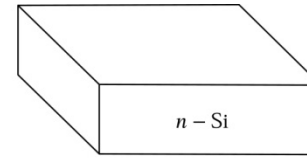

Semiconductor Devices

THIRD EDITION

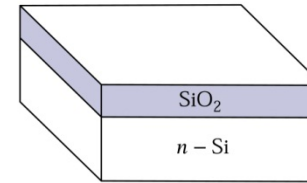
S. M. Sze and M. K. Lee

Chapter 3 *p-n* Junction

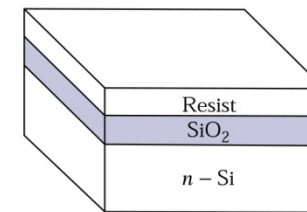
(a) A bare n -type Si wafer. (b) An oxidized Si wafer by dry or wet oxidation. (c) Application of resist. (d) Resist exposure through the mask.



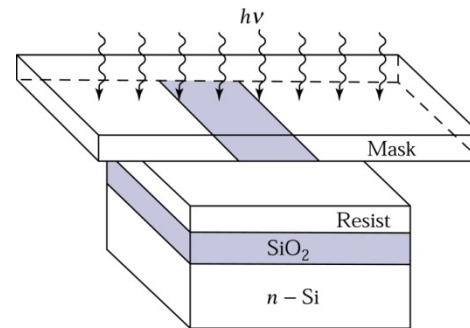
(a)



(b)

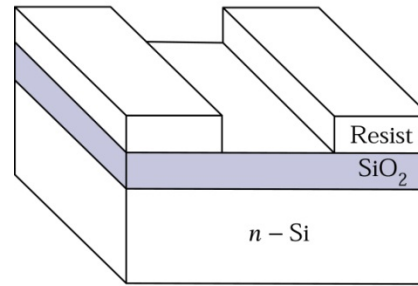


(c)

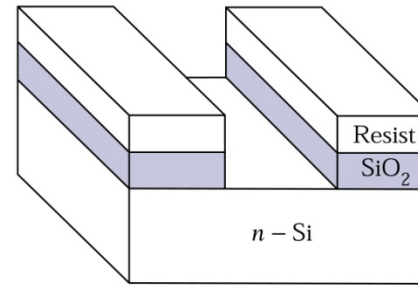


(d)

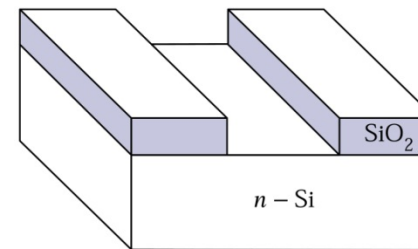
(a) The wafer after the development. (b) The wafer after SiO_2 removal. (c) The final result after a complete lithography process. (d) A p - n junction is formed in the diffusion or implantation process. (e) The wafer after metalization. (f) A p - n junction after the complete process.



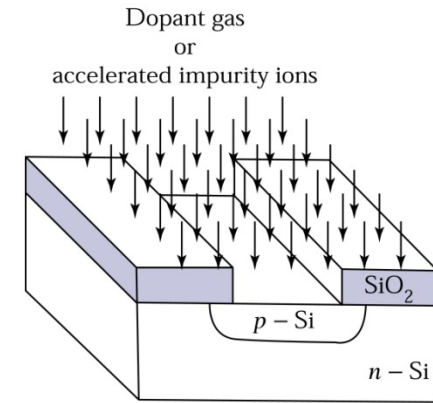
(a)



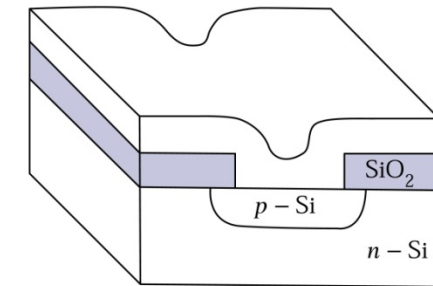
(b)



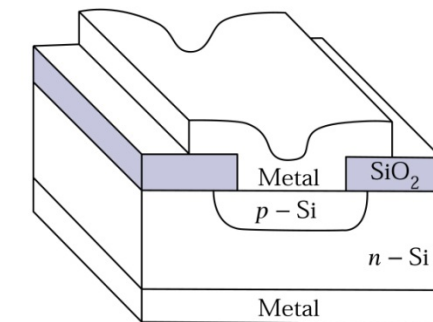
(c)



(d)



(e)



(f)

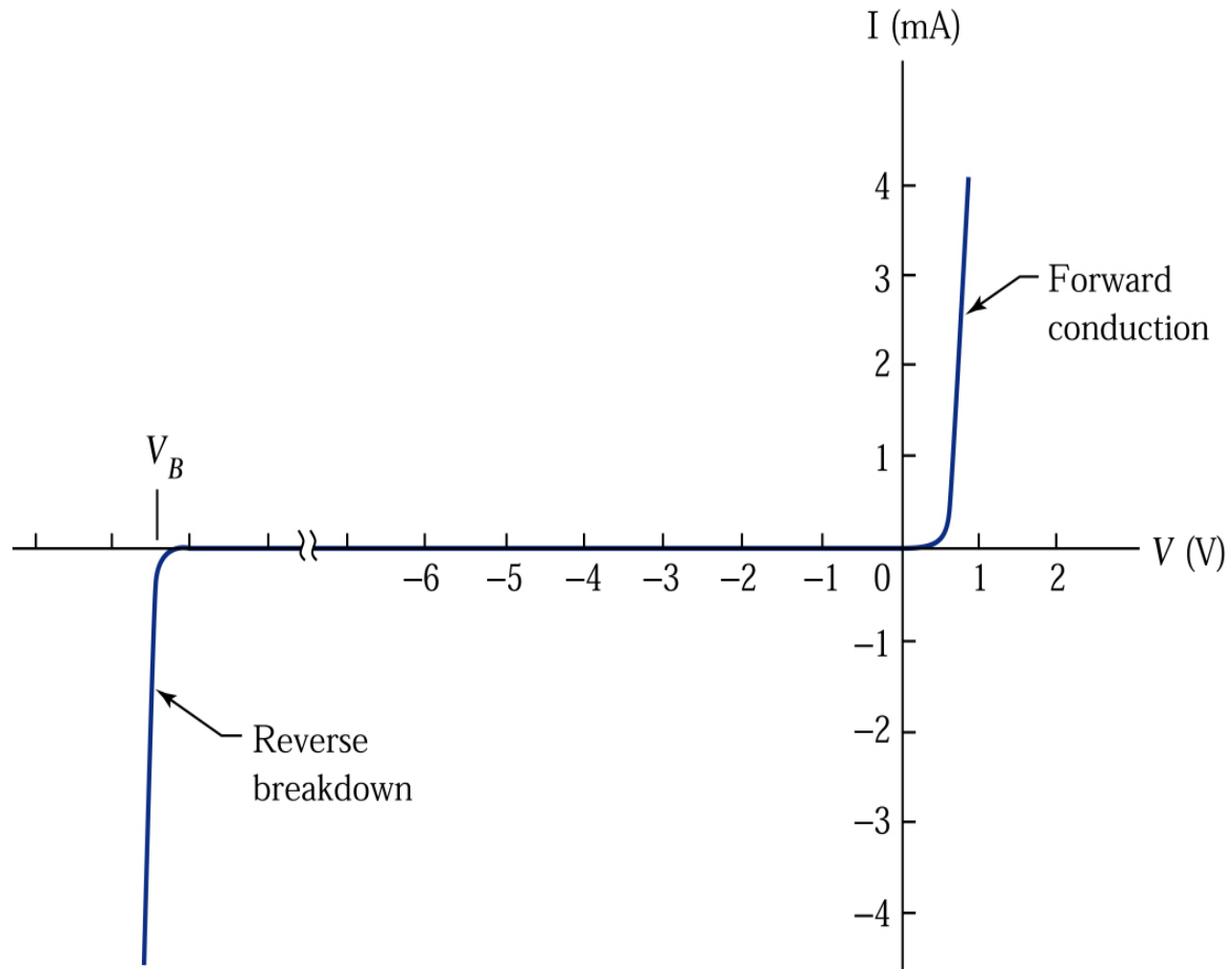


Figure 3.1 Current-voltage characteristics of a typical silicon p - n junction.

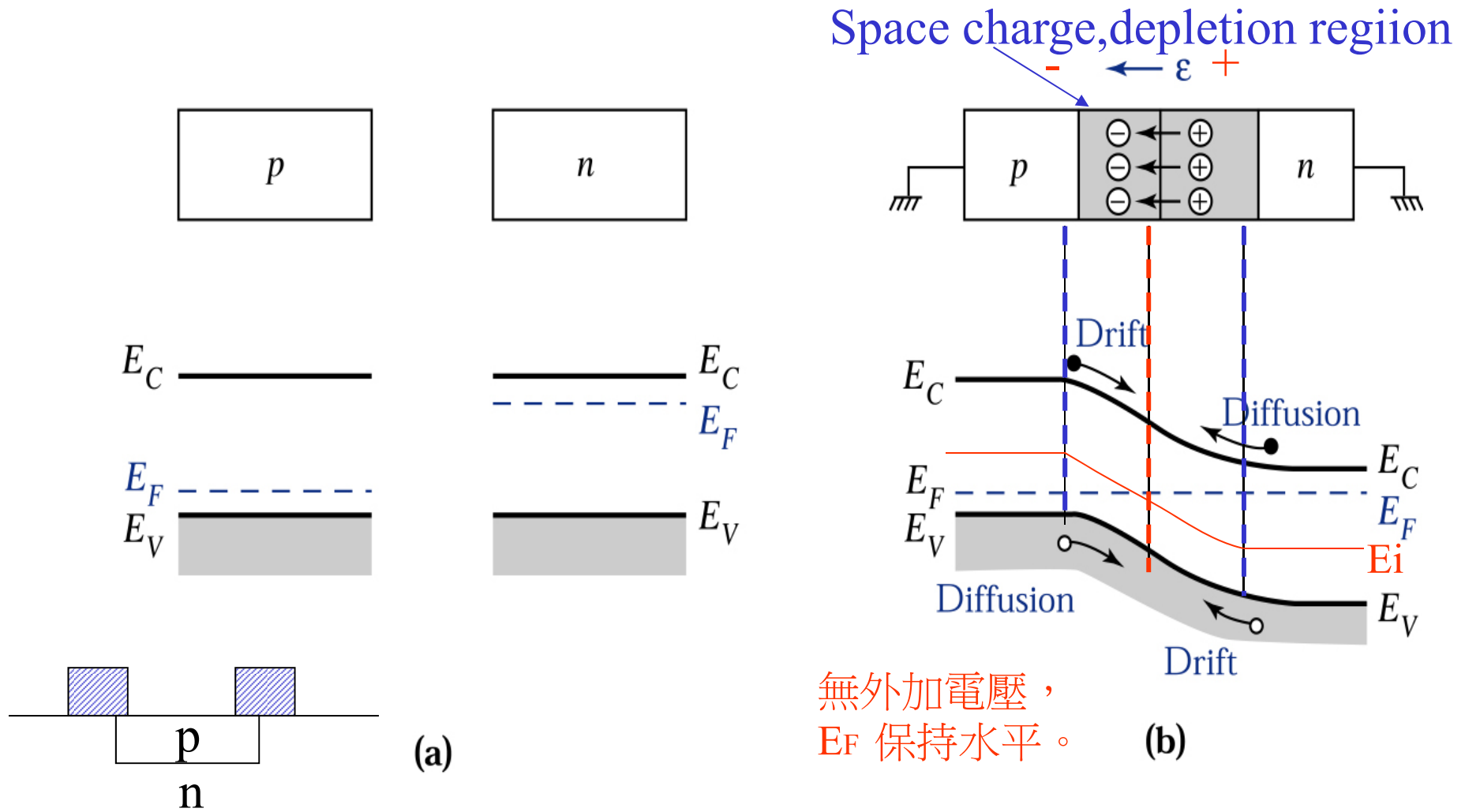
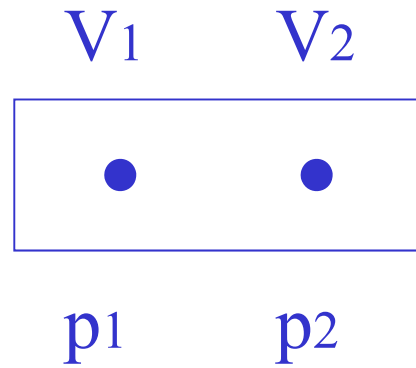


Figure 3.2. (a) Uniformly doped p -type and n -type semiconductors before the junction is formed. (b) The electric field in the depletion region and the energy band diagram of a p - n junction in thermal equilibrium.



$$J_p = J_p(\text{drift}) + J_p(\text{diffusion})$$

$$= q\mu_p p \mathcal{E} - qD_p \frac{dp}{dx} = 0$$

$$= q\mu_p p \left(\frac{1}{q} \frac{dE_i}{dx} \right) - kT\mu_p \frac{dp}{dx} = 0, \quad (1)$$

$$-\mu_p p \frac{dV}{dx} = D_p \frac{dp}{dx}$$

$$dV = -\frac{D_p}{\mu_p} \times \frac{dp}{p}$$

$$V_{21} = \frac{kT}{q} \ln \frac{p_1}{p_2}$$

又由已知 $p = n_i e^{(E_i - E_F)/kT}$ (2)

↓ 微分

$$\frac{dp}{dx} = \frac{p}{kT} \left(\frac{dE_i}{dx} - \frac{dE_F}{dx} \right) \quad (3)$$

(3)代入(1)

$$J_p = \mu_p p \frac{dE_F}{dx} = 0 \quad (4)$$

$$\frac{dE_F}{dx} = 0.$$

$$(5) \begin{cases} (1) \text{無外加電場} \\ (2) \text{熱平衡} \end{cases}$$

• 注意 $dE_i/dx \neq 0$

• 由Poisson's eq.

$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

$$(\nabla \cdot D = \rho)$$

$$\frac{d\Psi}{dx} = -E$$

$$\frac{d^2\psi}{dx^2} \equiv -\frac{d\mathcal{E}}{dx} = -\frac{\rho_s}{\epsilon_s} = -\frac{q}{\epsilon_s} (N_D - N_A + p - n).$$

(7)

正電 負電

中性區 $\rightarrow \frac{d^2\psi}{dx^2} = 0$ 即無 ρ_s 無 E (8)

所以 $N_D - N_A + p - n = 0.$ (9)

- $\overset{\text{p-type}}{\psi_p} \equiv -\frac{1}{q}(E_i - E_F) \Big|_{x \leq -x_p} = -\frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right).$ ($N_D = n = 0, \rho = N_A$) (10)

中性區相對 E_F 之電子電位

+ $\psi_n \equiv -\frac{1}{q}(E_i - E_F) \Big|_{x \geq x_n} = -\frac{kT}{q} \ln\left(\frac{N_D}{n_i}\right).$ (11)
n-type ($N_A = p = 0, \rho = N_D$)



$$V_{bi} = \psi_n - \psi_p = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right) = \frac{kT}{q} \ln \frac{n_{n0}}{n_{p0}} \quad (12)$$

Built in potential

$$\frac{n_{n0}}{n_i^2} = \frac{n_{n0}}{n_{p0}} \Rightarrow n_{n0} = n_{p0} e^{\frac{qV_{bi}}{kT}}$$

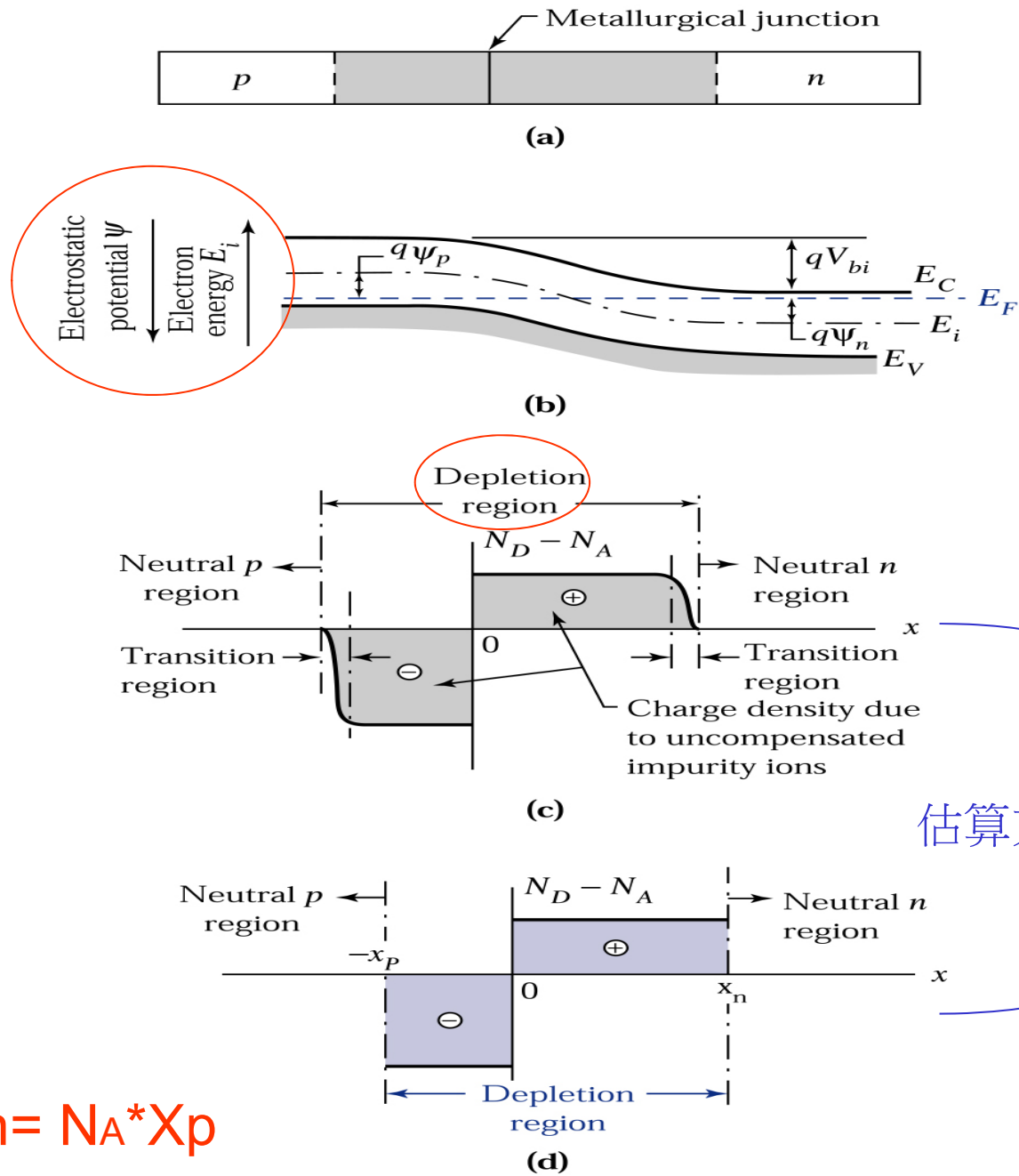
空乏區中，Poisson's eq. 為

$$\frac{d^2\psi}{dx^2} = \frac{q}{\epsilon_s} (N_A - N_D). \quad (13)$$

$$\frac{d^2\psi}{dx^2} \equiv -\frac{d\mathcal{E}}{dx} = -\frac{\rho_s}{\epsilon_s} = -\frac{q}{\epsilon_s} (N_D - N_A + p - n). \quad (7)$$



