Semiconductor Devices

THIRD EDITION

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Chapter 2

Carrier Transport Phenomena

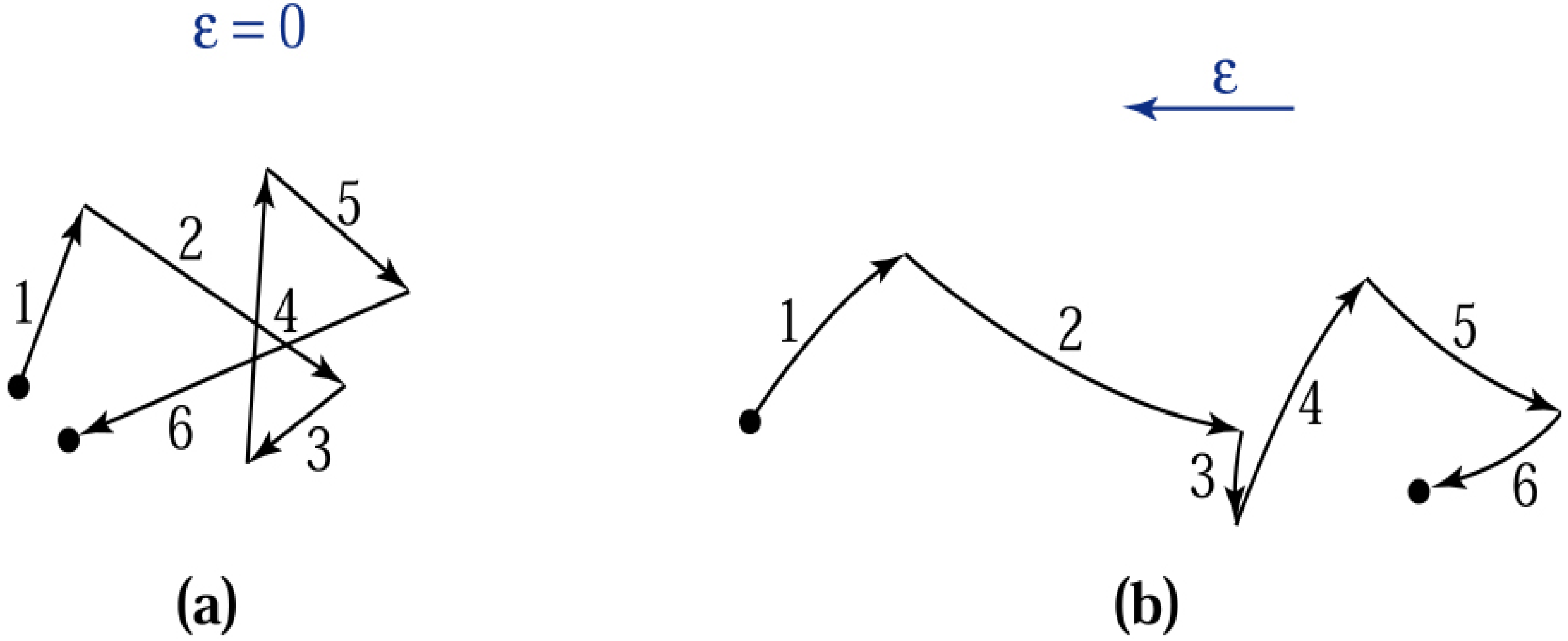


Figure 2.1. Schematic path of an electron in a semiconductor. (*a*) Random thermal motion. (*b*) Combined motion due to random thermal motion and an applied electric field.

$$F \bullet S = mv \qquad -q\varepsilon\tau_c = m_n v_n$$

$$v_n = -\left[\frac{q\tau_c}{m_n}\right]\varepsilon$$

Vth:thermal velocity

Tc: mean free time

Mean free path/Vth= τc

mobility

$$\mu_n \equiv \frac{q\tau_c}{m_n} \tag{3}$$

Effective mass

Unit: [g(cm/s^2)xcmxs]/Vg
=cm^2/sV

Drift velocity

$$v_n = -\mu_n\varepsilon \tag{4}$$

⟷ 即電流之發生

外加電場

$$v_p = \mu_p\varepsilon \tag{5}$$

Unit: (cm/s)/(V/cm)
=cm^2/sV

Figure 2.2.
Electron mobility in silicon versus temperature for various donor concentrations. Insert shows the theoretical temperature dependence of electron mobility.[3]

μn受scattering mech.

