n with the rate dictated by the Debye length. In a 3 to 5 Debye lengths from x_n the increment becomes negligible. In conclusion, if Debye length is negligible with respect to the depletion width than the depletion approximation is adequate.