0



估算方便以方形計算

$$N_D \cdot x_n = N_A \cdot x_p$$

Figure 3.3

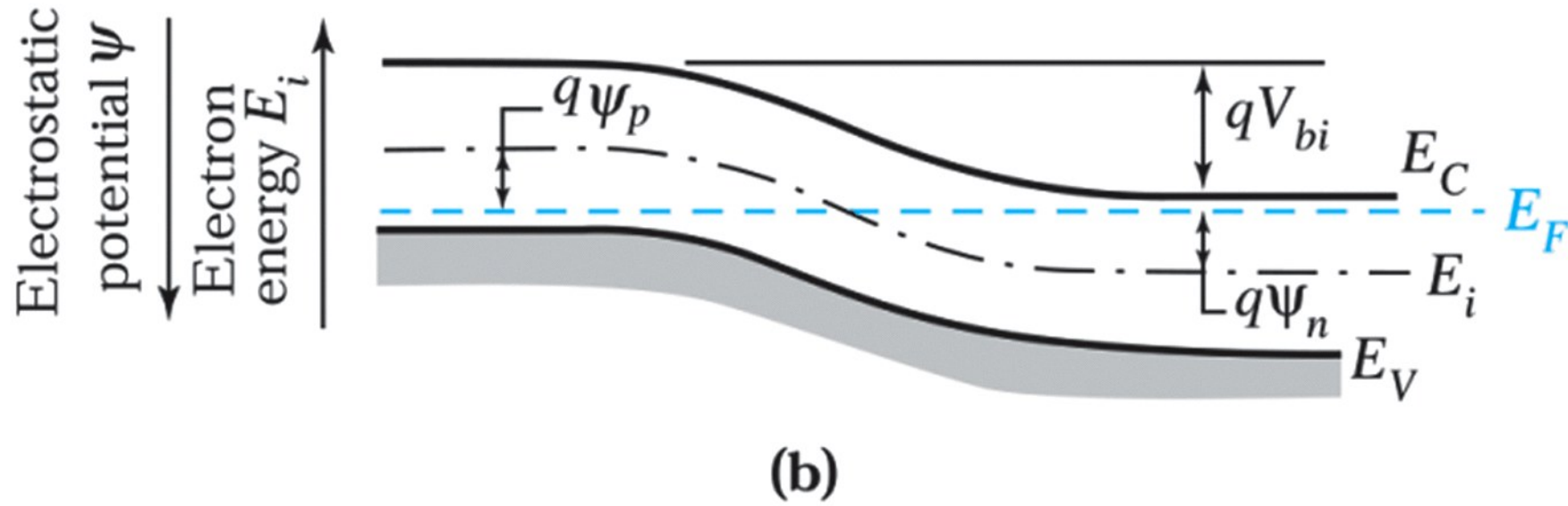
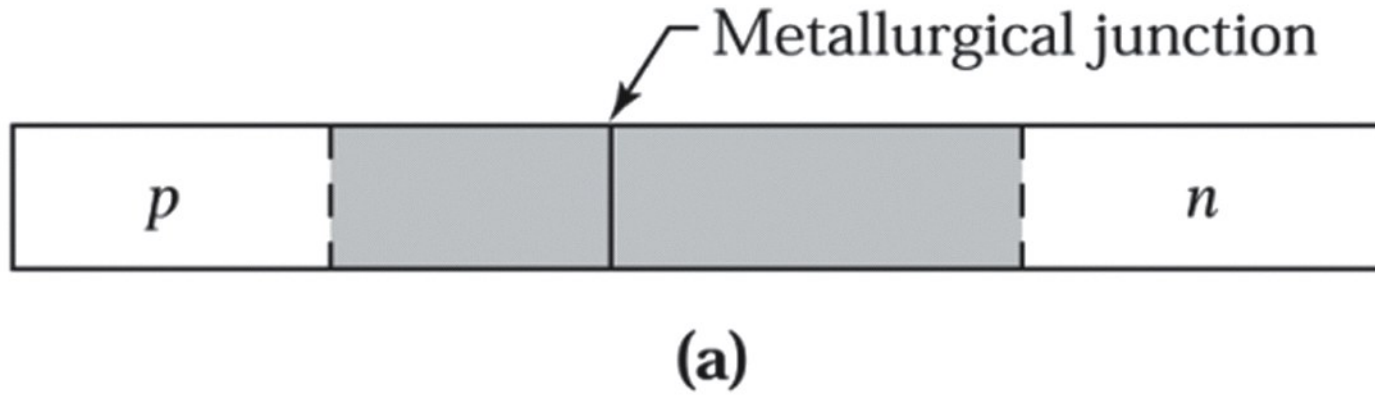
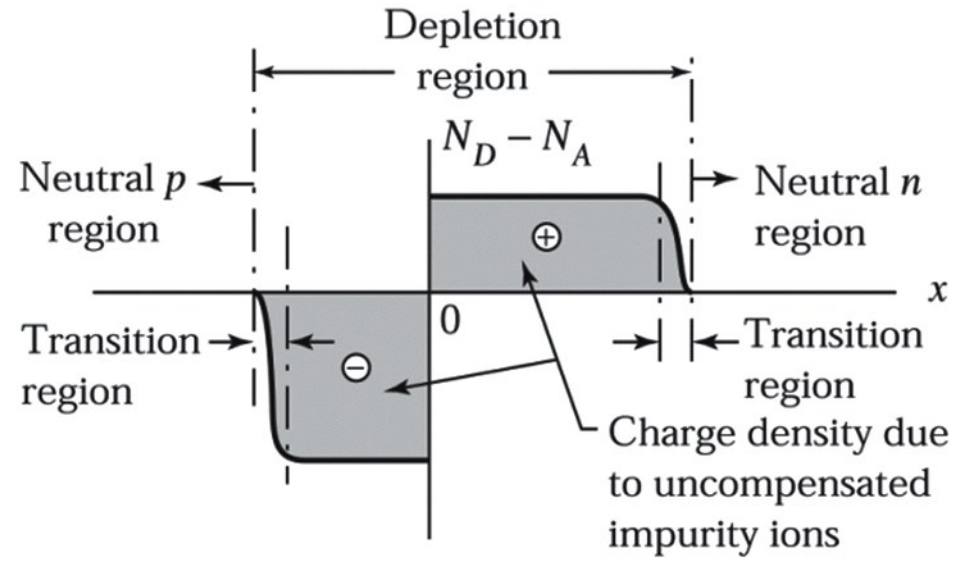
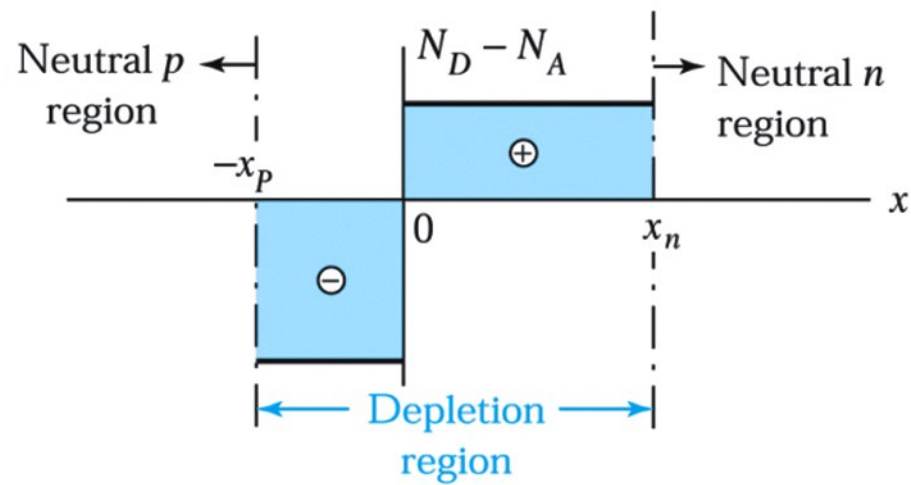


Figure 3.3ab
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(c)



(d)

Figure 3.3cd
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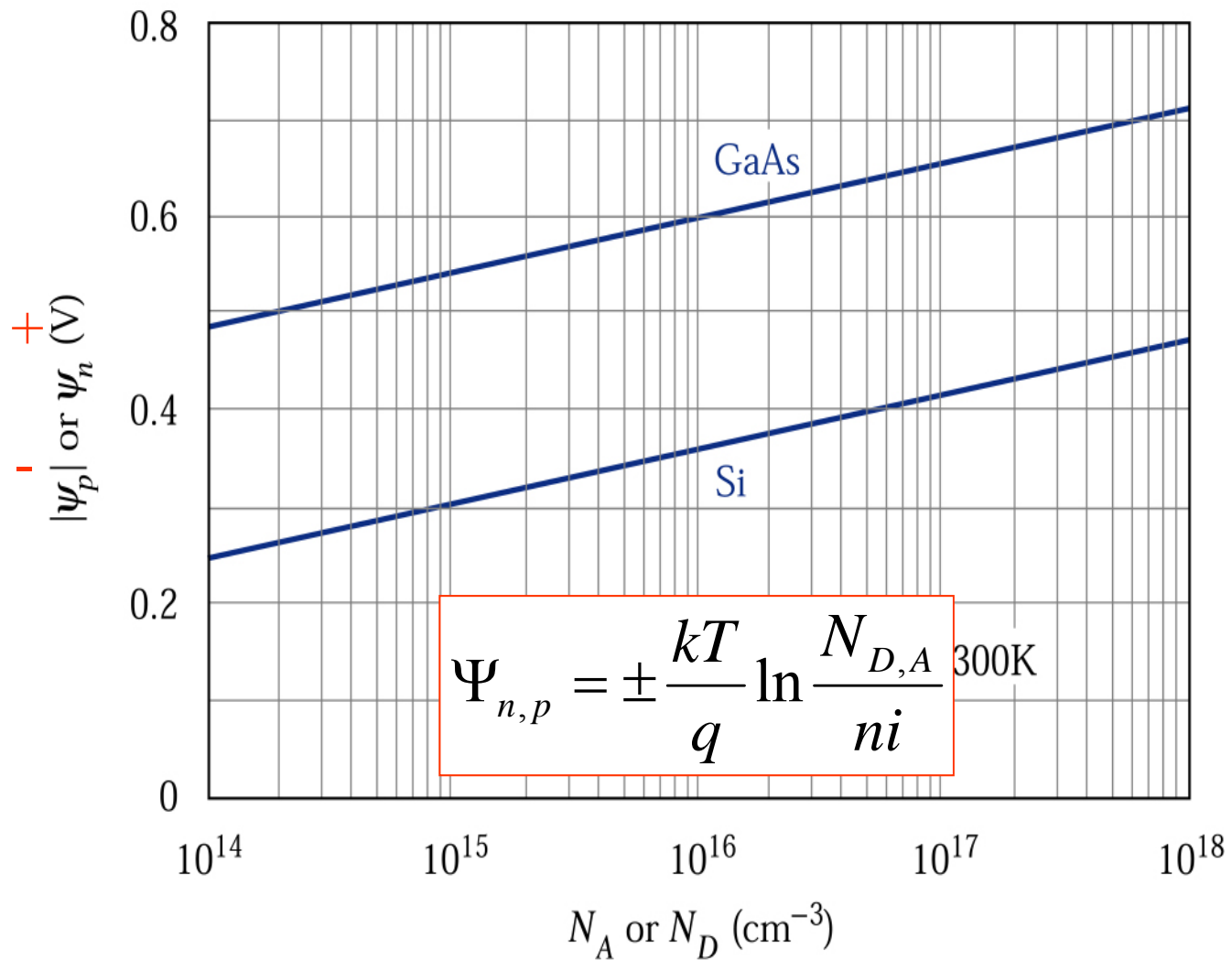


Figure 3.4. Built-in potentials on the p -side and n -side of abrupt junctions in Si and GaAs as a function of impurity concentration.

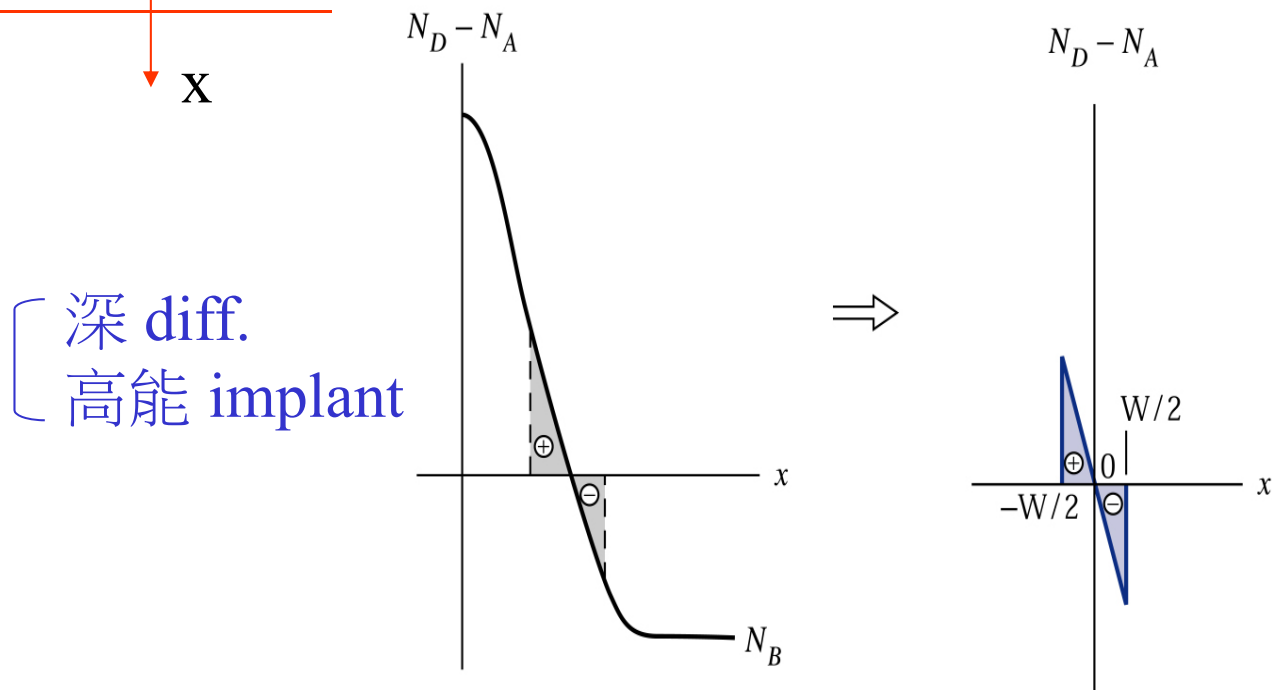
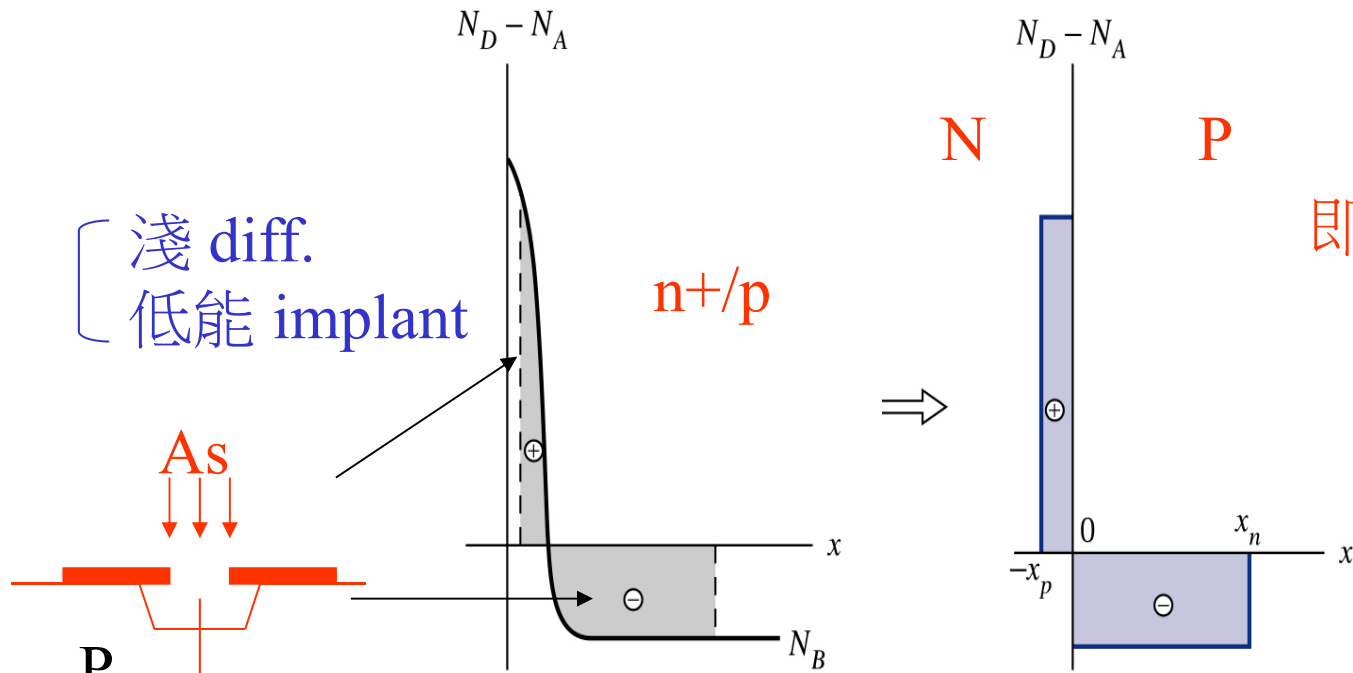


Figure 3.5.
Approximate doping profiles.
(a) **Abrupt** junction.
(b) Linearly graded junction.

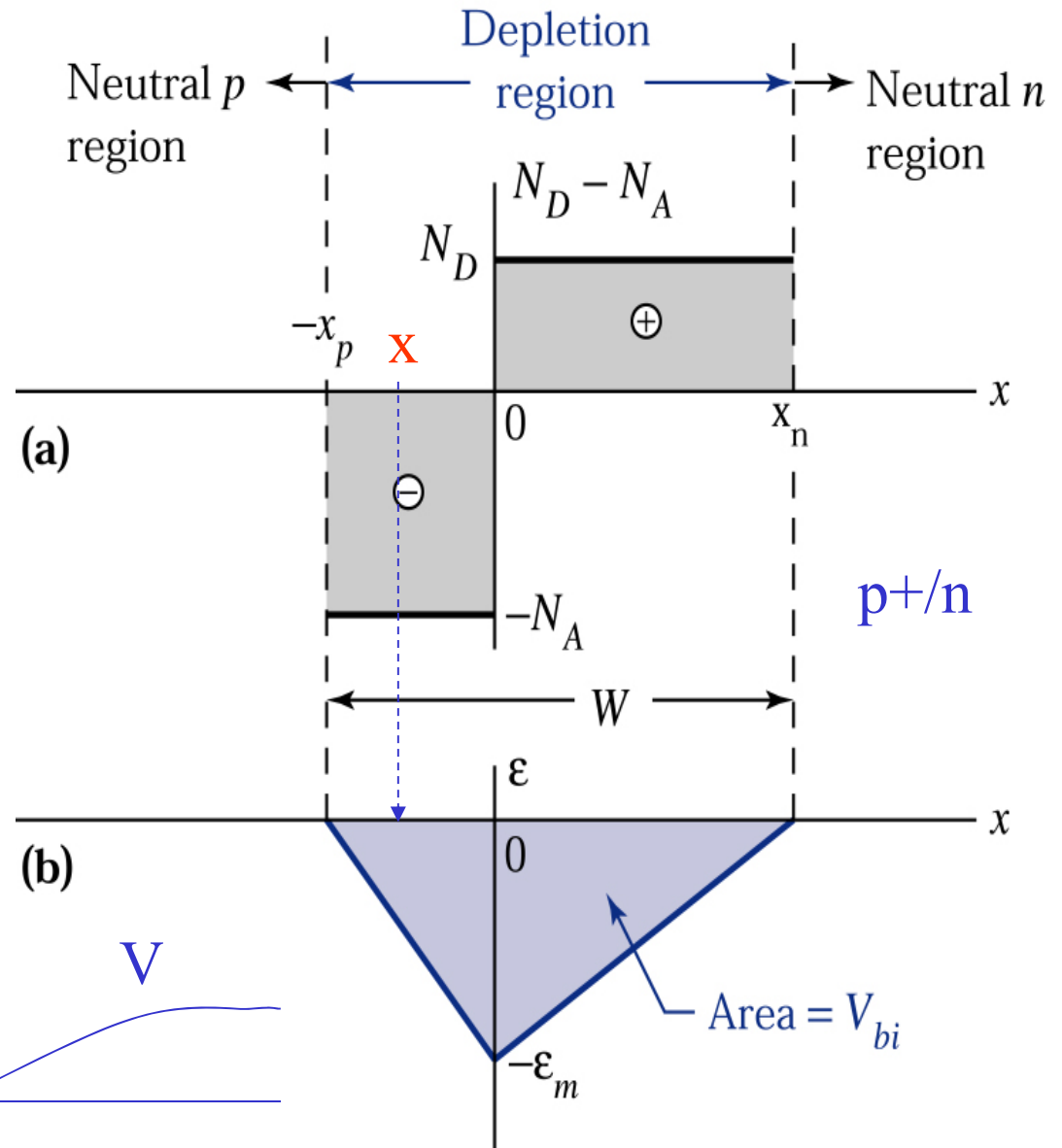
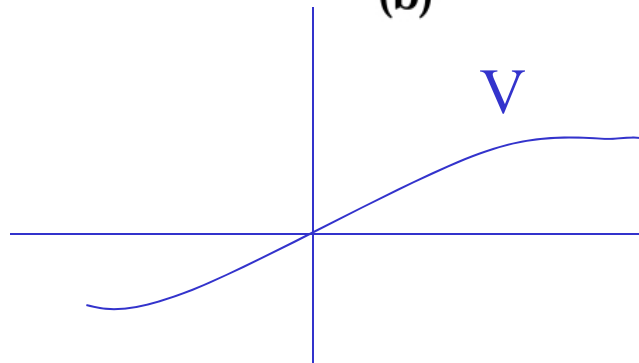
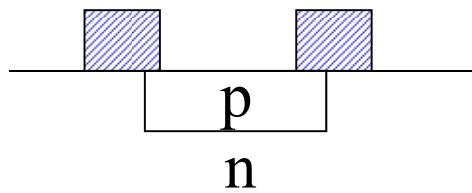
Abrupt Junction

計算方便，亦大都屬於此種Junction

$$\left\{ \begin{array}{l} \frac{d^2\psi}{dx^2} = +\frac{qN_A}{\epsilon_s} \quad \text{for } -x_p \leq x < 0, \quad (14a) \\ \frac{d^2\psi}{dx^2} = -\frac{qN_D}{\epsilon_s} \quad \text{for } 0 < x \leq x_n. \quad (14b) \end{array} \right.$$

Figure 3.6.