Lattice, T↑ 得 μ ↓
(thermal vibration)
Impurity, T↑ 得 μ ↑
(Coulomb force)

$$\frac{dt}{\tau} = \frac{dt}{\tau_I} + \frac{dt}{\tau_L}$$

een any scattering even
with the definitions of m

欲μ_n ↑
則ND↓ T↓

$$\boxed{\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}}$$

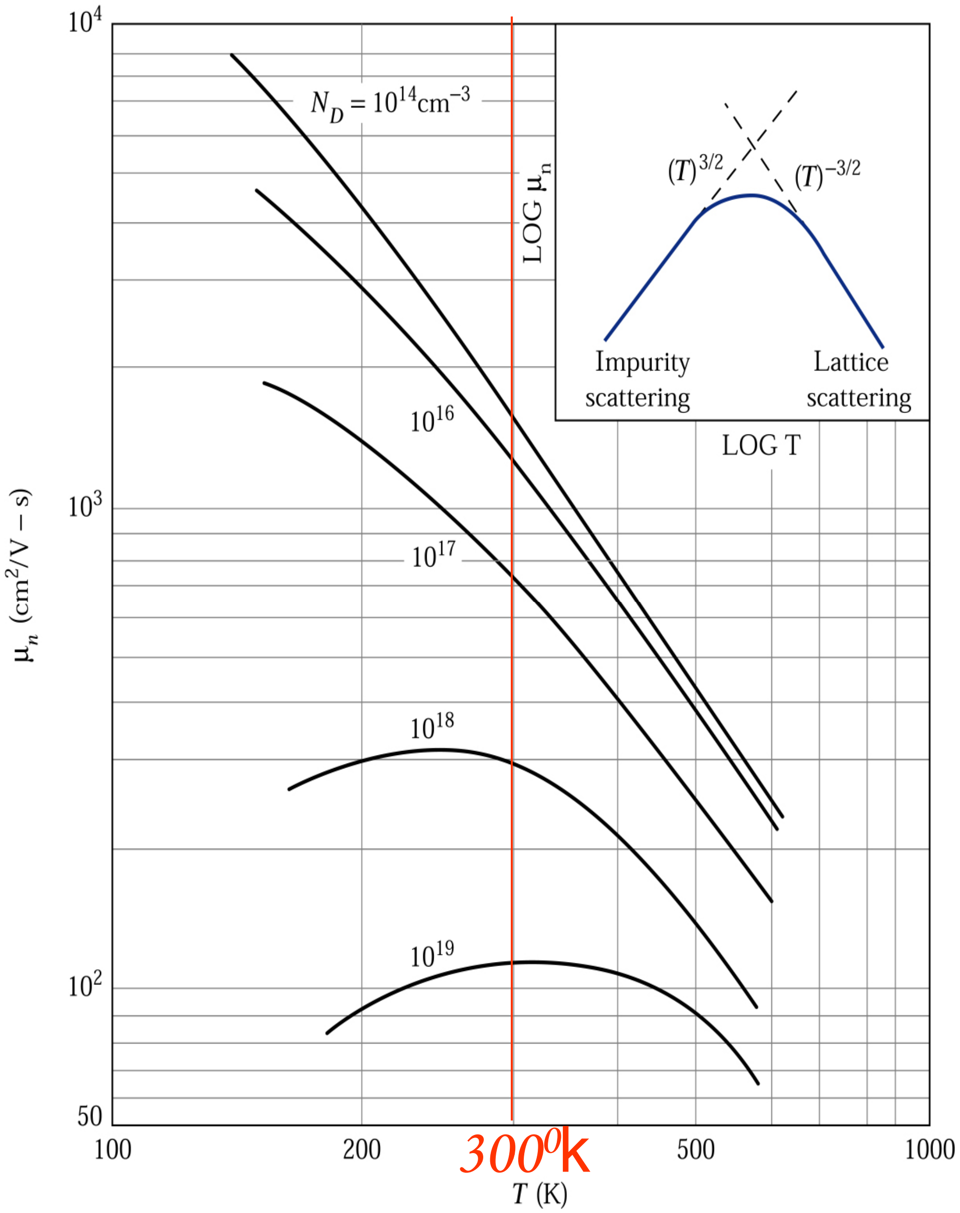
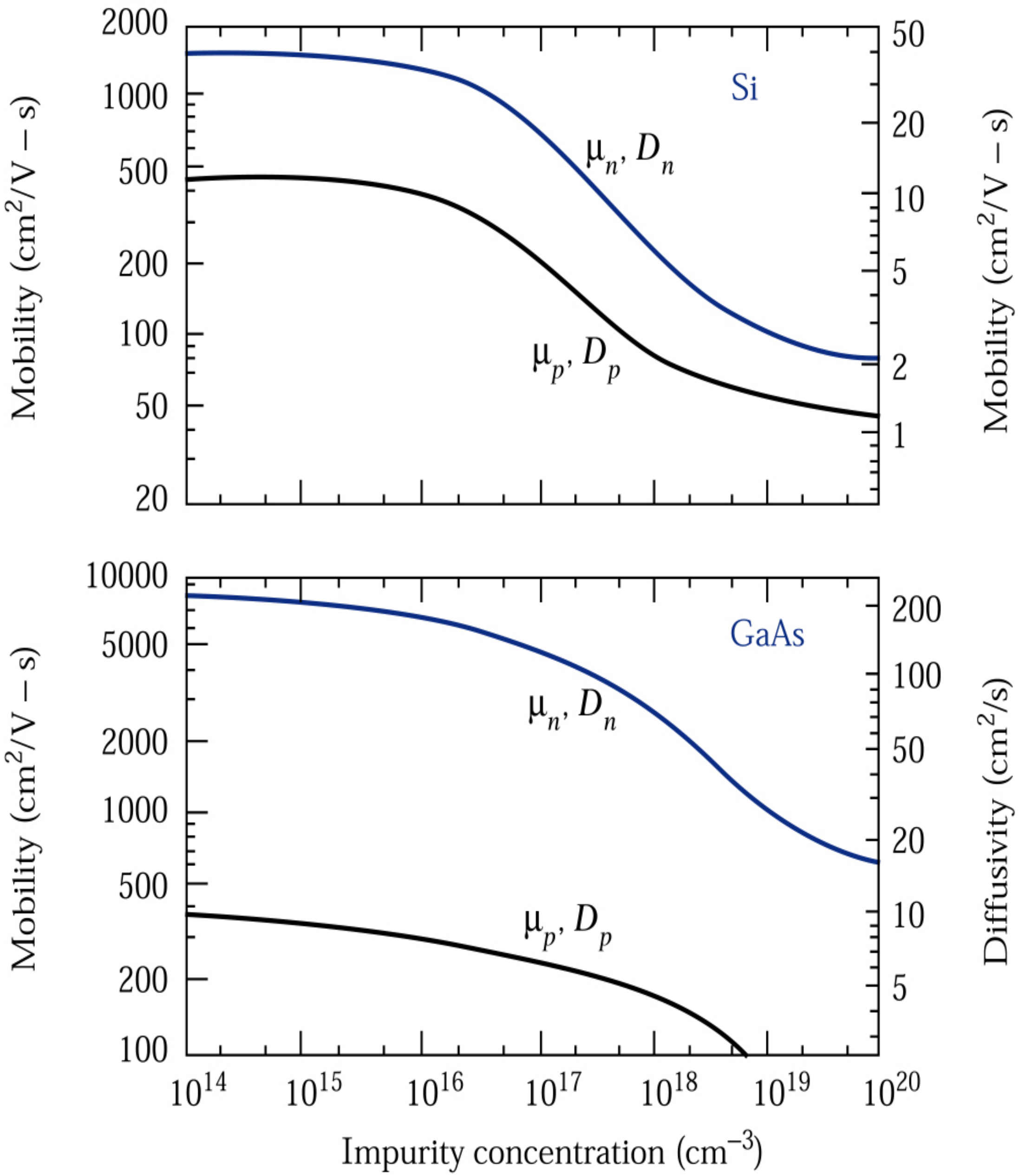