(a) Space charge distribution in the depletion region at thermal equilibrium. (b) Electric-field distribution. The shaded area corresponds to the built-in potential.



$$\left\{ \begin{array}{l} N_A x_p = N_D x_n. \end{array} \right. \quad (15)$$

電中性

$$\left\{ \begin{array}{l} W = x_p + x_n. \end{array} \right. \quad (16)$$

Depletion width

$$\mathcal{E}(x) = -\frac{d\psi}{dx} = -\frac{qN_A(x+x_p)}{\epsilon_s} \quad \text{for } -x_p \leq x < 0 \quad (17a)$$

$$\therefore \frac{d^2\Psi}{dx^2} = \frac{qN}{\epsilon_s}$$

$$\mathcal{E}(x) = -\mathcal{E}_m + \frac{qN_D x}{\epsilon_s} = \frac{qN_D}{\epsilon_s} (x - x_n) \quad \text{for } 0 < x \leq x_n, \quad (17b)$$

$$\mathcal{E}_m = \frac{qN_D x_n}{\epsilon_s} = \frac{qN_A x_p}{\epsilon_s}. \quad (18)$$

area

$$\begin{aligned} V_{bi} &= -\int_{-x_p}^{x_n} \mathcal{E}(x) dx = -\int_{-x_p}^0 \mathcal{E}(x) dx \Big|_{p\text{-side}} - \int_0^{x_n} \mathcal{E}(x) dx \Big|_{n\text{-side}} \\ &= \frac{qN_A x_p^2}{2\epsilon_s} + \frac{qN_D x_n^2}{2\epsilon_s} = \frac{1}{2} \mathcal{E}_m W. \end{aligned} \quad (19)$$

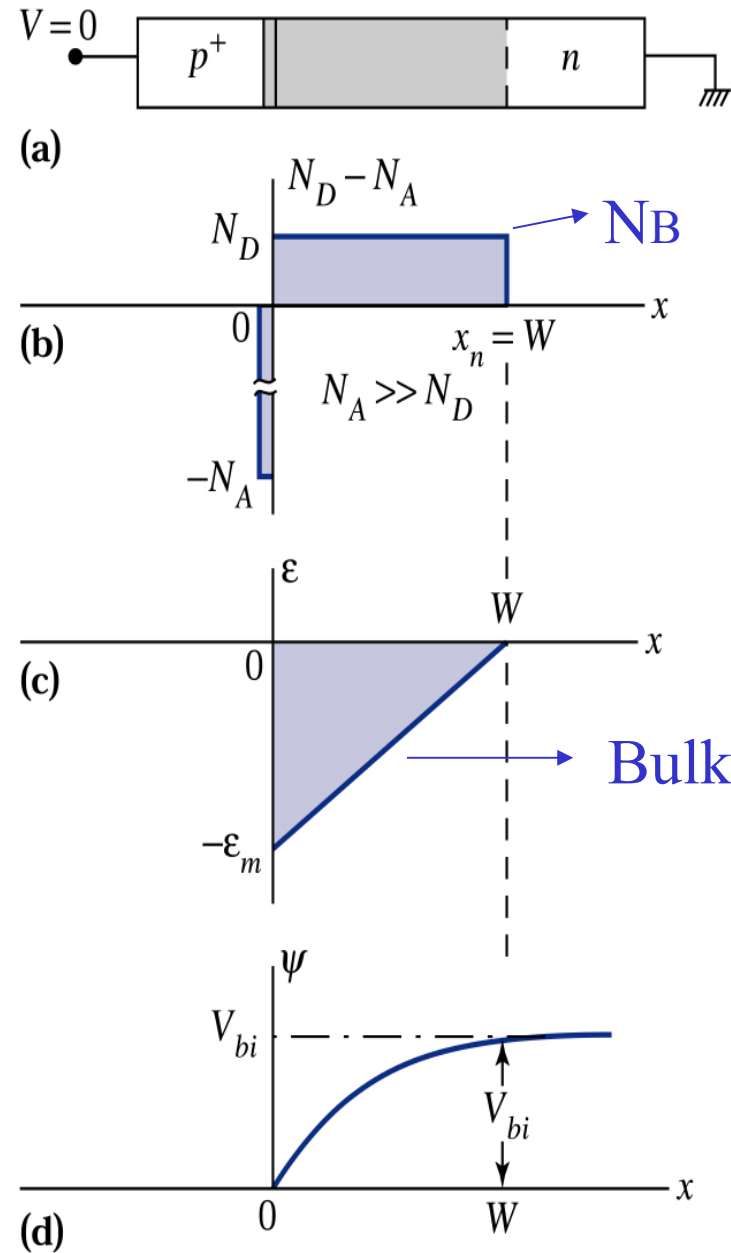
由(15)(19)

$$W = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) V_{bi}}. \quad (20)$$

Figure 3.7.

- (a) One-sided abrupt junction (with $N_A \gg N_D$) in thermal equilibrium.
- (b) Space charge distribution.
- (c) Electric-field distribution.
- (d) Potential distribution with distance, where V_{bi} is the built-in potential.

p⁺/n



p+/n $N_A \gg N_D \rightarrow$
$$W \cong x_n = \sqrt{\frac{2\epsilon_s V_{bi}}{qN_D}}. \quad (21)$$

• 實際p/n junction亦多為abrupt

$$\mathcal{E}_m = \frac{qN_B W}{\epsilon_s} \quad (23)$$

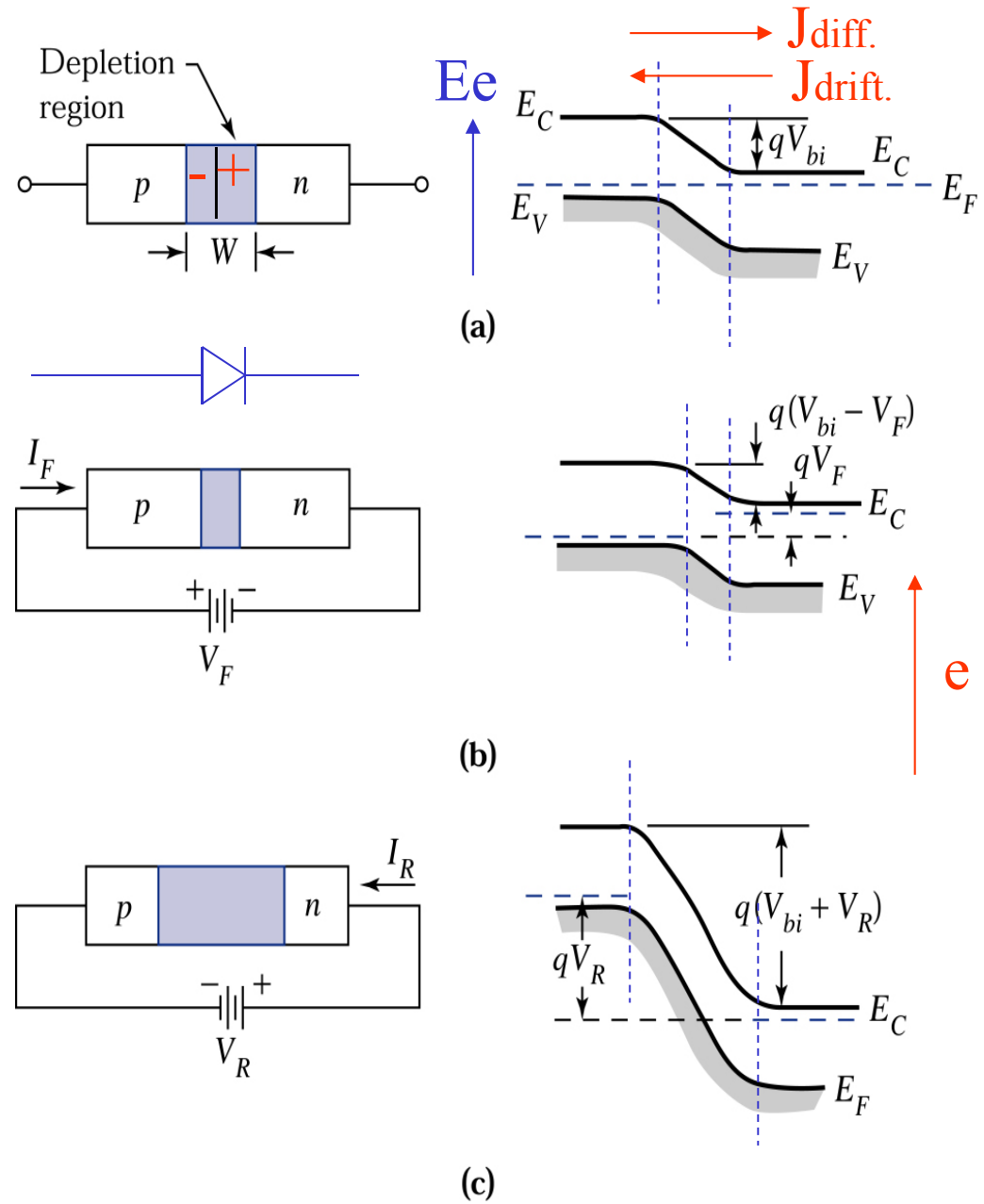
$$\mathcal{E}(x) = \frac{qN_B}{\epsilon_s} (-W + x) = -\mathcal{E}_m \left(1 - \frac{x}{W} \right), \quad (24)$$

$$\psi(x) = \frac{V_{bi} x}{W} \left(2 - \frac{x}{W} \right). \quad (x=W, \psi(W)=V_{bi}) \quad (26)$$

Figure 3.8.

Schematic representation of depletion layer width and energy band diagrams of a p - n junction under various biasing conditions.

a) Thermal-equilibrium condition. (b) Forward-bias condition. (c) Reverse-bias condition.



•若外加偏壓

$$W = \sqrt{\frac{2\epsilon_s(V_{bi} - V)}{qN_B}},$$

(27)

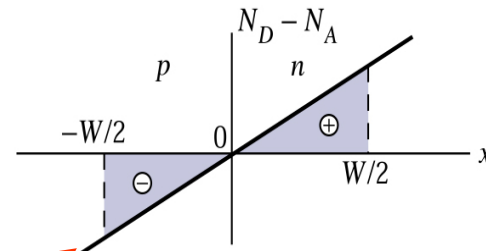
☆ V :forward 為正，reverse 為負

Figure 3.9.

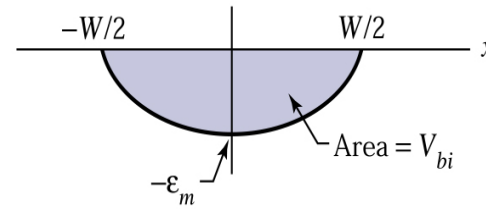
Linearly graded junction in thermal equilibrium.

- (a) Impurity distribution.
- (b) Electric-field distribution.
- (c) Potential distribution with distance.
- (d) Energy band diagram.

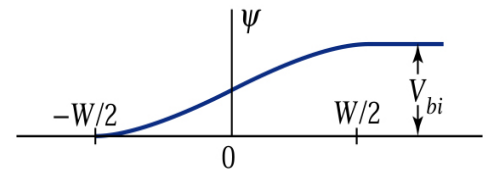
Slope = a



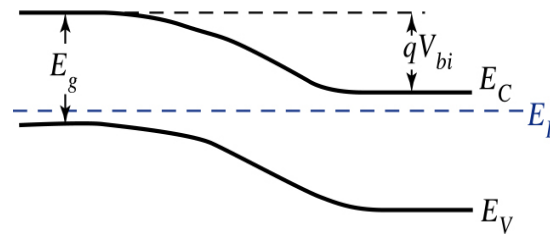
(a)



(b)



(c)



(d)

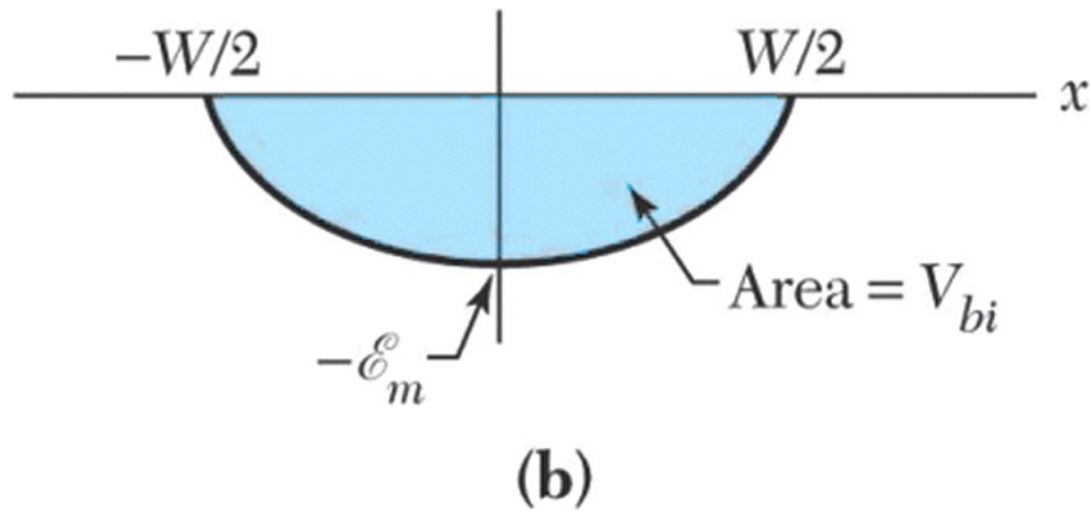
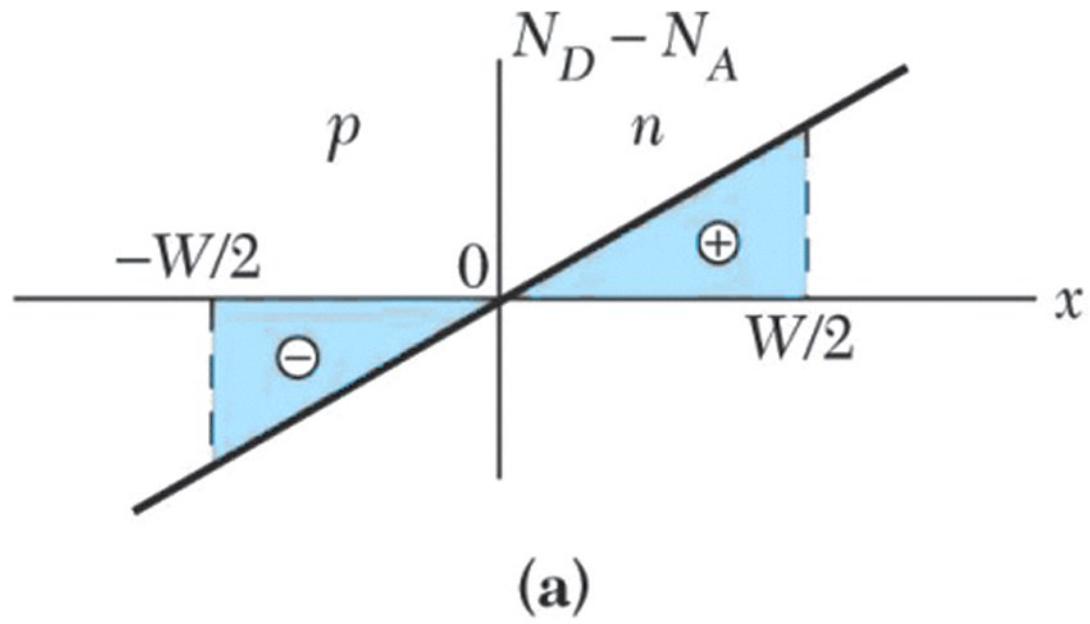
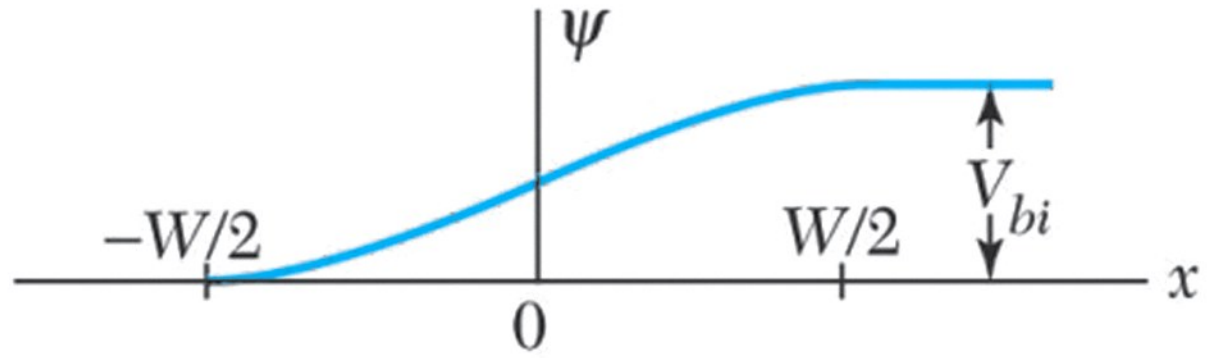
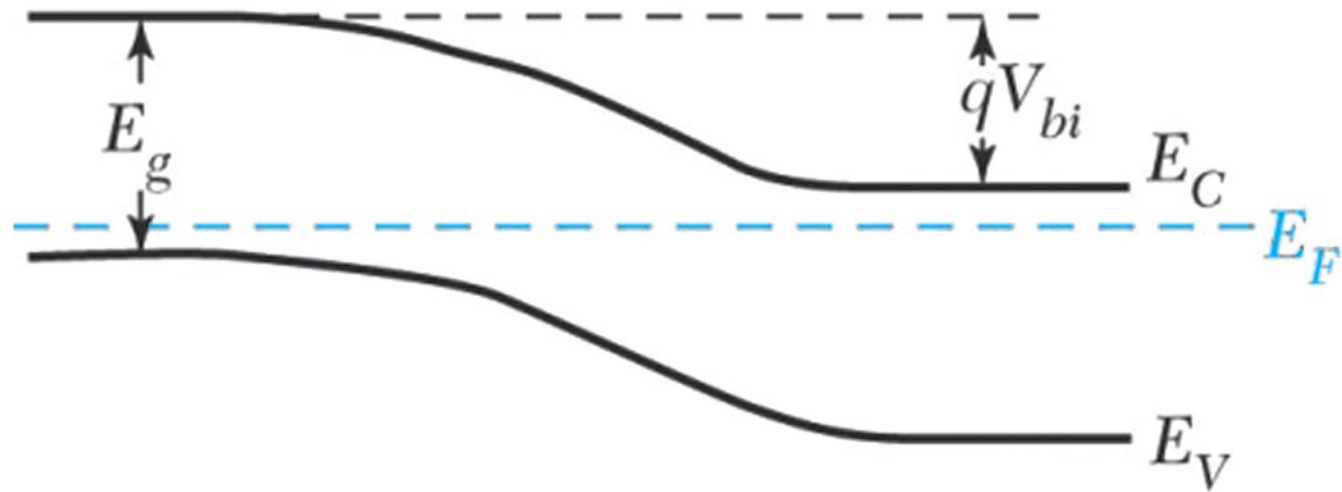


Figure 3.9ab
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(c)



(d)

Figure 3.9cd
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- Linearly Graded Junction

$$\frac{d^2\psi}{dx^2} = \frac{-d\mathcal{E}}{dx} = \frac{-\rho_s}{\epsilon_s} = \frac{-qax}{\epsilon_s} \quad -\frac{W}{2} \leq x \leq \frac{W}{2}, \quad (28)$$

$$\mathcal{E}(x) = -\frac{qa}{\epsilon_s} \left[\frac{(W/2)^2 - x^2}{2} \right]. \quad (29)$$

$$\mathcal{E}_m = \frac{qaW^2}{8\epsilon_s}. \quad (29a)$$

$$W = \left(\frac{12\epsilon_s V_{bi}}{qa} \right)^{1/3}. \quad (31)$$

V_{bi} 正比 ln(a)

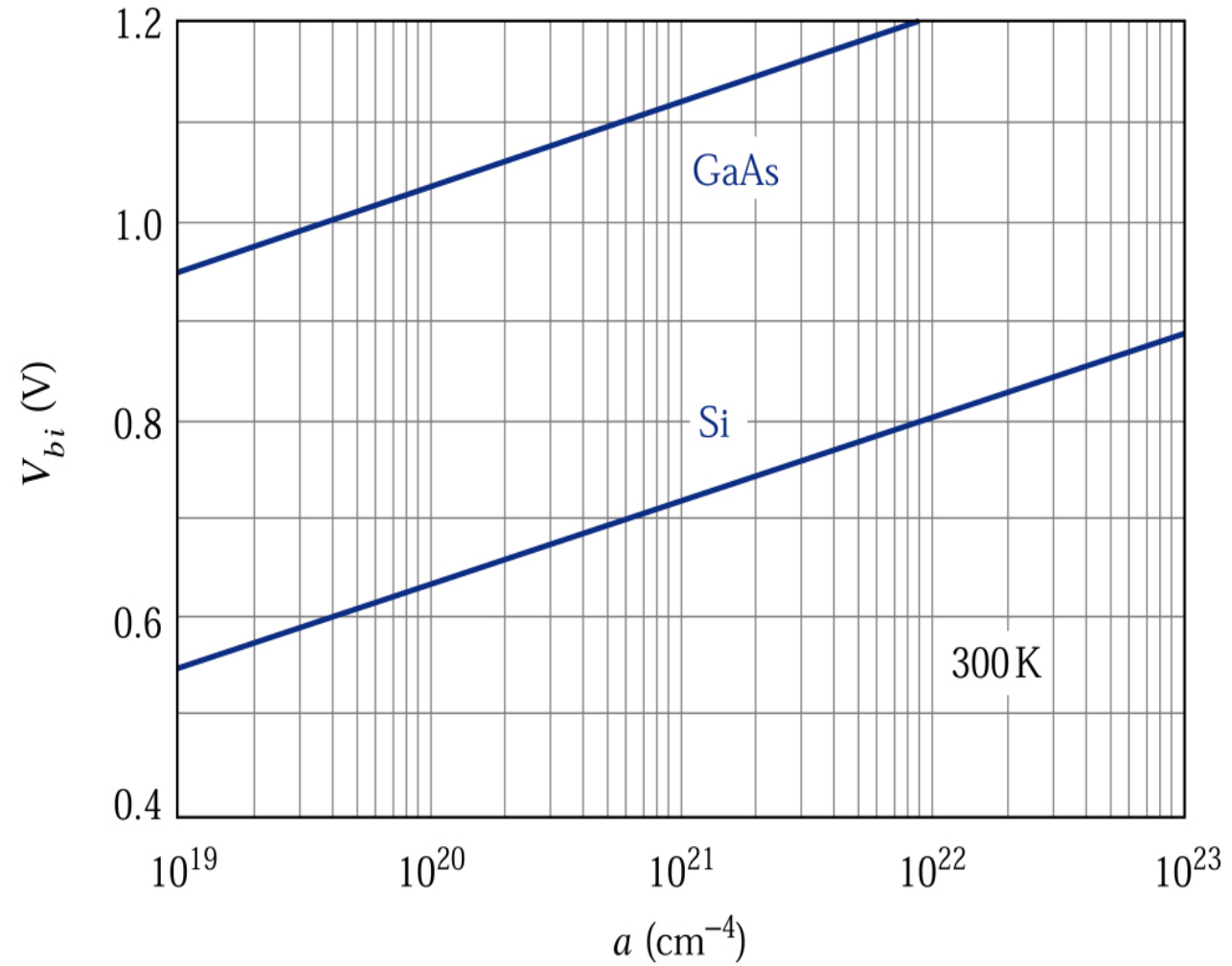


Figure 3.10. Built-in potential for a linearly graded junction in Si and GaAs as a function of impurity gradient.

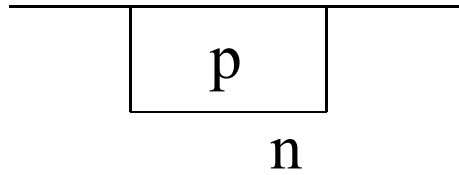
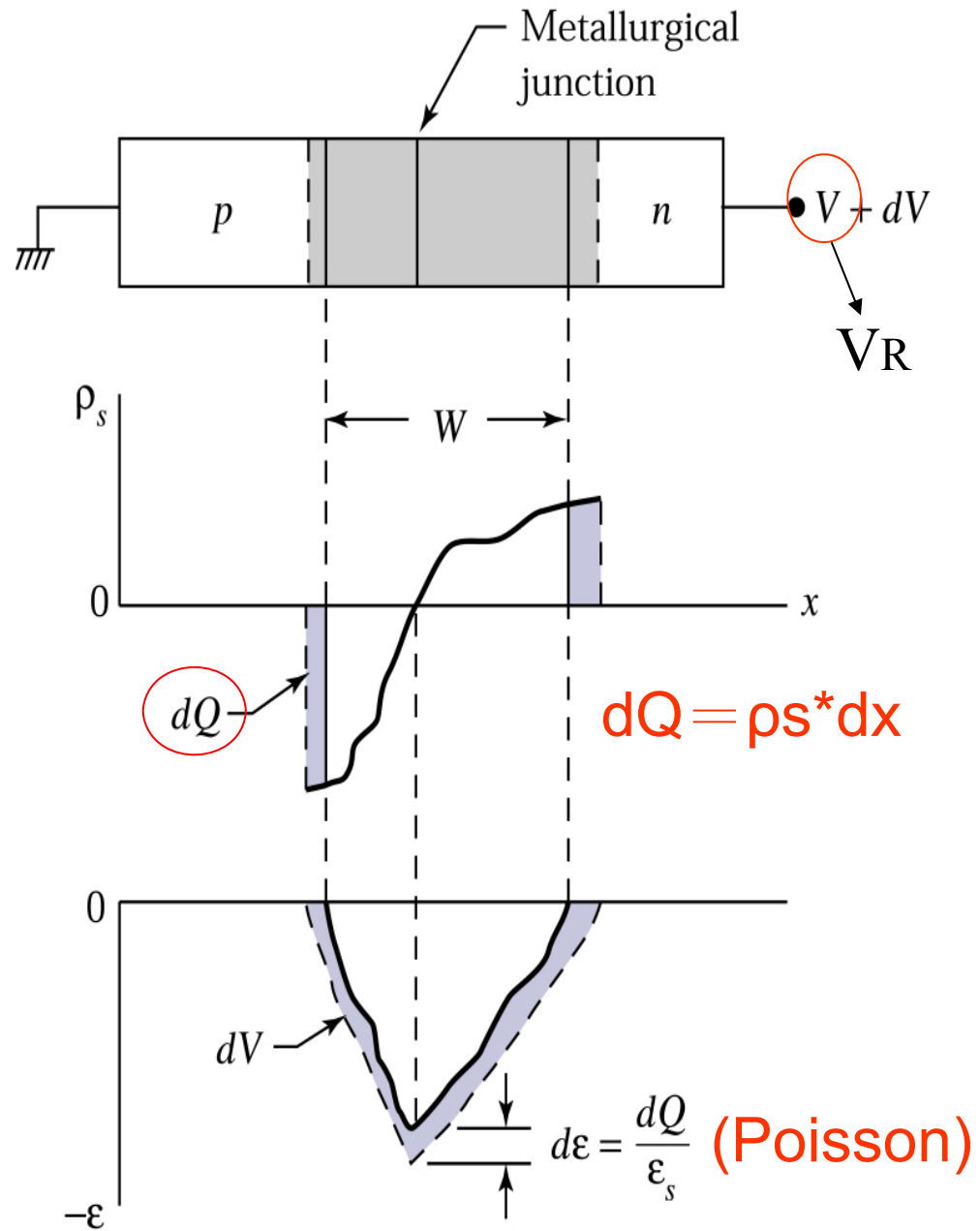


Figure 3.11.

(a) *p-n* junction with an arbitrary impurity profile under reverse bias. (b) Change in space charge distribution due to change in applied bias. (c) Corresponding change in electric-field distribution.



$$C_j \equiv \frac{dQ}{dV} = \frac{dQ}{W \frac{dQ}{\epsilon_s}} = \frac{\epsilon_s}{W} \quad (32)$$

For a one-sided abrupt junction

$$C_j = \frac{\epsilon_s}{W} = \sqrt{\frac{q \epsilon_s N_B}{2(V_{bi} - V)}}$$

$$\frac{1}{C_j^2} = \frac{2(V_{bi} - V)}{q \epsilon_s N_B}$$

* $1/C_j^2$ vs V 得到一直線

1: 斜率可算 N_B (bulk concentration)

2: 交點可算 V_{bi}

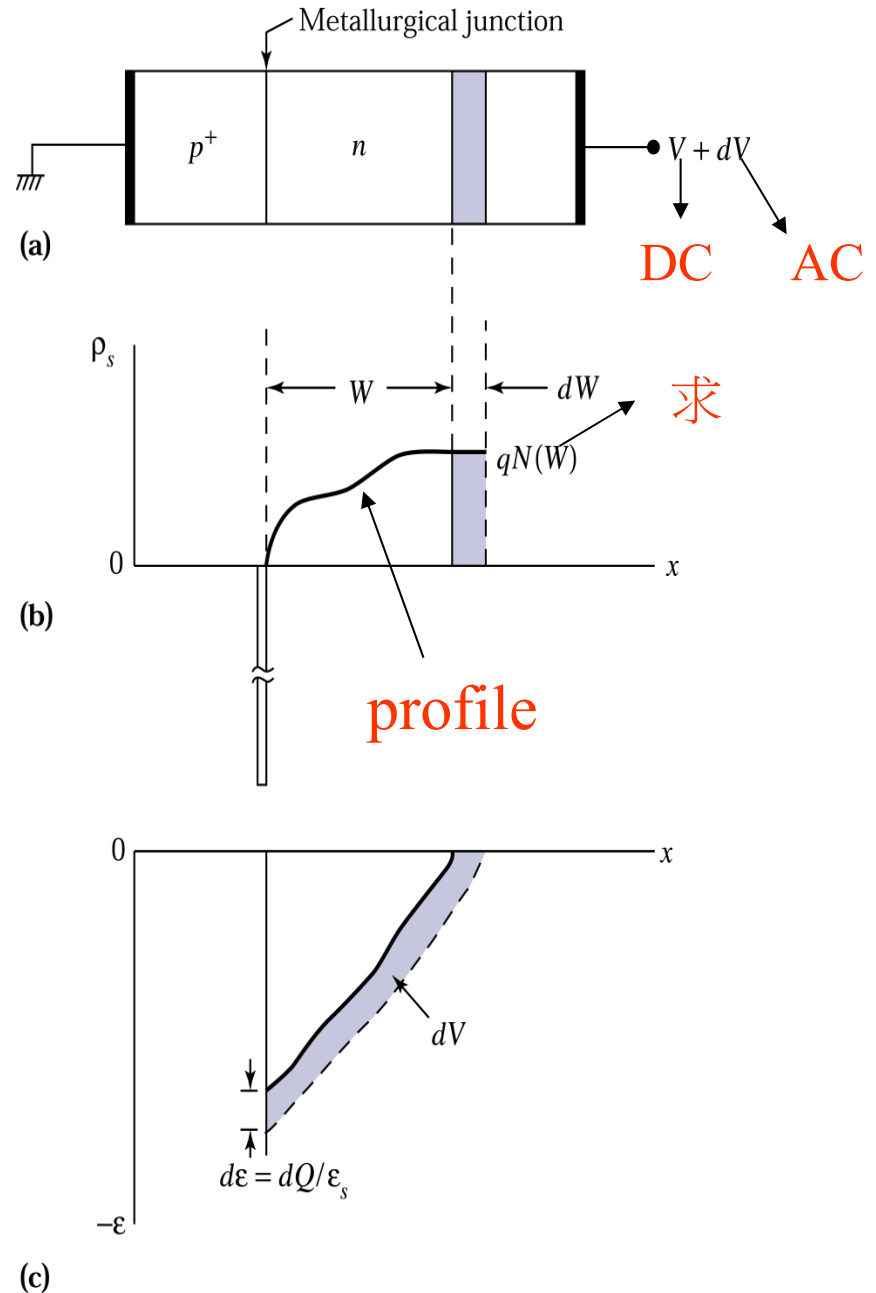
△ 測量 doping profile 用 C-V 法

○ abrupt junction

Figure 3.12.

(a) p^+ - n junction with an arbitrary impurity distribution. (b) Change in space charge distribution in the lightly doped side due to a change in applied bias. (c) Corresponding change in the electric-field distribution.

$$N(W) = \frac{2}{q\epsilon_s} \left(\frac{1}{\frac{d(1/C_j^2)}{dV}} \right)$$





C_j 正比 (V_R)⁻ⁿ
n = 1/2 for abrupt
n = 1/3 for linear graded

C_j 正比 (V_R)^{-1/(m+2)}
for hyperabrupt,
Epitaxial growth

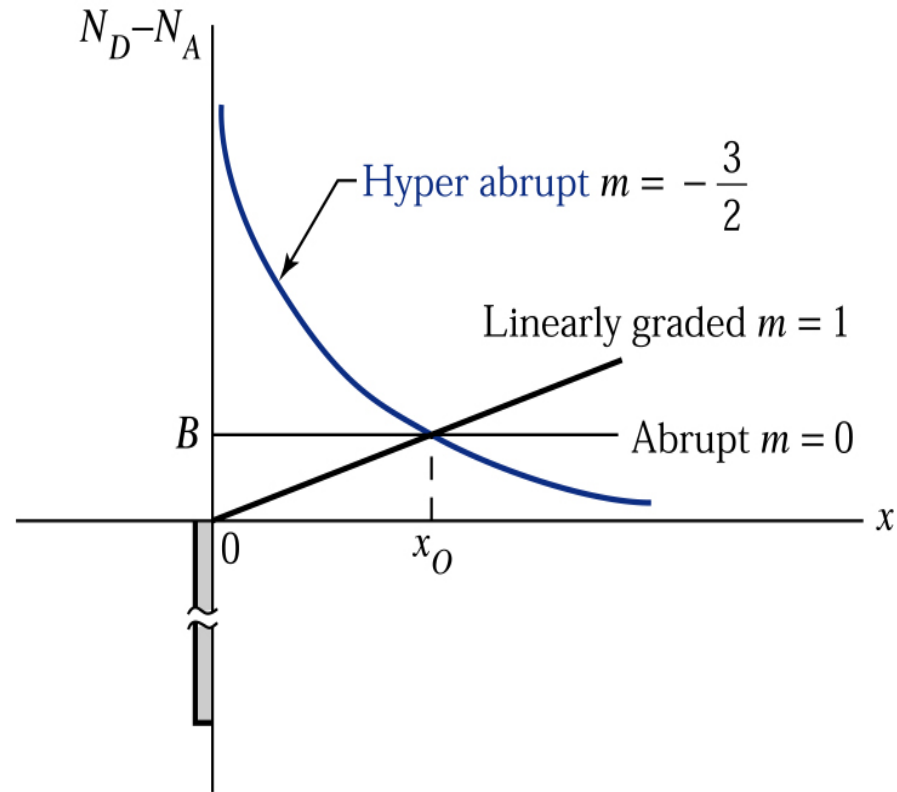
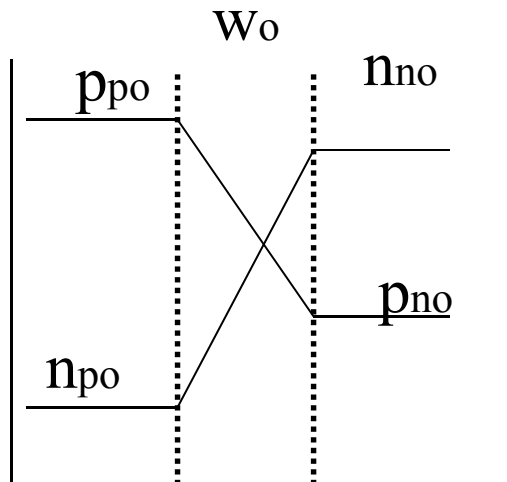
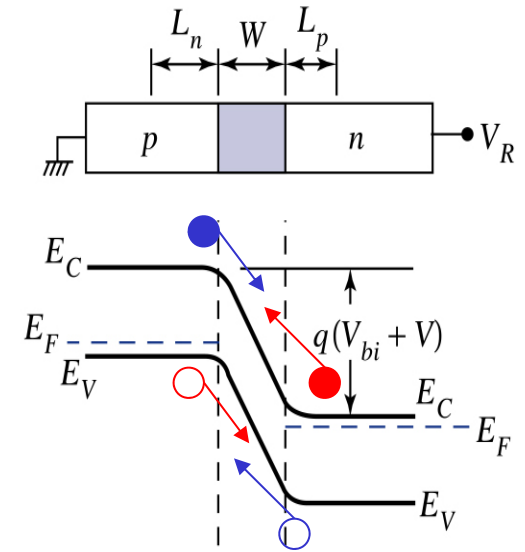
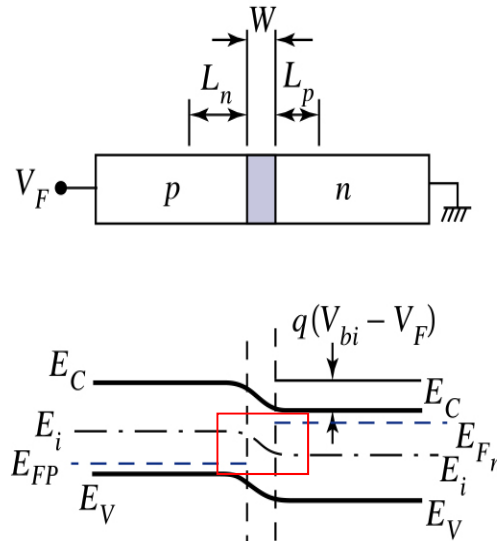


Figure 3.13. Impurity profiles for hyperabrupt, one-sided abrupt, and one-sided linearly graded junctions.

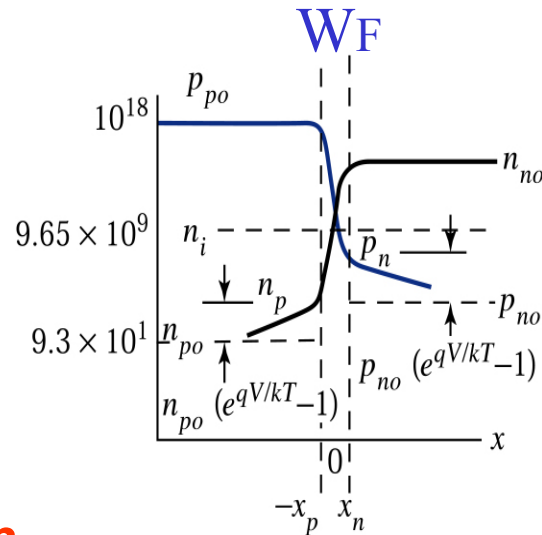
Figure 3.14.

Depletion region, energy band diagram and carrier distribution.

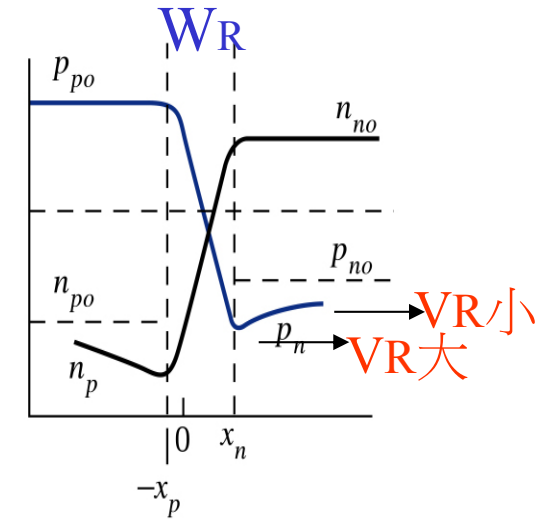
- (a) **Forward** bias.
- (b) **Reverse** bias.