Figure 2.3.
Mobilities and diffusivities in Si and GaAs at 300 K as a function of impurity concentration.[3]



1. $\mu n > \mu p$
2. N ↑ , μ ↓
3. μ_n (GaAs)>> μ_n (Si)

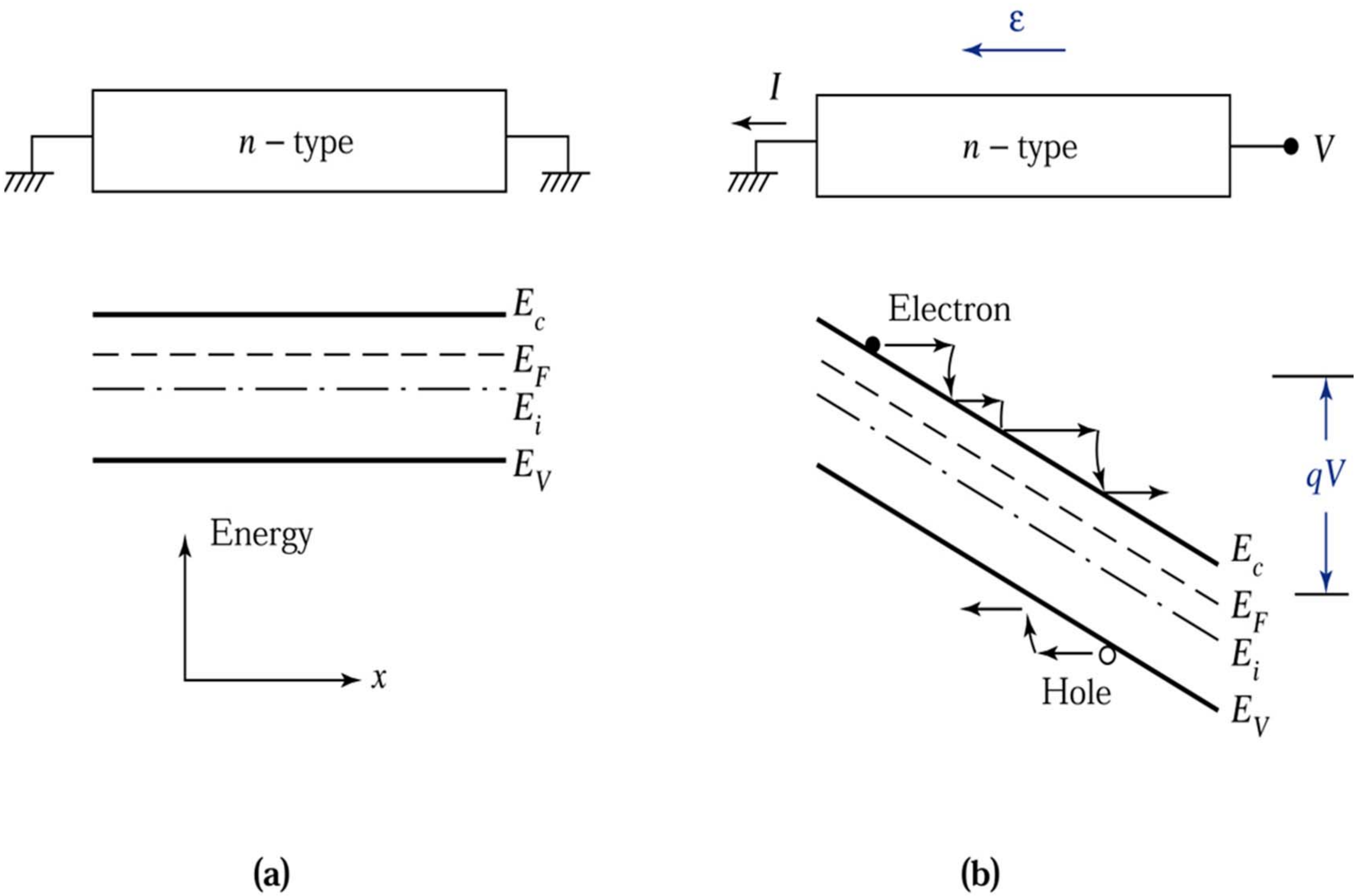
電位能(eV)