Thermal equilibrium



(a)



(b)

Excess carrier

$$n_p - n_{po} = n_{po} \left(e^{qV/kT} - 1 \right).$$

$$p_n - p_{no} = p_{no} \left(e^{qV/kT} - 1 \right)$$

$$\therefore n_{no} = n_{po} e^{qV_{bi}/kT}$$

$$n_n = n_p e^{q(V_{bi} - V)/kT}, \quad n_n \sim n_{no}$$

$$n_p = n_{po} e^{qV/kT}$$

Excess carrier

$$J_p(x_n) = -qD_p \left. \frac{dp_n}{dx} \right|_{x_n} = \frac{qD_p p_{no}}{L_p} (e^{qV/kT} - 1).$$

$$J_n(-x_p) = qD_n \left. \frac{dn_p}{dx} \right|_{-x_p} = \frac{qD_n n_{po}}{L_n} (e^{qV/kT} - 1)$$

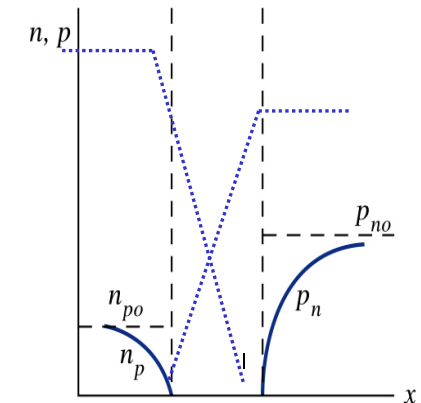
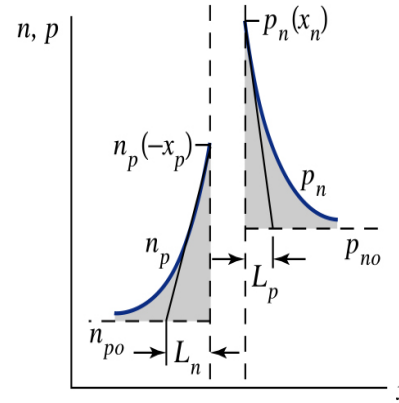
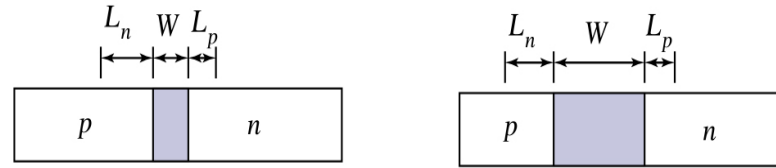
$$\therefore \frac{d^2 p_n}{dx^2} - \frac{p_n - p_{no}}{D_p \tau_p} = 0$$

$$p_n = p_{no} + p_{no} (e^{qV/kT} - 1) e^{-(x-x_n)/L_p}$$

Figure 3.15.

Injected minority carrier distribution and electron and hole currents. (a) Forward bias. (b) Reverse bias. The figure illustrates idealized currents. For practical devices, the currents are not constant across the space charge layer.

$p+ / n$
 $J_p \gg J_n$



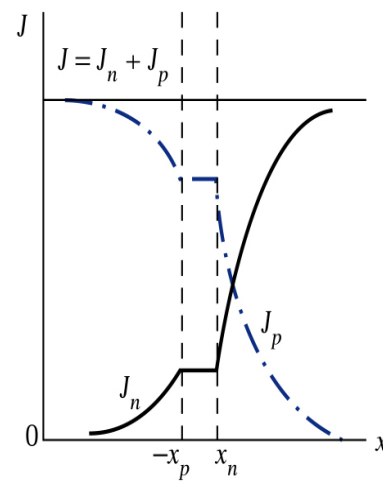
Ideal diode eq. (僅求 diff. current)

$$J = J_p(x_n) + J_n(-x_p) = J_s \left(e^{qV/kT} - 1 \right),$$

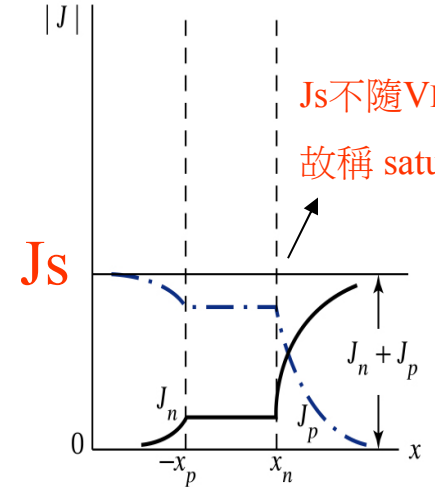
V 為負時, 可略 $J = J_s$ 為定值

$$J_s \equiv \frac{qD_p p_{no}}{L_p} + \frac{qD_n n_{po}}{L_n},$$

: sat. current density



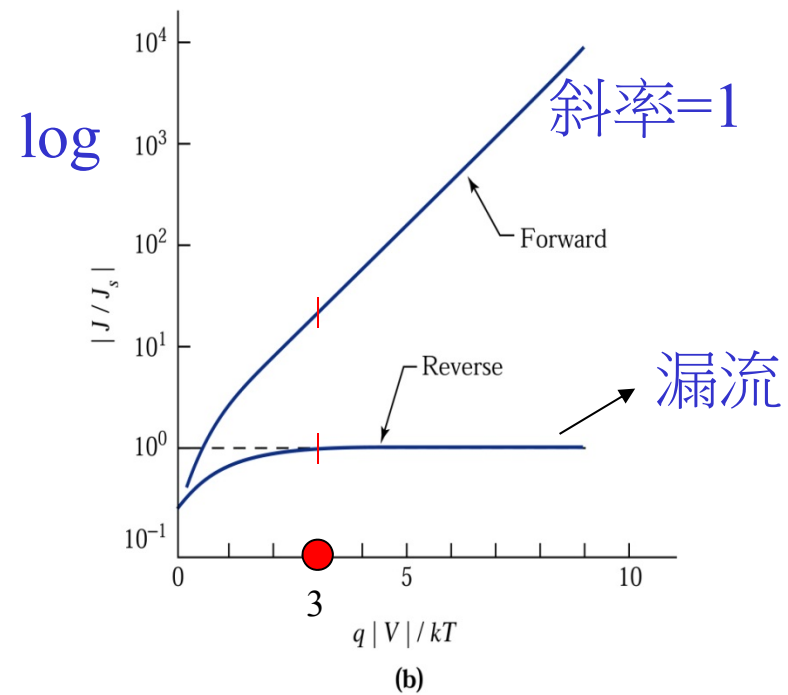
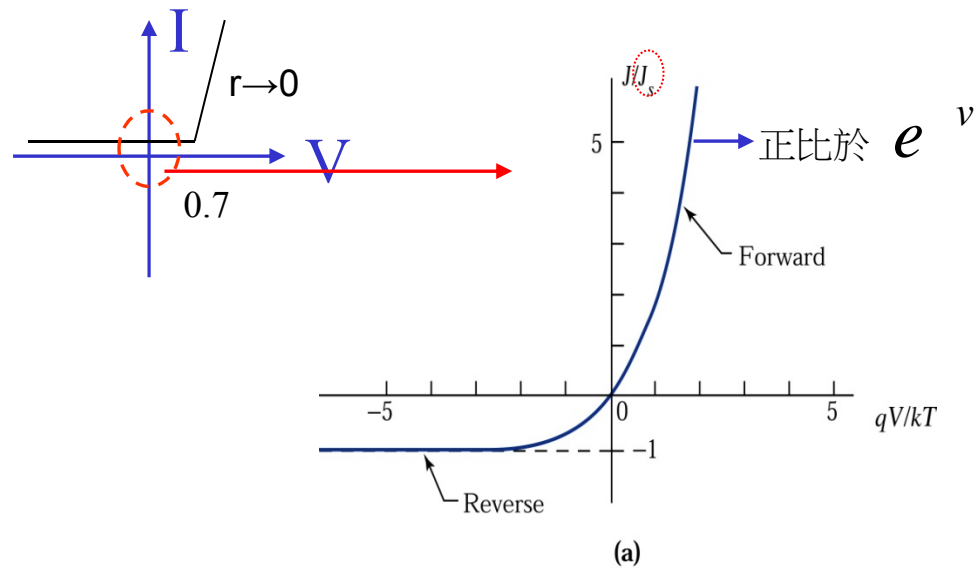
(a)



(b)

J_s 不隨 VR 而變
 故稱 saturate

Figure 3.16.
Ideal current-voltage characteristics.
 (a) Cartesian plot.
 (b) Semilog plot.



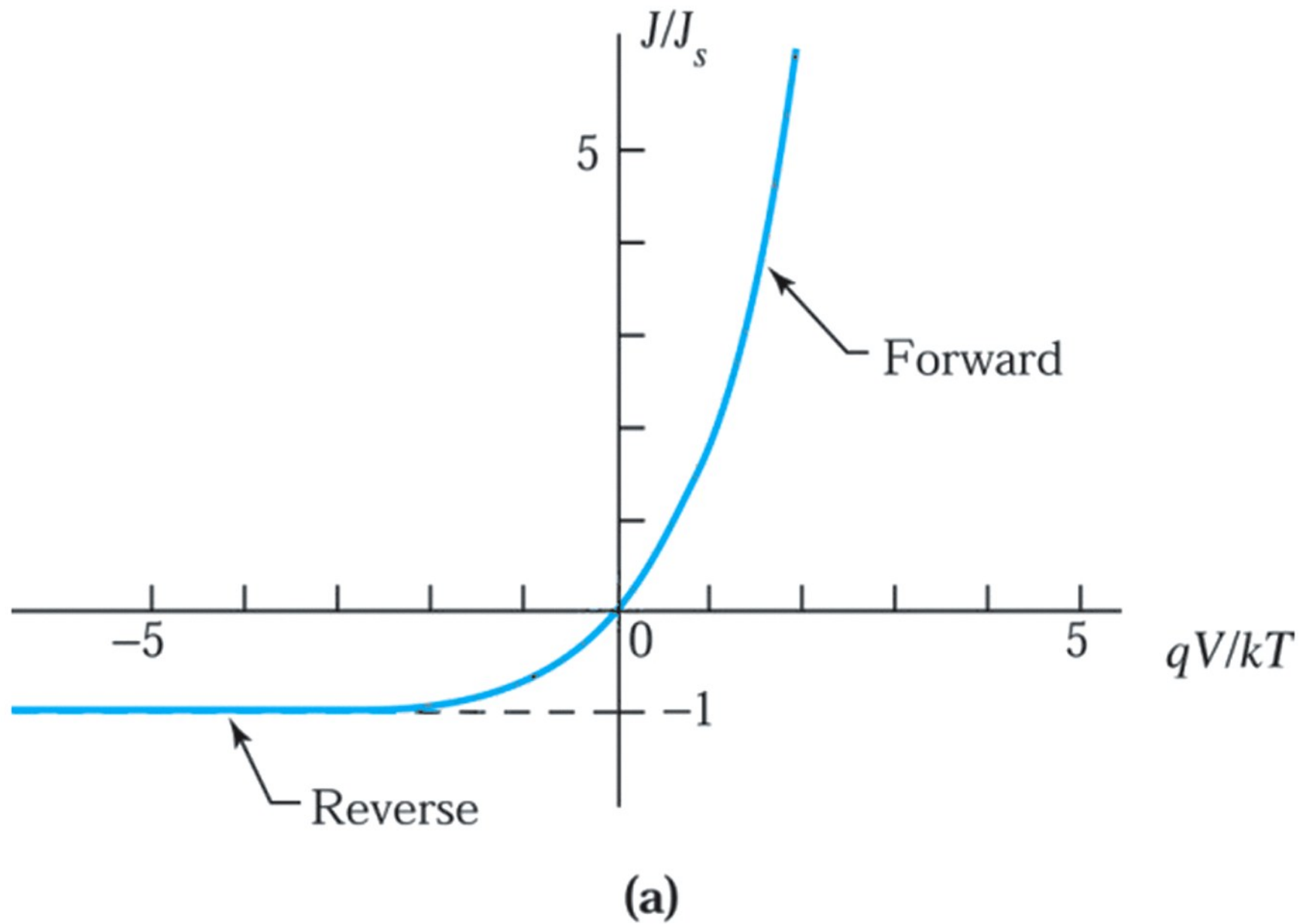


Figure 3.16a
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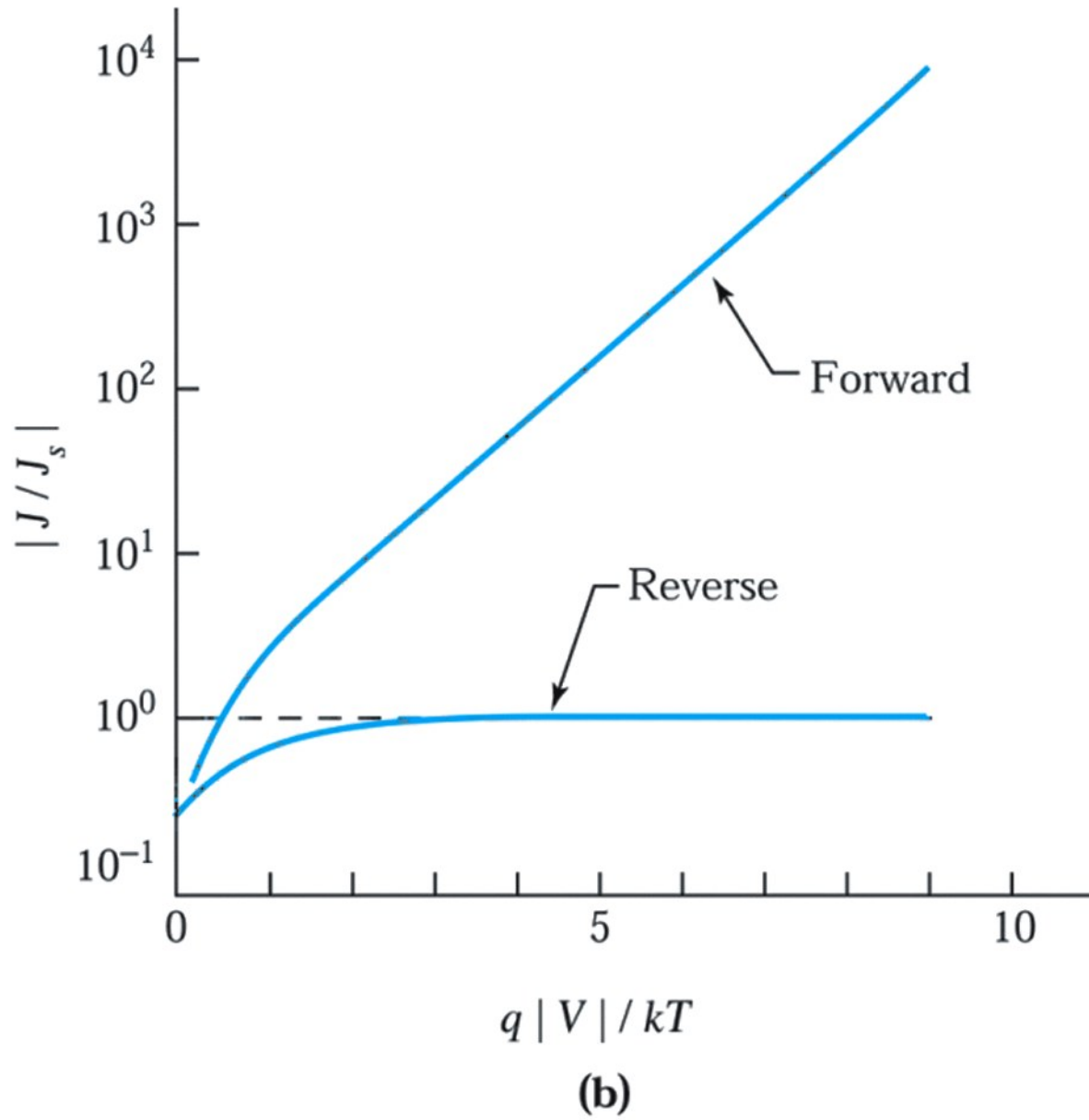


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Generation-Recombination and high-injection Effect*

$$G = -U \approx \left[\frac{\sigma_p \sigma_n V_{th} N_t}{\sigma_n \exp\left[\frac{E_t - E_i}{kT}\right] + \sigma_p \exp\left[\frac{E_i - E_t}{kT}\right]} \right] = \frac{n_i}{\tau_g}$$

空乏區之產生電流

(因為 $G = n_i / \tau_g$)

Assume $\sigma_n = \sigma_p = \sigma_0$

$$G \approx \frac{\sigma_0 V_{th} N_t n_i}{2 \cosh\left[\frac{E_t - E_i}{kT}\right]}$$

G = Max at $E_t = E_i$

Generation-Recombination and high-injection Effect*

空乏區之產生電流

(因為 $G = n_i / \tau_g$)

$$J_{gen} = \int_0^W qGdx \cong qGW = \frac{qn_i W}{\tau_g}$$

For p+/n ($N_A \gg N_D$) (55a)

(J_{diff} 為 $V_R \geq 3kT/q$
下由(55)得到)

反向 $J_R \cong q \sqrt{\frac{D_p}{\tau_p} \frac{n_i^2}{N_D}} + \frac{qn_i W}{\tau_g}$

Ge: n_i 大, **Jdiff** dominate
Si: n_i 小, **Jgen** dominate

Recombination Effect*

$$n_n p_n \cong n_{n0} p_{n0} e^{qV/kT} = n_i^2 e^{qV/kT}$$

Assume $\sigma_n = \sigma_p = \sigma_0$

$$U \approx \frac{\sigma_0 V_{th} N_t n_i^2 (e^{qV/kT} - 1)}{n_n + p_n + 2n_i \cosh \frac{E_i - E_t}{kT}}$$

at $E_t = E_i$

$$U \approx \sigma_0 V_{th} N_t \frac{n_i^2 (e^{qV/kT} - 1)}{n_n + p_n + 2n_i}$$

Recombination Effect*

At $n_n + p_n = \text{min.}$, $U = \text{Max.}$

$$n_n p_n = \text{常數}$$

$$dn_n p_n + n_n dp_n = 0$$

$$-dn_n = \frac{n_n}{p_n} dp_n$$

$$d(p_n + n_n) = 0$$

$$dp_n = -dn_n = \frac{n_n}{p_n} dp_n$$

$$p_n = n_n = n_i e^{qV/2kT}$$

$$U \text{ max} \approx \sigma_0 V_{th} N_t \frac{n_i^2 (e^{qV/kT} - 1)}{2n_i (e^{qV/2kT} + 1)}$$

$$U \text{ max} \approx \frac{1}{2} \sigma_0 V_{th} N_t n_i e^{qV/2kT}$$

Recombination Effect*

$$J_{rec} = \int_0^W qUdx \cong \frac{qW}{2} \sigma_o v_{th} N_t n_i e^{qV/2kT} = \frac{qWn_i}{2\tau_r} e^{qV/2kT}$$

$$J_F = q \sqrt{\frac{D_p}{\tau_p} \frac{n_i^2}{N_D}} e^{qV/kT} + \frac{qWn_i}{2\tau_r} e^{qV/2kT}$$

正向

J_{diff} J_{rec}

$$J_F \approx \exp\left(\frac{qV}{\eta kT}\right), \quad 1 < \eta < 2$$

Where the factor η is called the ideality factor

p / n junction 好壞之重要參數

Figure 3.17.

Comparison of the forward current-voltage characteristics of Si and GaAs diodes² at 300 K. Dashed lines indicate slopes of different ideality factors η .

ref:(69)

***high level injection**:(p+-n為例)

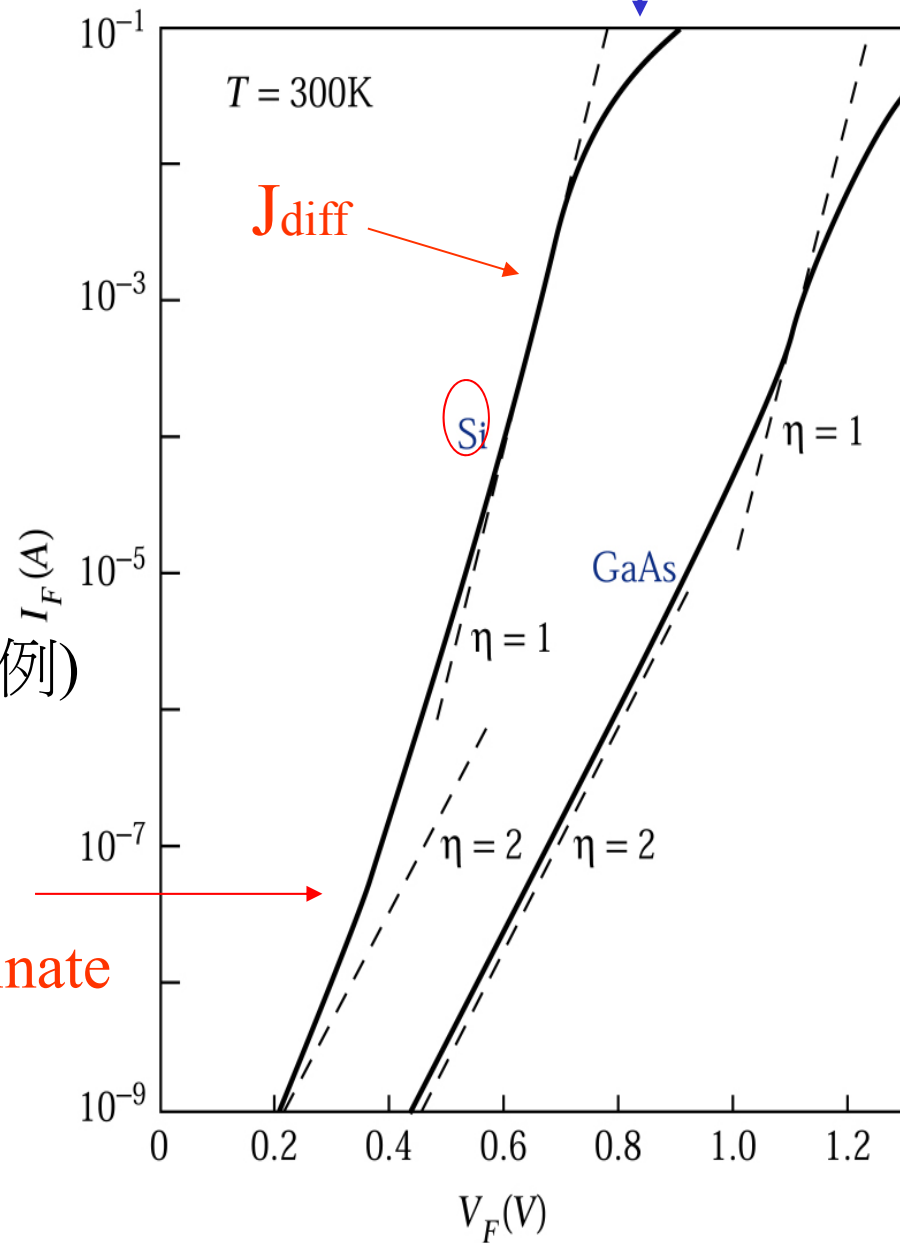
即 $p_n(x_n) \approx n_{no}$

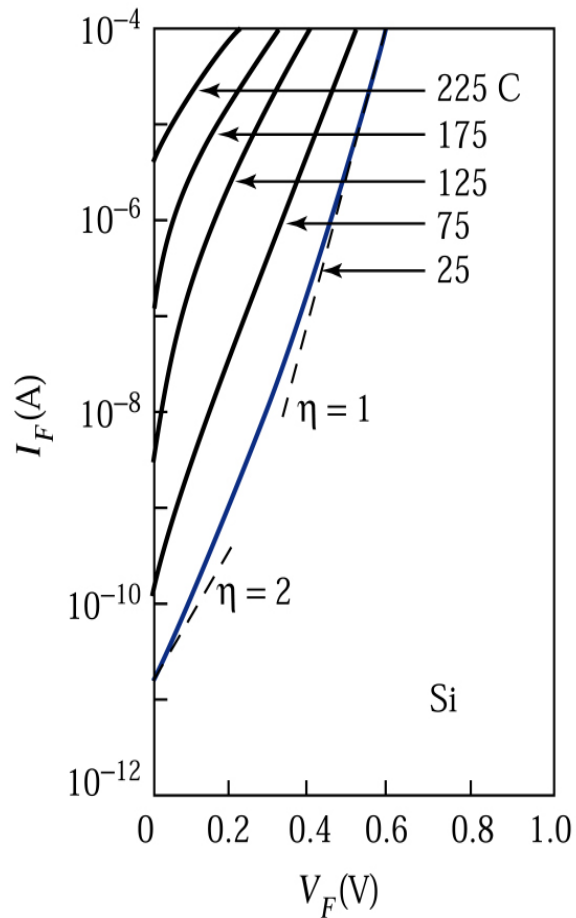
由(60) $p_n n_n = n_i^2 e^{qv/kt}$ **Jrec**

故 $p_n \approx n_n \approx n_i e^{2qv/kt}$ **dominate**

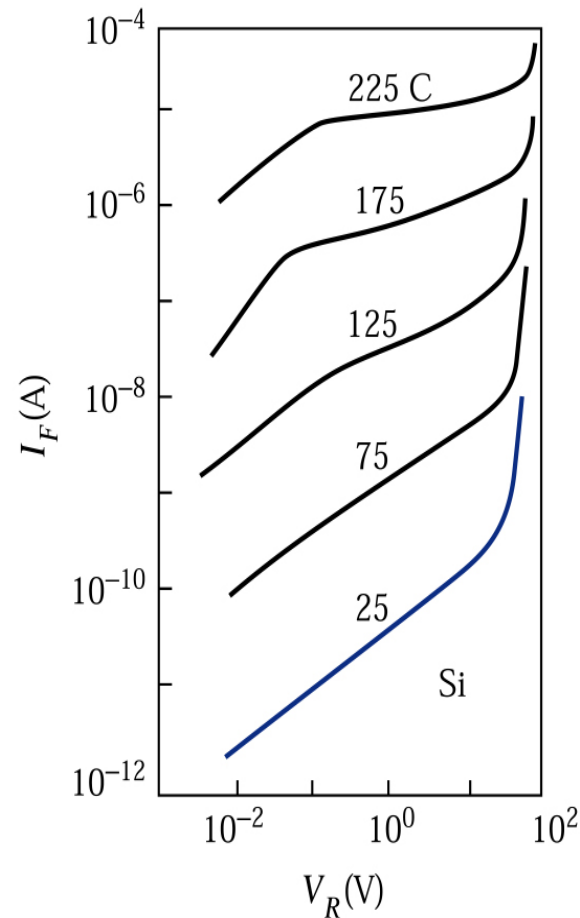
電流增加較緩

串聯電阻和 **high level inj.**





(a)



(b)

(a) With increasing temp, increase of $I_{diff} > I_{recom}$ due to J_s

(b) Low temp, J_g dominate $\sim (V_R)^{1/2}$; high temp, J_s dominate

Trap analysis should be low temp

Figure 3.18. Temperature dependence of the current-voltage characteristics of a Si diode². (a) Forward bias. (b) Reverse bias.

Minority Carrier Storage

☆在Forward bias 會有

$$Q_p = q \int_{x_n}^{\infty} (\underbrace{p_n - p_{no}}_{\text{excess}}) dx,$$
$$= q \int_{x_n}^{\infty} p_{no} \left(e^{qV/kT} - 1 \right) e^{-(x-x_n)/L_p} dx,$$
$$= qL_p p_{no} \left(e^{qV/kT} - 1 \right).$$

$$Q_p = \frac{L_p^2}{D_p} J_p(x_n) = \tau_p \underbrace{J_p(x_n)}_{\text{電流}}.$$

Minority life time

電流

Diffusion Capacitance

☆在 **forward bias** 下：

*反向有 C_j

*順向有 C_d

definition $C_d = AdQ_p / dV$ dV 指 A.C.

$$C_d = \frac{Aq^2 L_p p_{no}}{kT} e^{qV/kT}$$

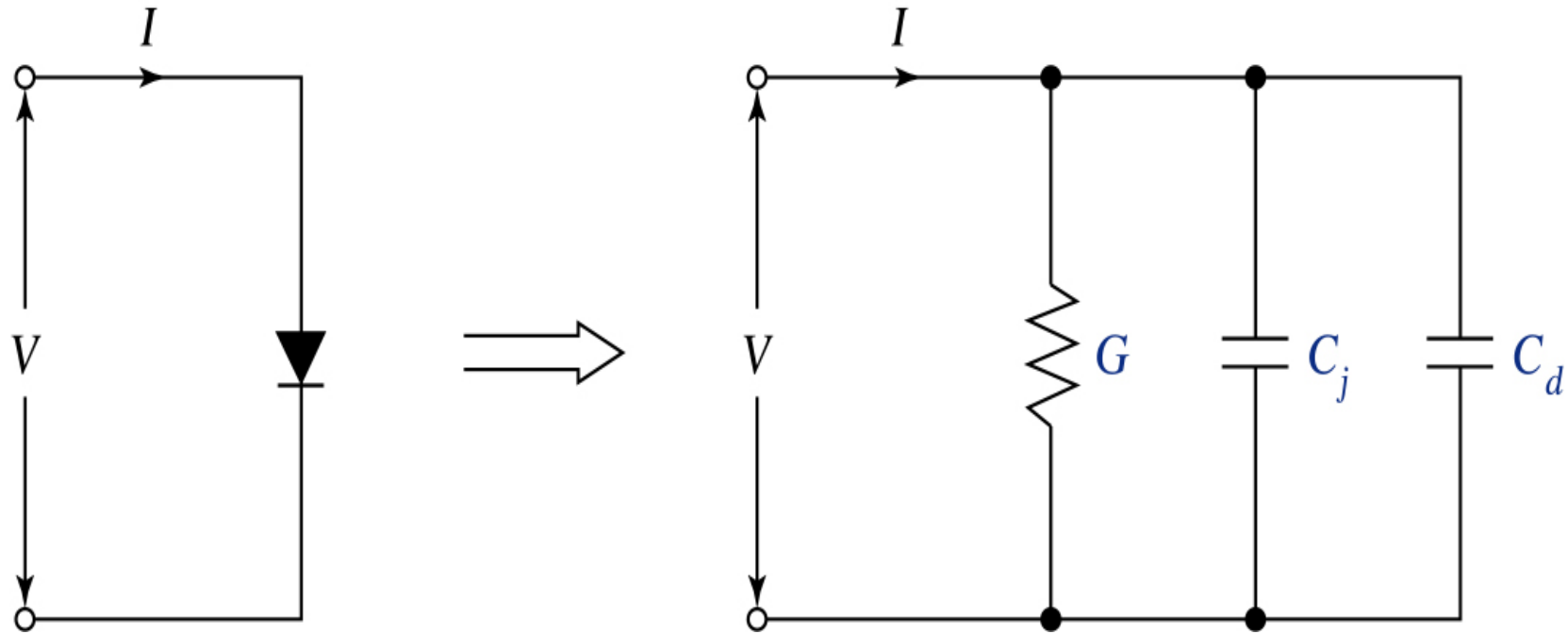


Figure 3.19. Small-signal equivalent circuit of a p - n junction.

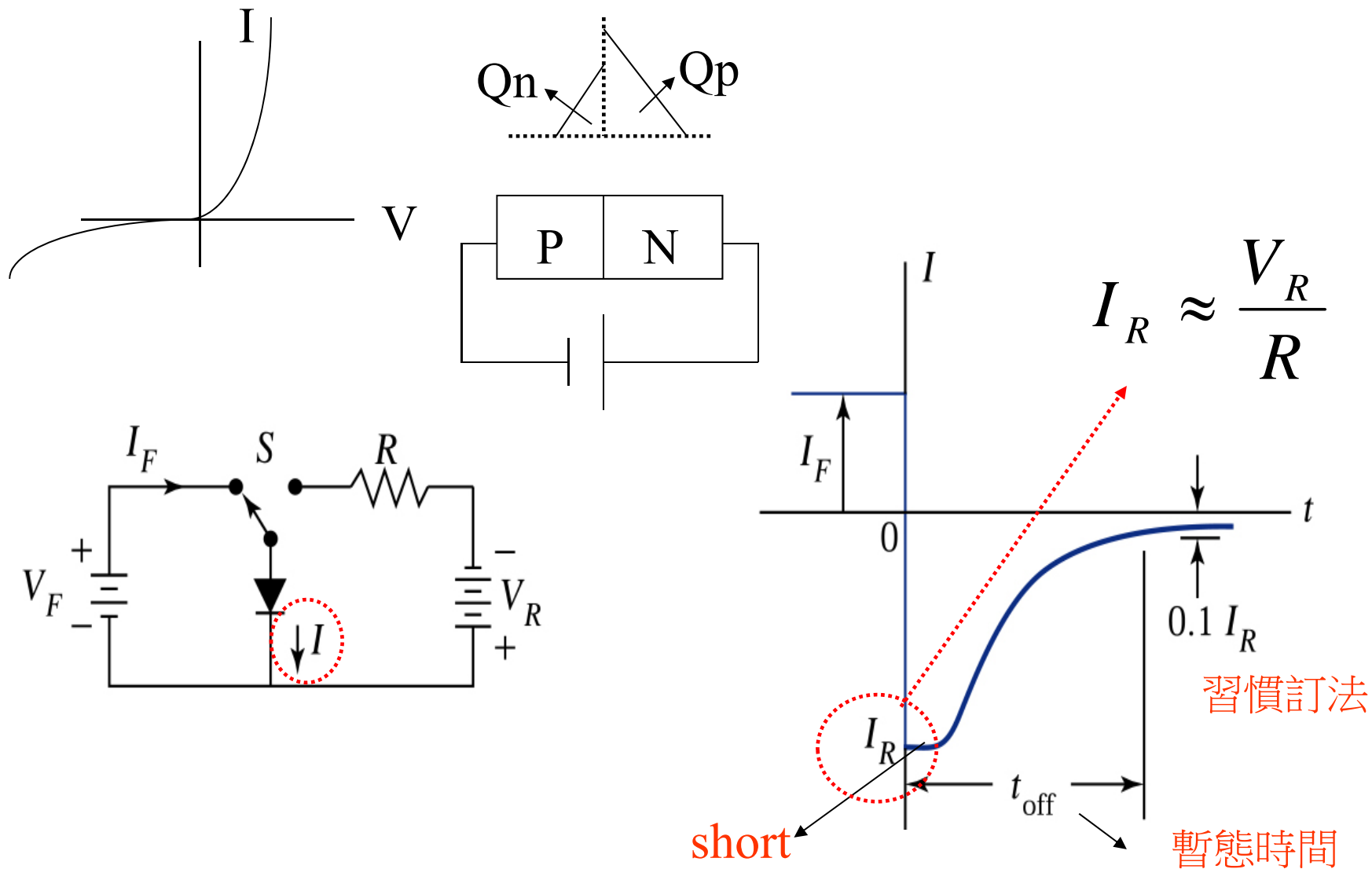


Figure 3.20. Transient behavior of a p - n junction (a) Basic switching circuit. (b) Transient response of the current switched from forward bias to reverse bias.

• $t_{off} \downarrow$ 越好

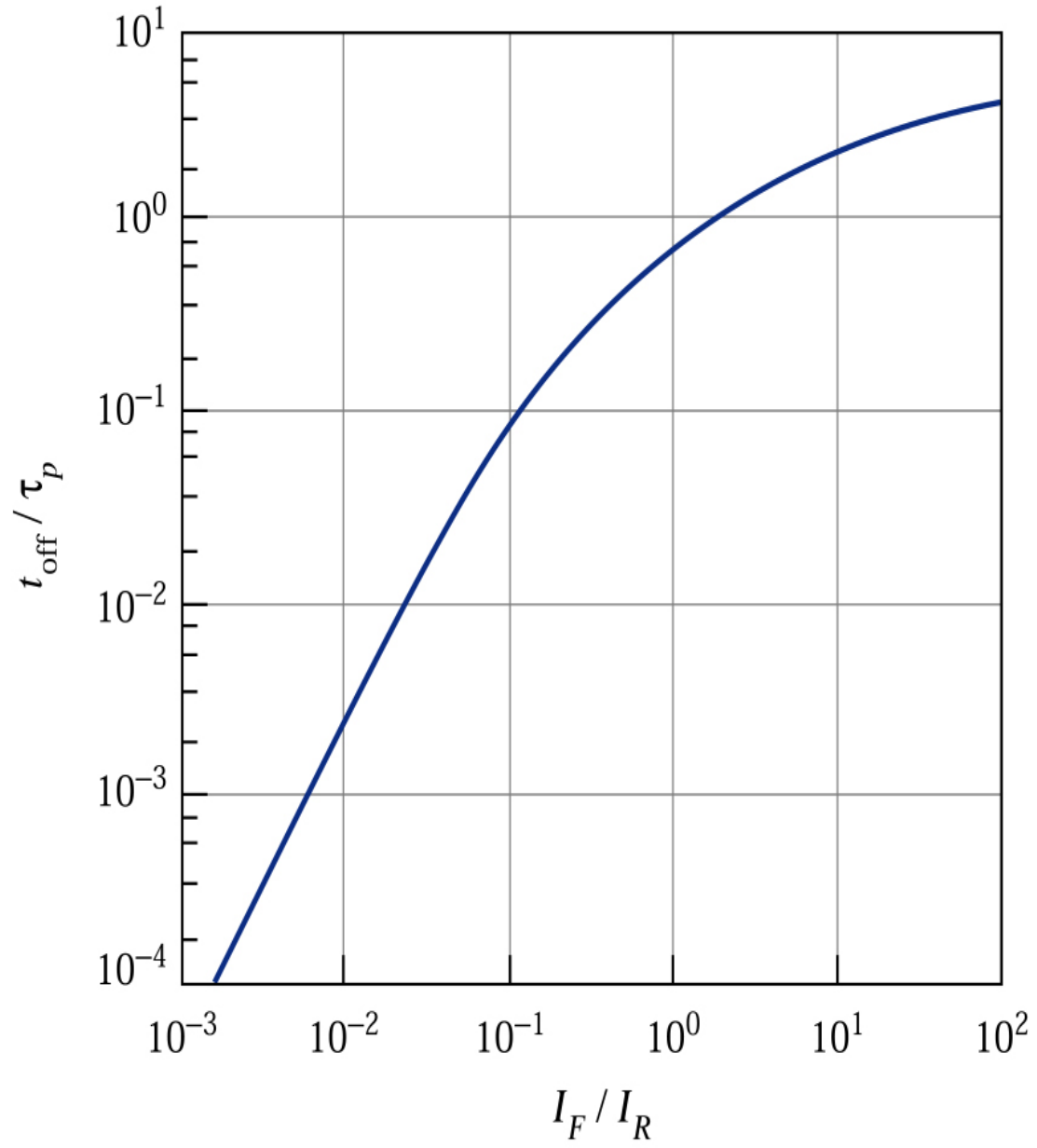
$$t_{off} \approx \frac{Q_p A}{I_{R,ave}} = \tau_p \left[\frac{I_F}{I_{R,ave}} \right]$$

☆ For 快速switch，使 $\tau_p \downarrow$

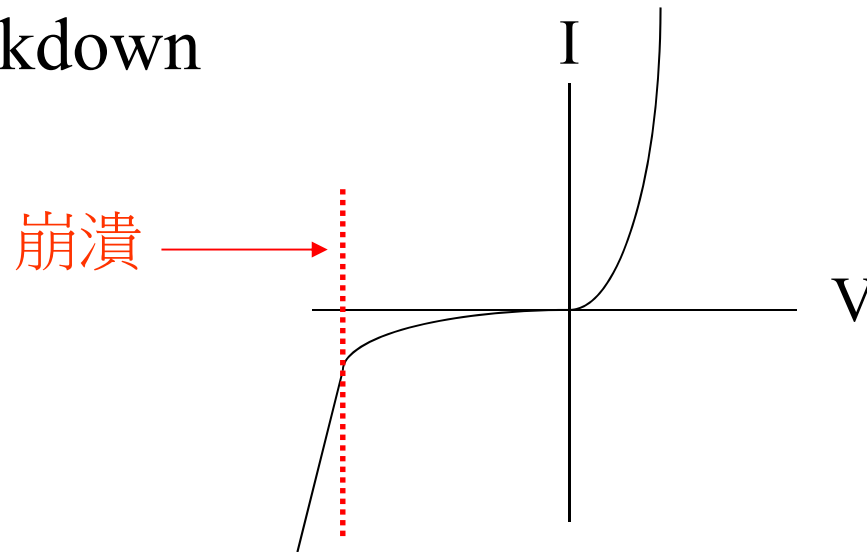
⇒ 在 E_i 增加 G-R center

(doping Au 等)

Figure 3.21.
Normalized transient time
versus the ratio of forward
current to reverse current.³



* Junction breakdown



* Tunneling Effect

☆ p , n 濃度都很高(W 小)