$$\psi = -\frac{E_i}{q}$$

電位(V)

$$\varepsilon = -\frac{d\psi}{dx}$$

$$\varepsilon = \frac{1}{q}\frac{dE_i}{dx}$$



外加偏壓,Ψ電位與Ei呈現性關係

Figure 2.4. Conduction process in an *n*-type semiconductor (a) at thermal equilibrium and (b) under a biasing condition.

Ref. Fig 2.5

$$J_n = \frac{I_n}{A} = \sum_{i=1}^{n} (-q v_i) = -qnv_n = qn\mu_n\varepsilon \tag{11}$$

$$J = J_n + J_p = (qn\mu_n + qp\mu_p)\varepsilon \tag{13}$$

$$\sigma = (qn\mu_n + qp\mu_p) \tag{14}$$

Unit: (V/Ω)s(1/cm^3)(cm^2/sV)
=1/cmΩ

$$\boxed{\rho = \frac{1}{\sigma}} = \frac{1}{q(n\mu_n + p\mu_p)} \tag{15}$$

Unit:cm-Ω

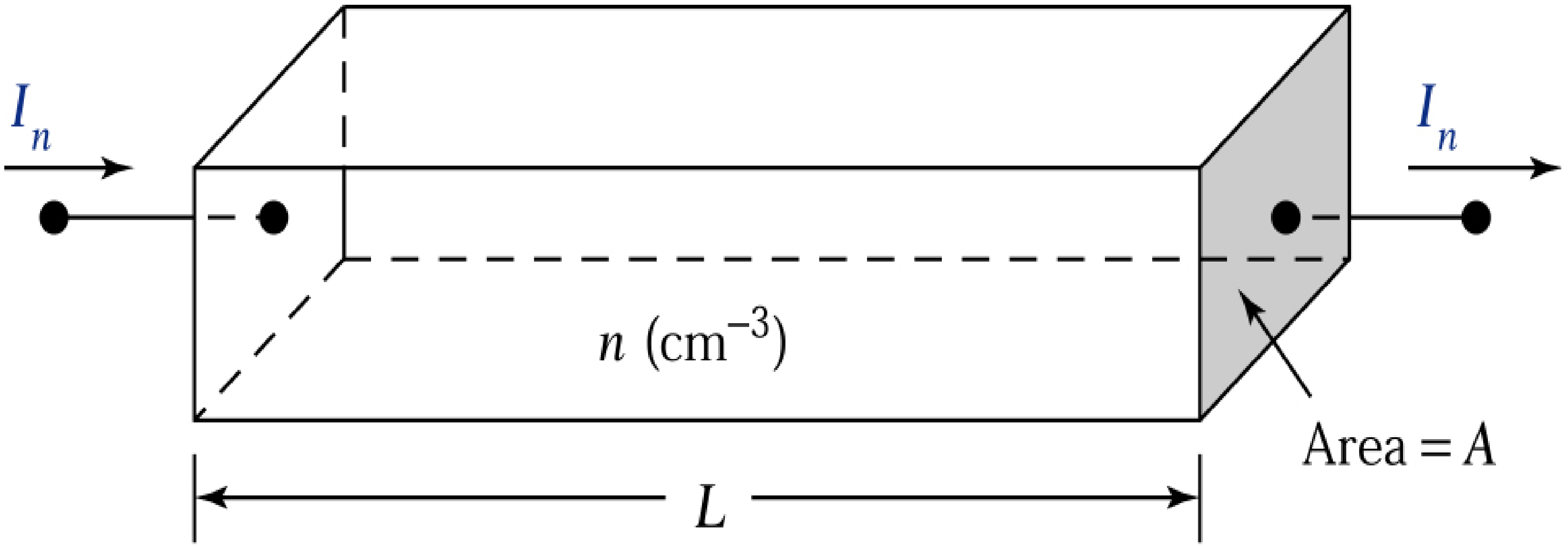
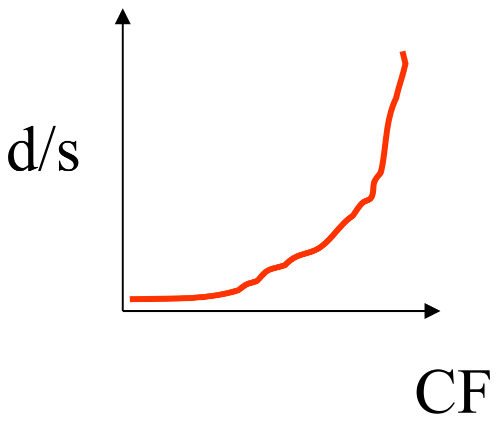
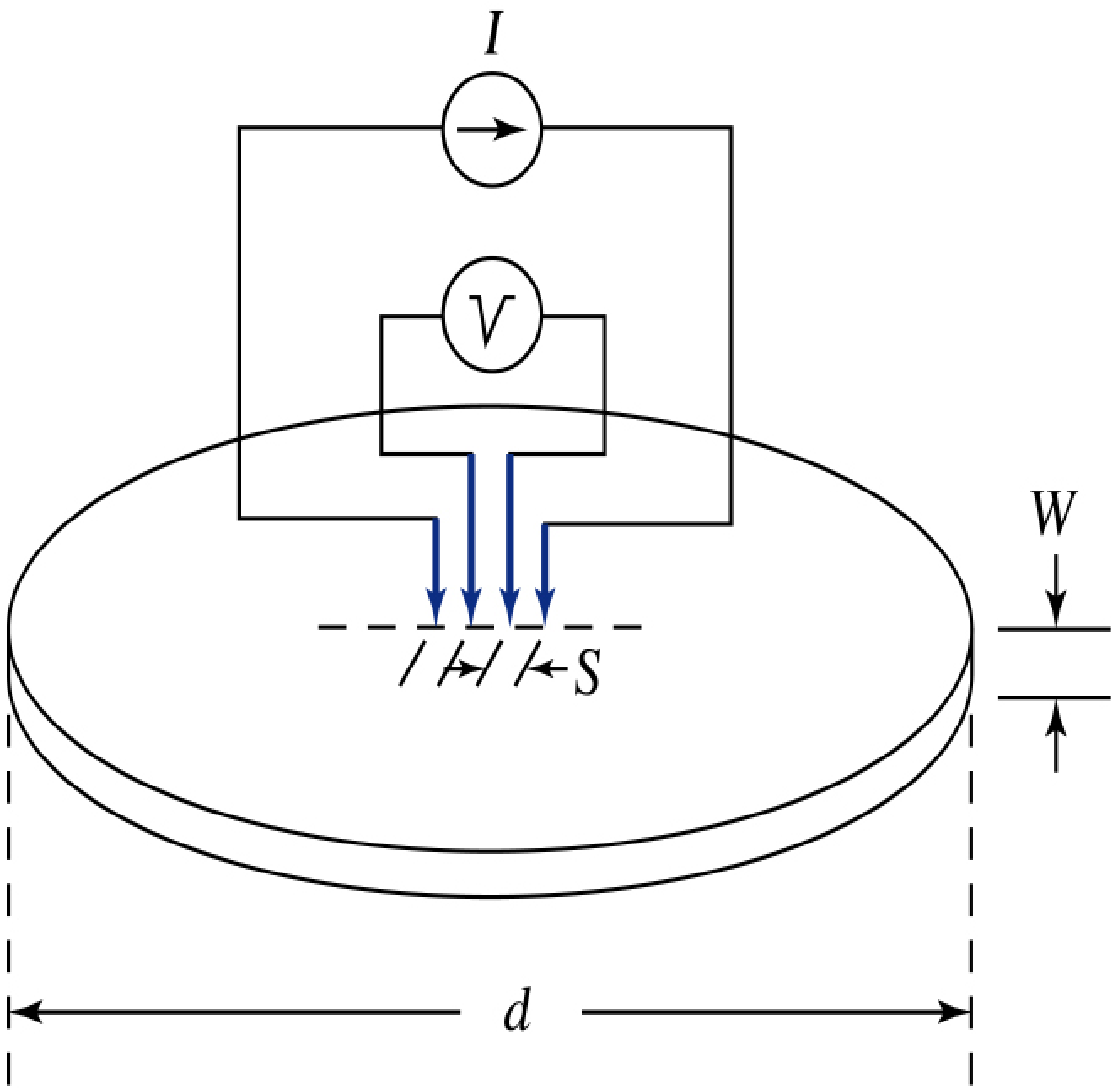