* Avalanche Multiplication

impact ionization

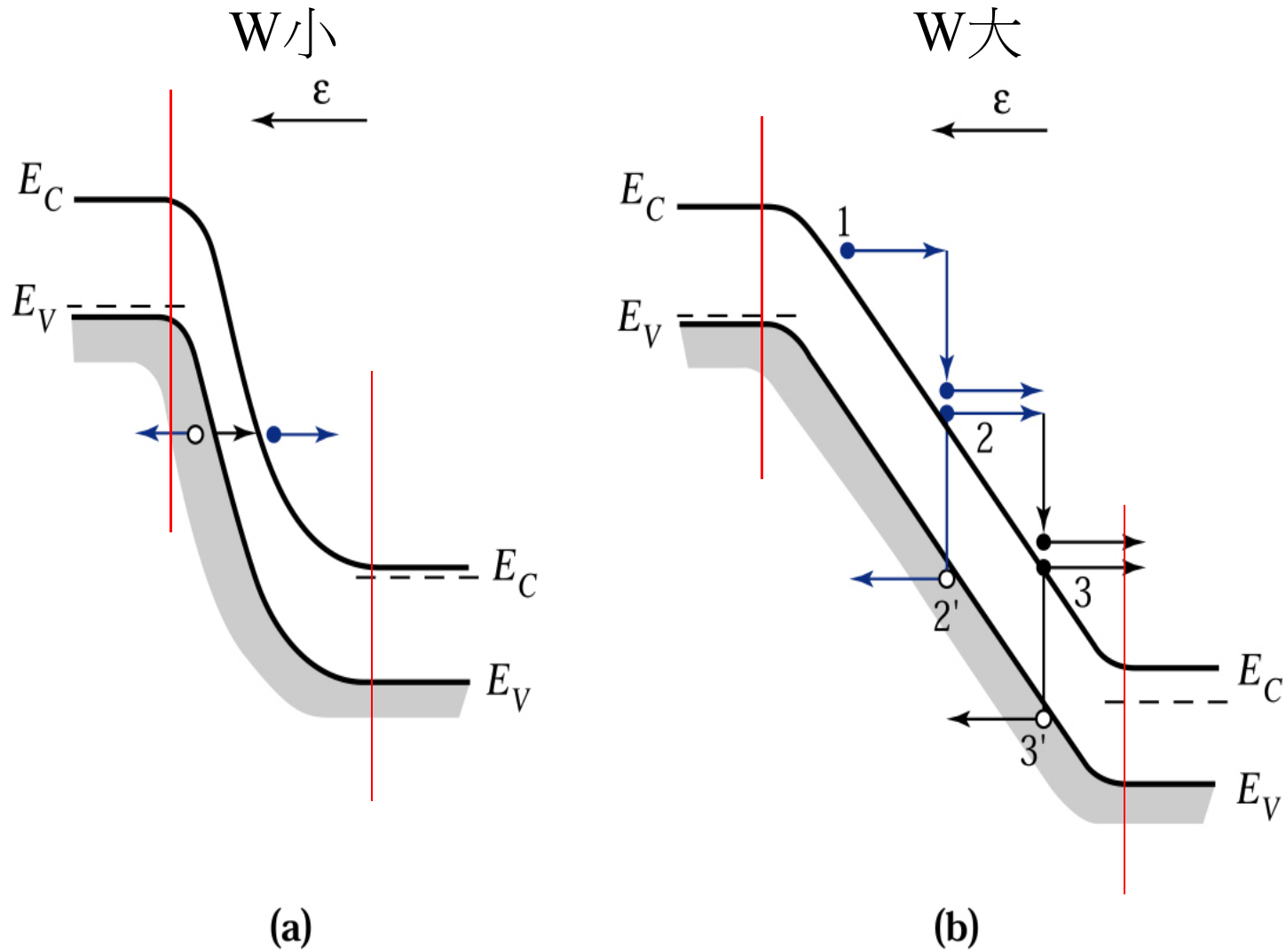


Figure 3.22. Energy band diagrams under junction-breakdown conditions. (a) Tunneling effect (b) Avalanche multiplication.

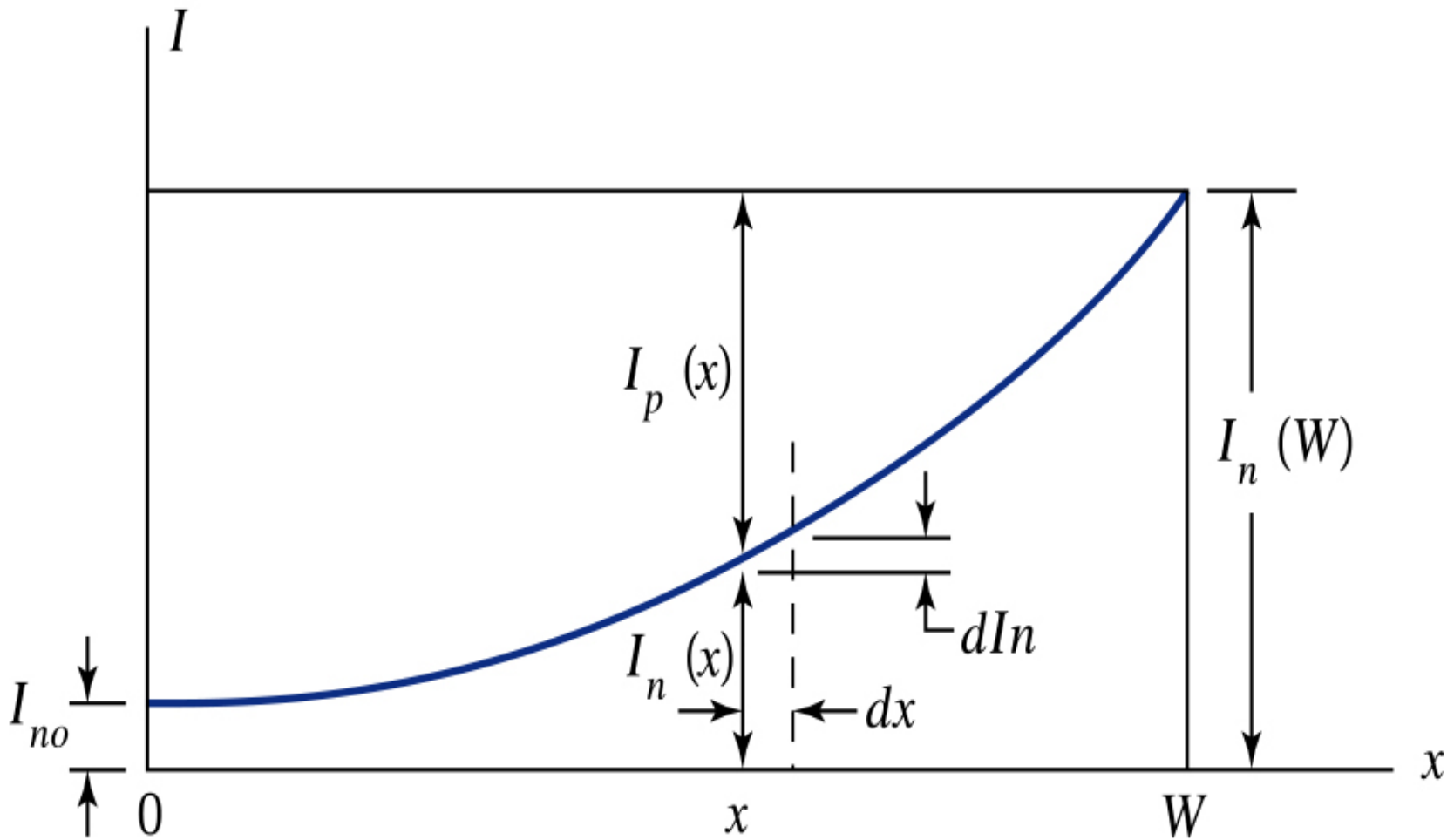


Figure 3.23. Depletion region in a p - n junction with multiplication of an incident current.

$$V_B \text{ (breakdown voltage)} = \frac{E_c W}{2} = \frac{\epsilon_s E_c^2}{2q} (N_B)^{-1} \quad W = \frac{\epsilon_s E_c}{qN_B}$$

$N_B \uparrow \rightarrow V_B \downarrow$

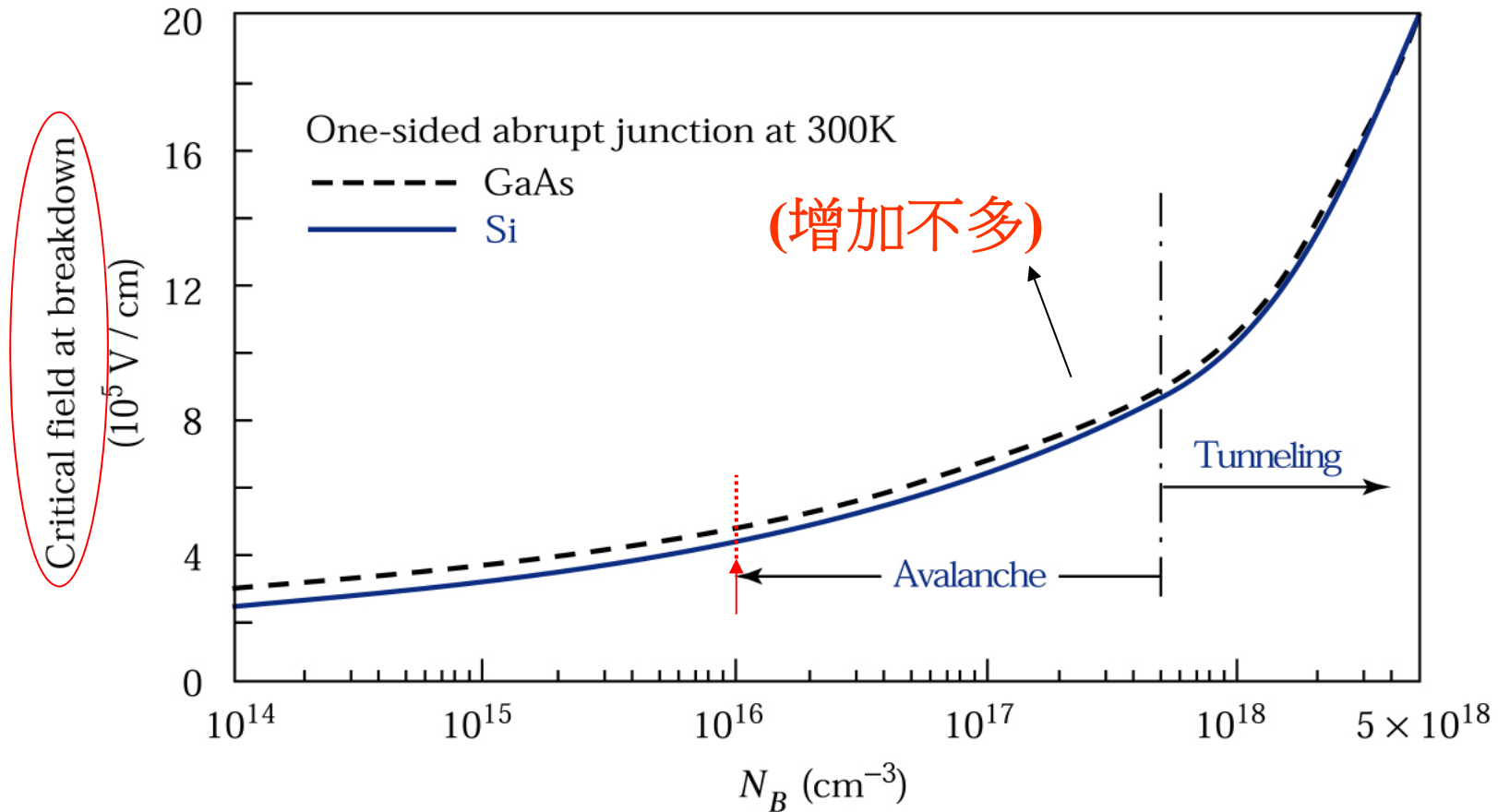


Figure 3.24. Critical field at breakdown versus background doping for Si and GaAs one-sided abrupt junctions.⁵

Figure 3.25. Avalanche breakdown voltage versus impurity concentration for one-sided abrupt junction and avalanche breakdown voltage versus impurity gradient for linearly graded junction in Si and GaAs. **Dash-dot line indicates the onset of the tunneling mechanism.**⁵

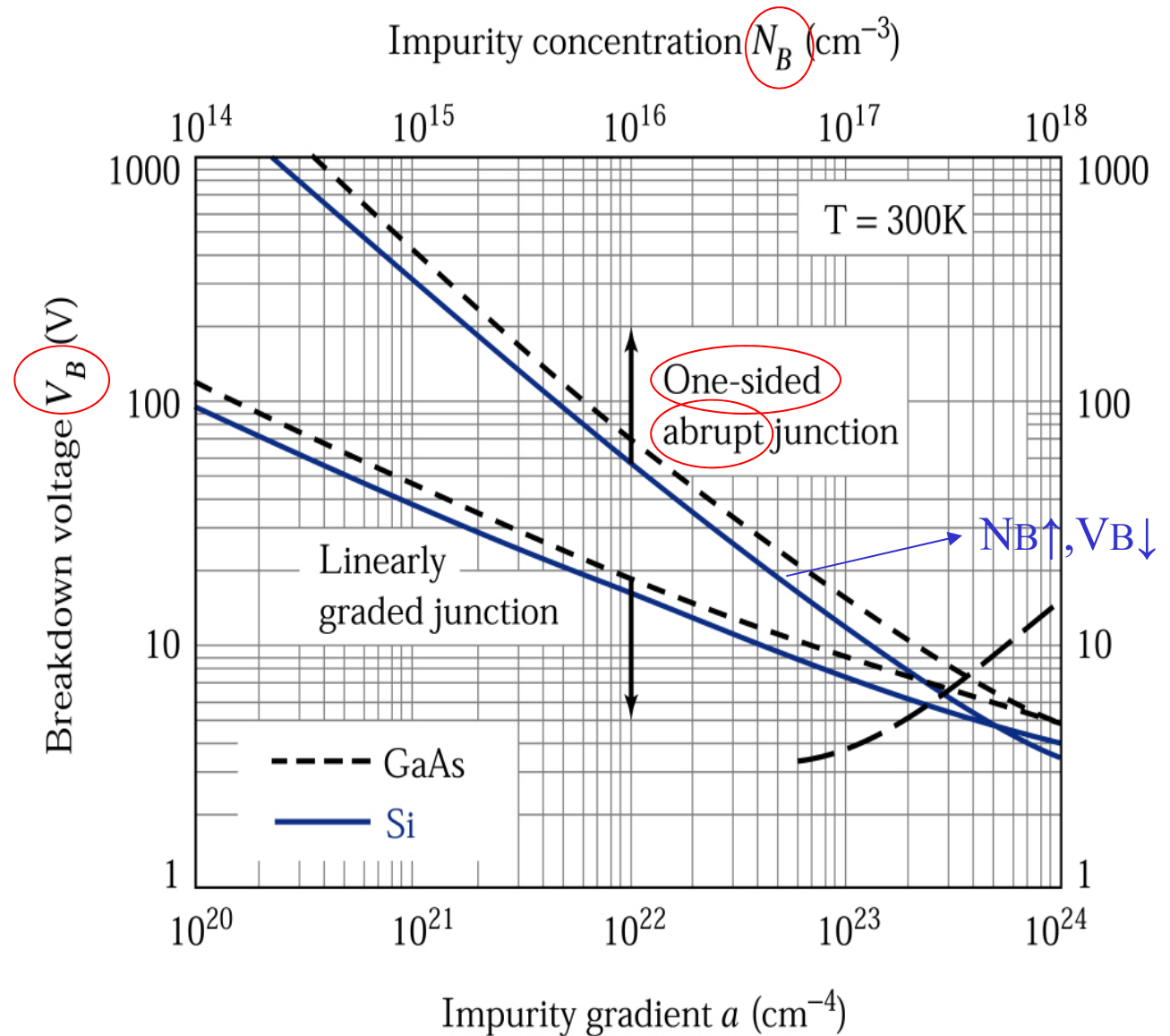
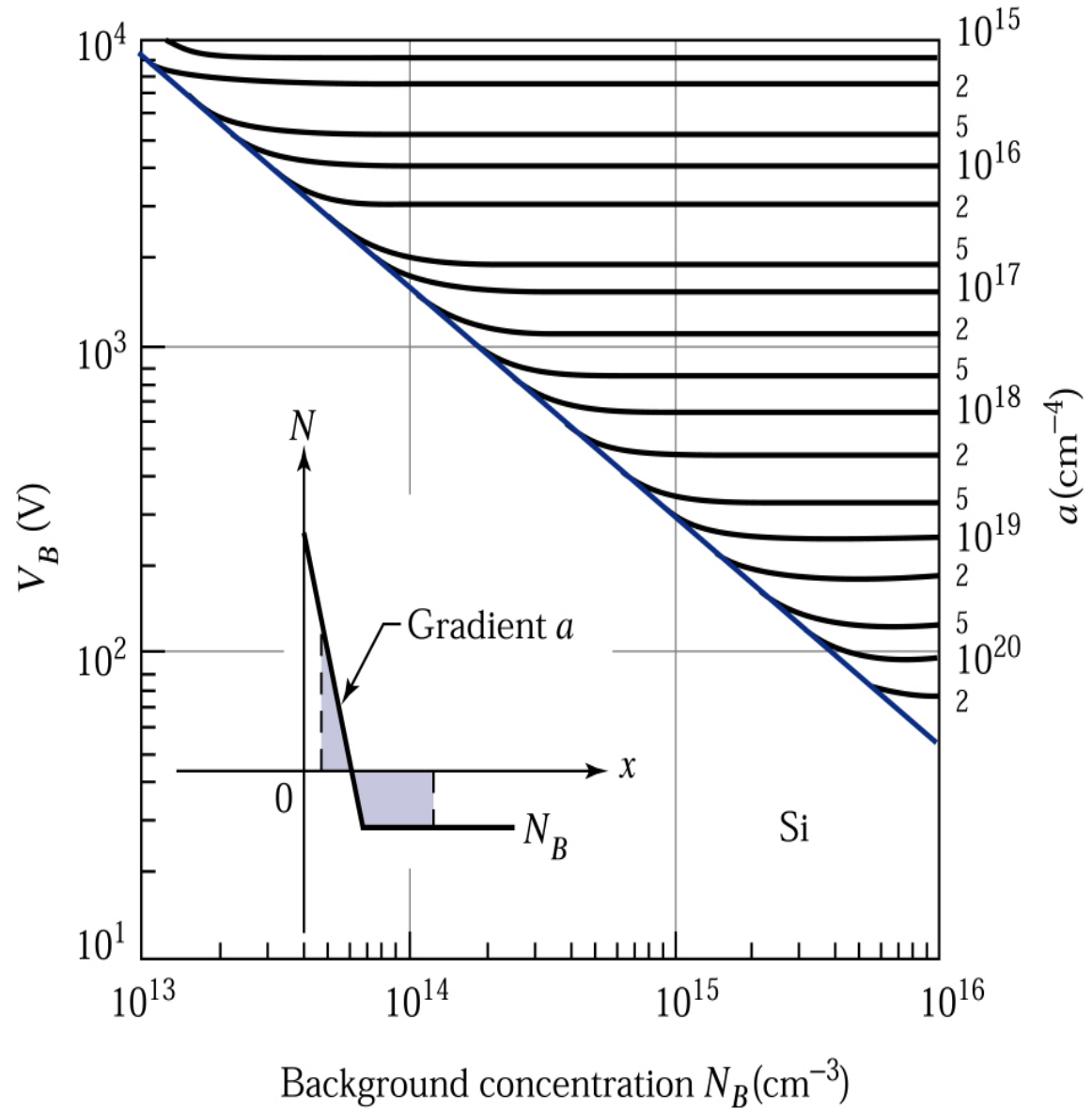


Figure 3.26.
Breakdown voltage for
diffused junctions.
Inset shows the space
charge distribution.⁶



$N_B \downarrow, a \downarrow, V_B \uparrow$

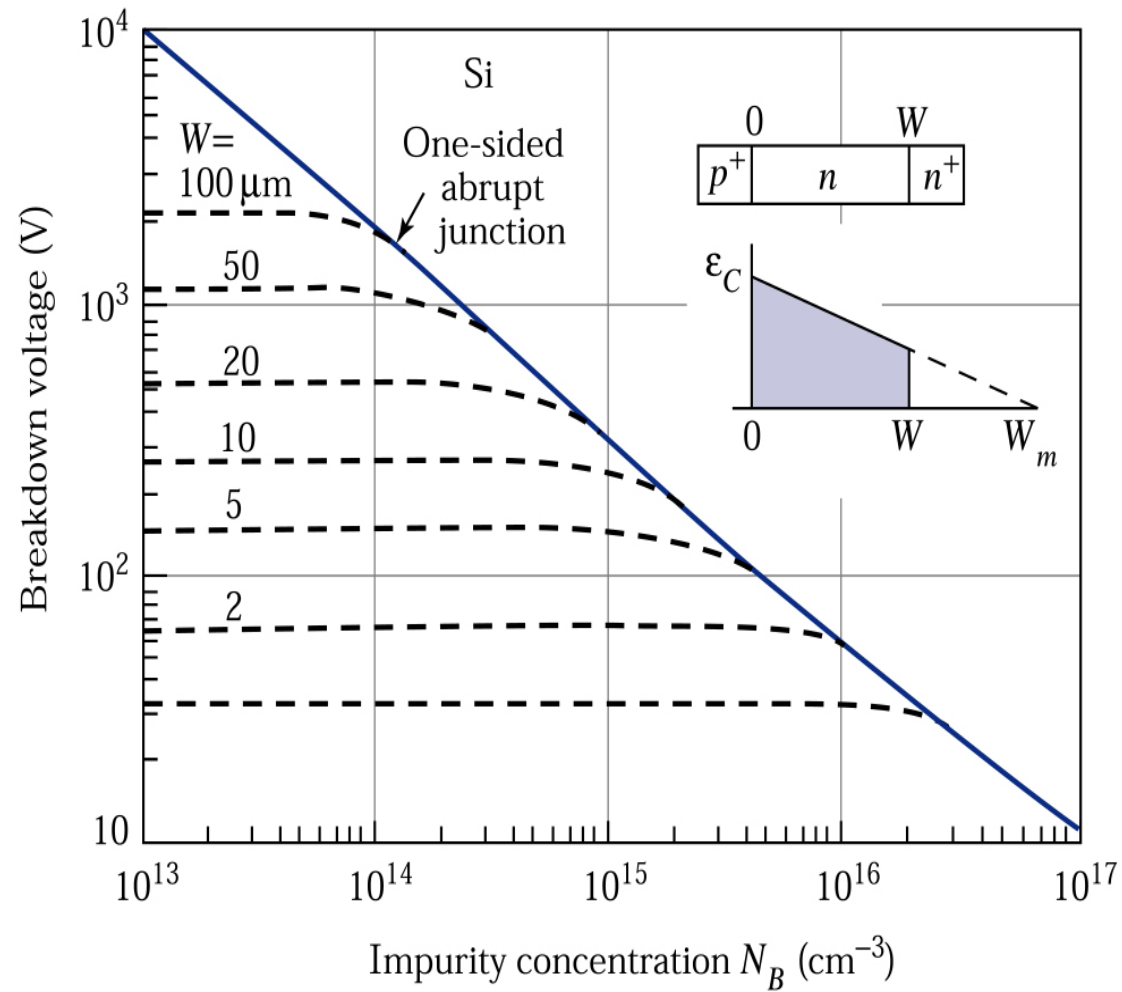
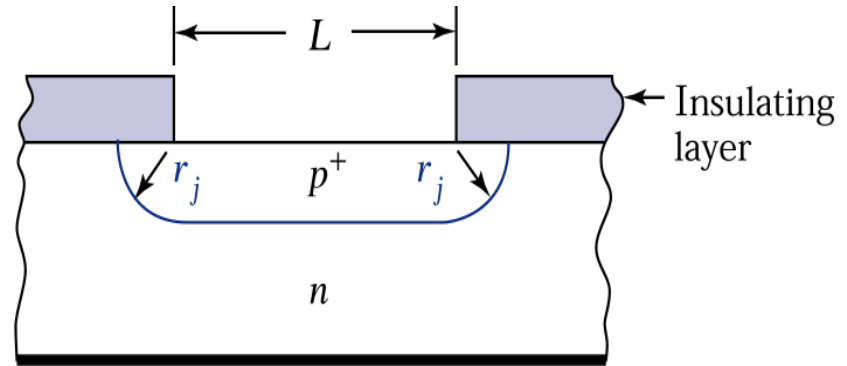


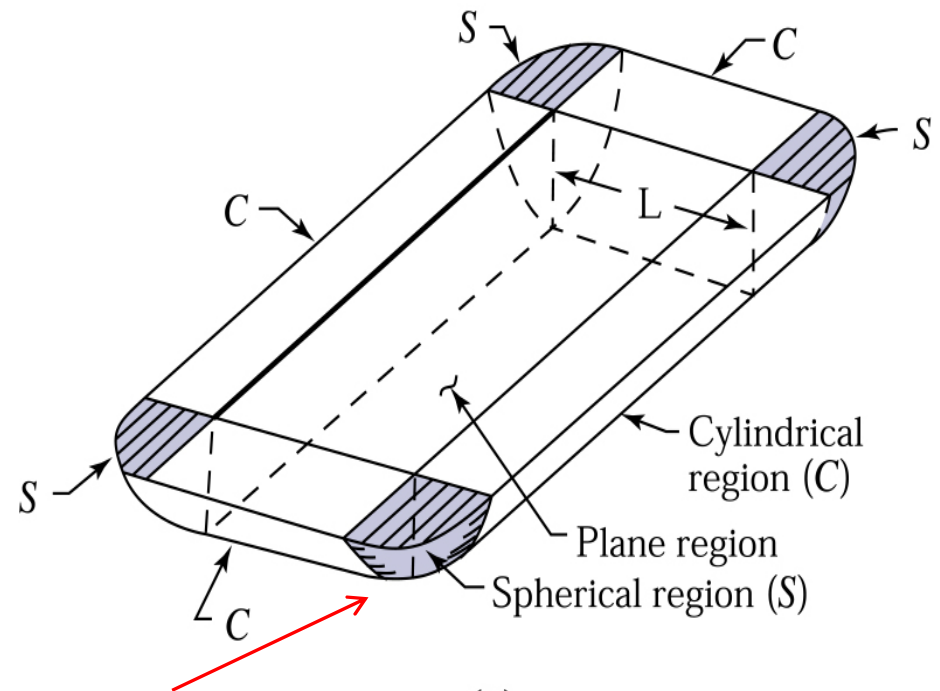
Figure 3.27. Breakdown voltage for $p^+ - \pi - n^+$ and $p^+ - v - n^+$ junctions. W is the thickness of the **lightly doped p -type (π)** or the **lightly doped n -type (v)** region.

Figure 3.28.

(a) Planar diffusion process that forms junction curvature near the edge of the diffusion mask, where r_j is the radius of curvature. (b) Cylindrical and spherical regions formed by diffusion through a rectangular mask.



(a)



(b)

Breakdown occurs in spherical regions

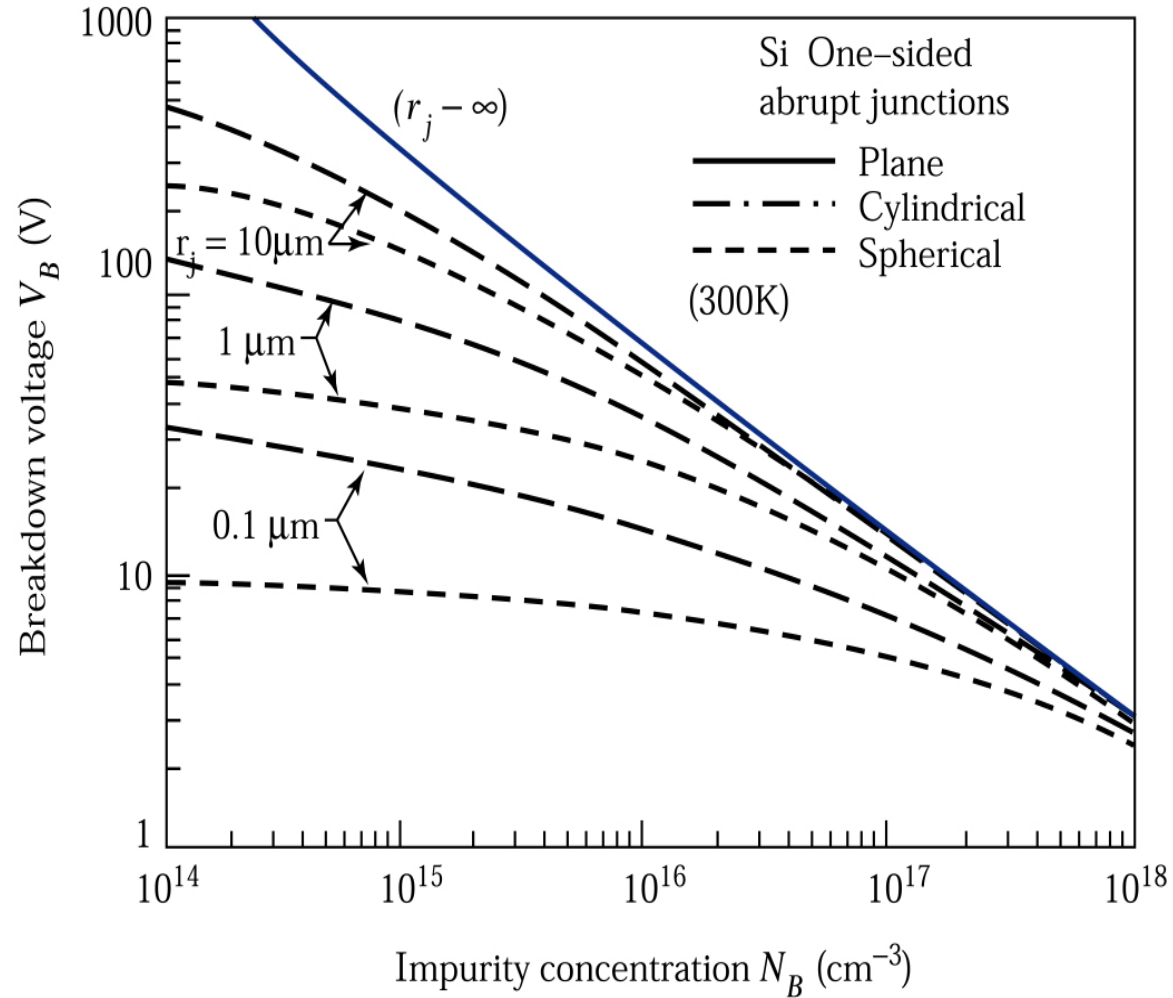
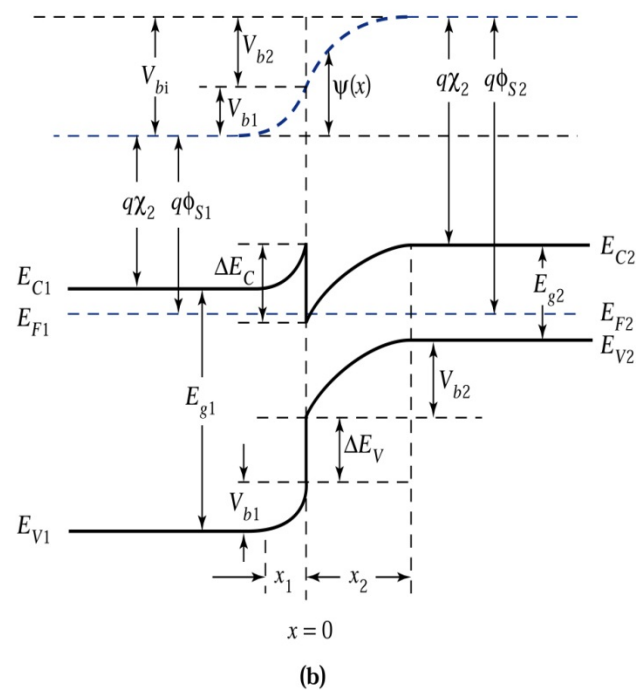
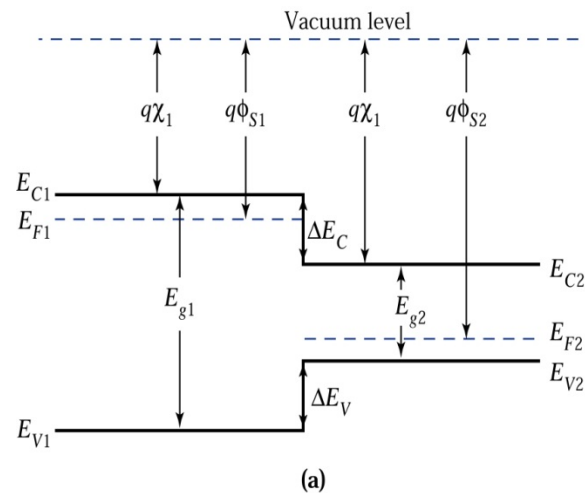


Figure 3.29. Breakdown voltage versus impurity concentration for one-sided abrupt doping profile with cylindrical and spherical junction geometries,⁷ where r_j is the radius of curvature indicated in Fig. 3.28..

Figure 3.30.

(a) Energy band diagram of two isolated semiconductors.
 b) Energy band diagram of an ideal *n-p heterojunction* at thermal equilibrium.



Band offset: ΔE_C , ΔE_V
 Band alignment

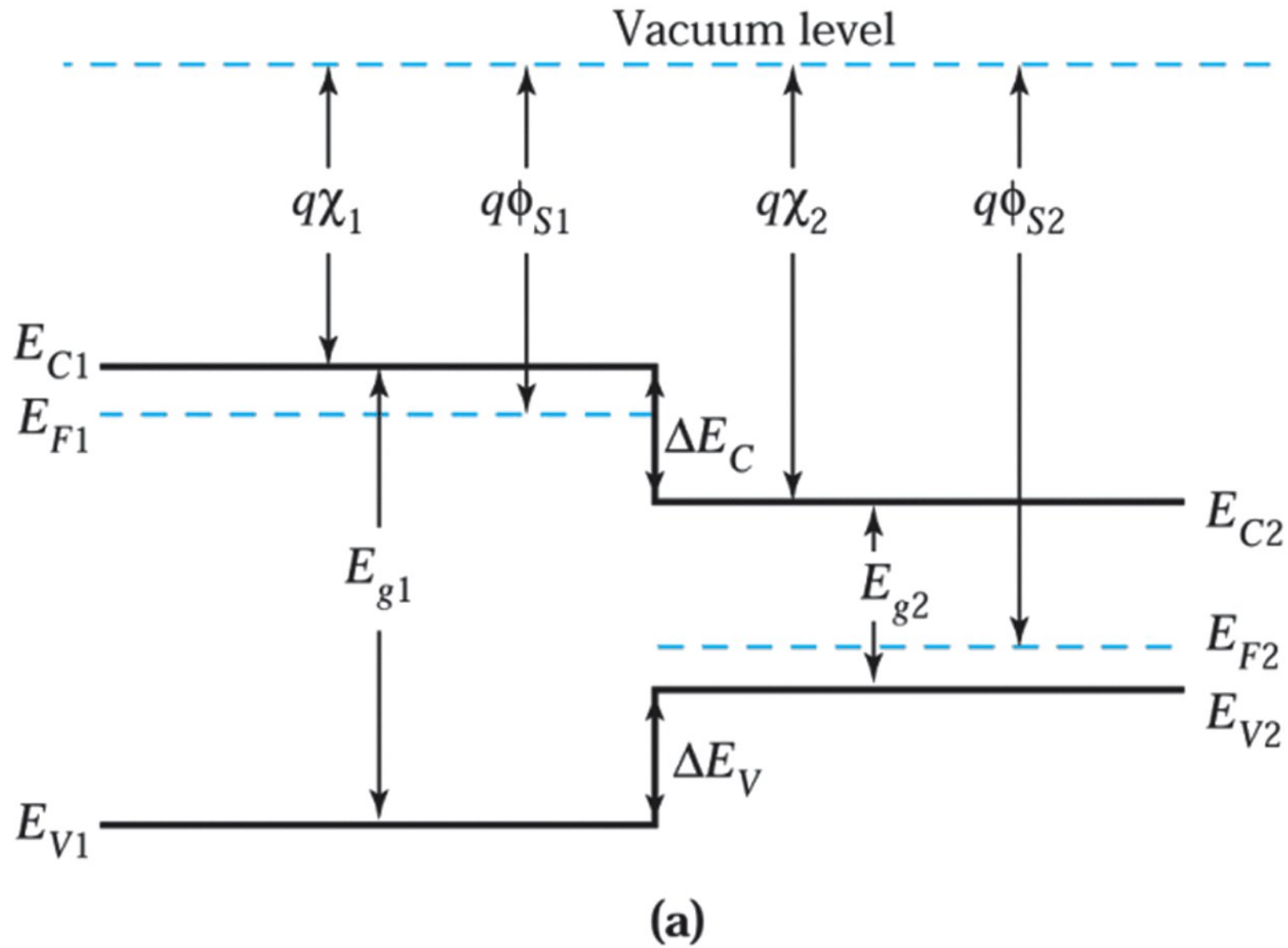
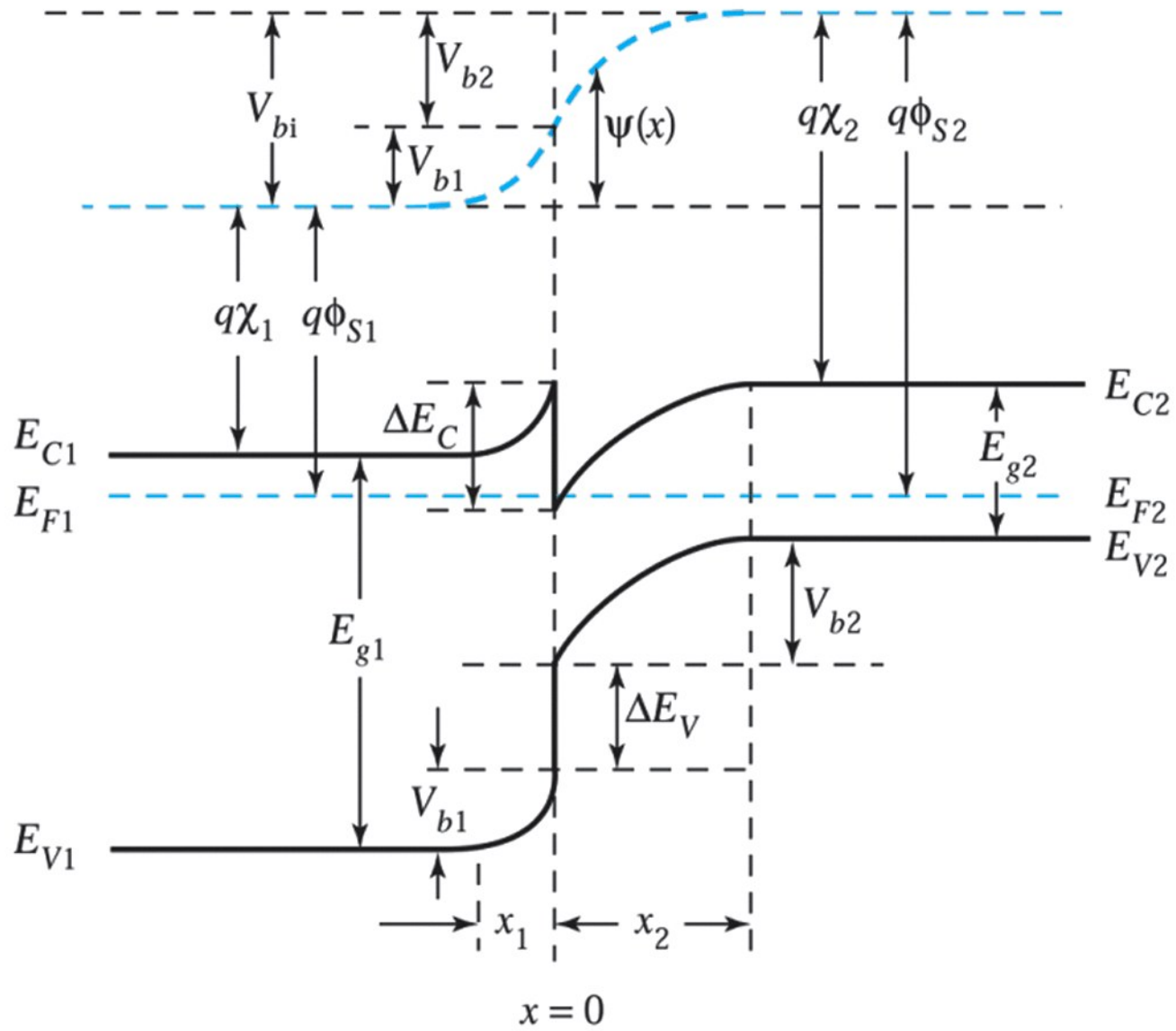


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(b)

Figure 3.30b
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