Figure 2.5. Current conduction in a uniformly doped semiconductor bar with length *L* and cross-sectional area *A*.



d/s >> 20, CF= 4.54

ρ= (V/I)xWxCF



用四點探針(4-point probe) 量

Figure 2.6. Measurement of resistivity using a four-point probe.[3]

Problem,
$N_D=10^{16}$,
$\rho=0.48$
(Si原來
$\rho>10^3$)

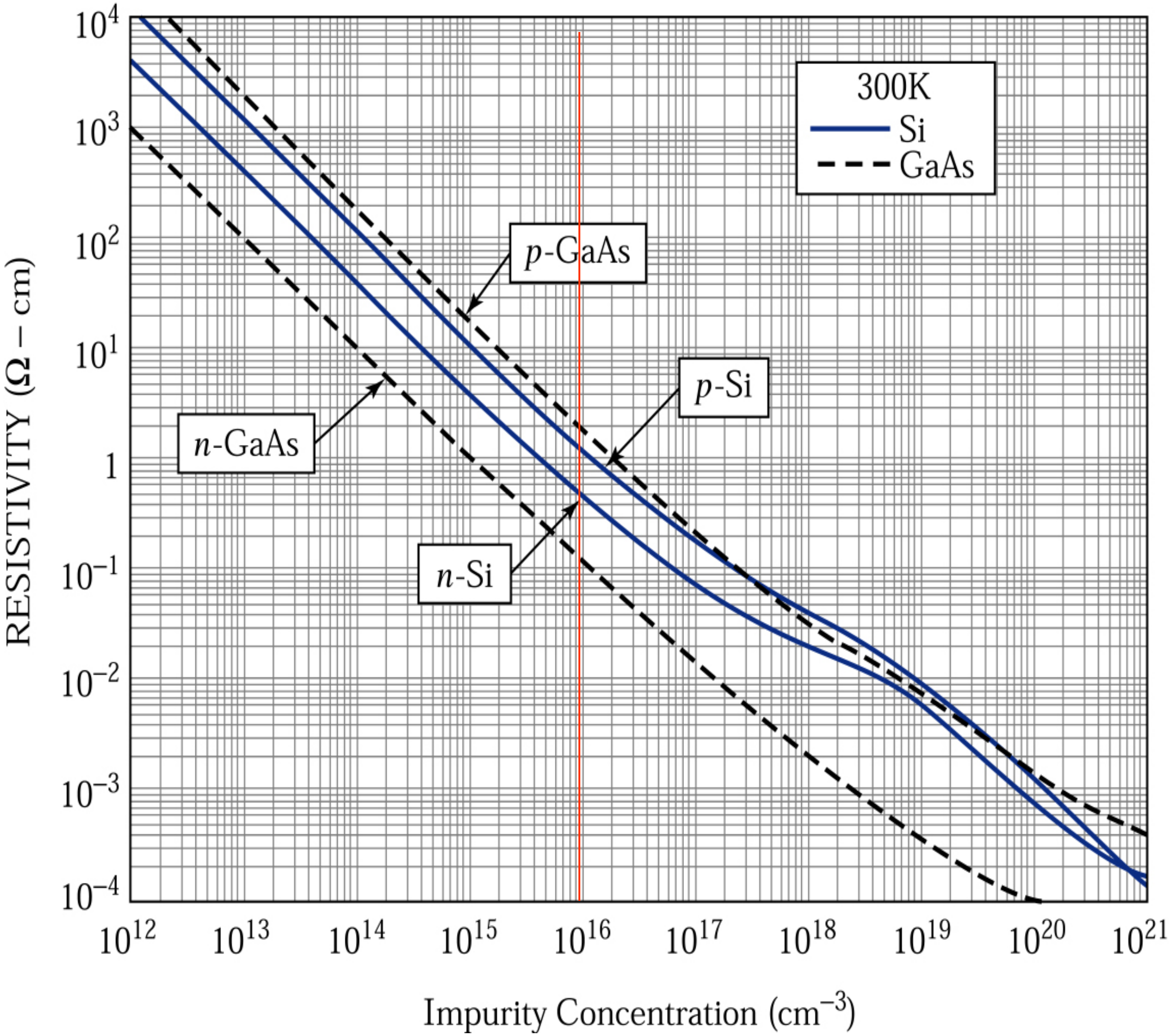


Figure 2.7. Resistivity versus impurity concentration[3] for Si and GaAs.

重要工程資料,一般**wafer ρ~1,N_A~10^{16}**

Lorenz force= qVx X Bz

$Ey=(V_H/W)= R_H JxBz$

N>>p

$R_H= (-1/qn)$

P>>n

$R_H= (1/qp)$

Thus, $p=IBzW/qV_HA$

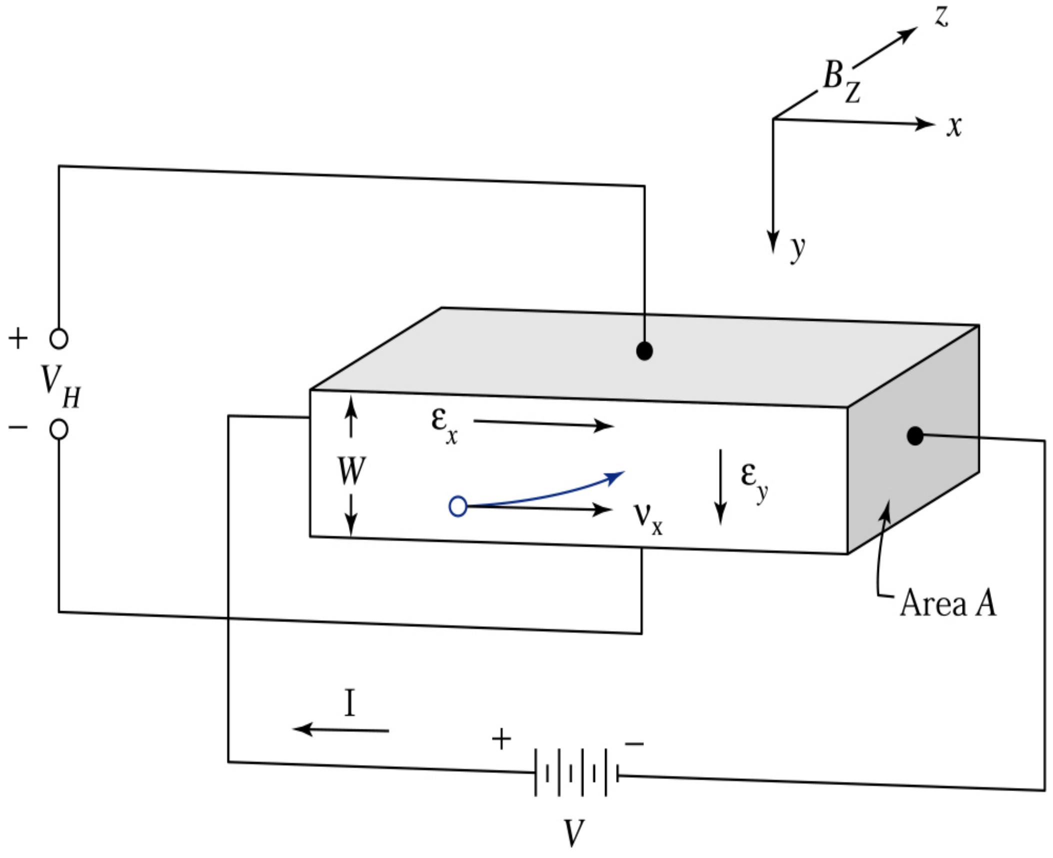
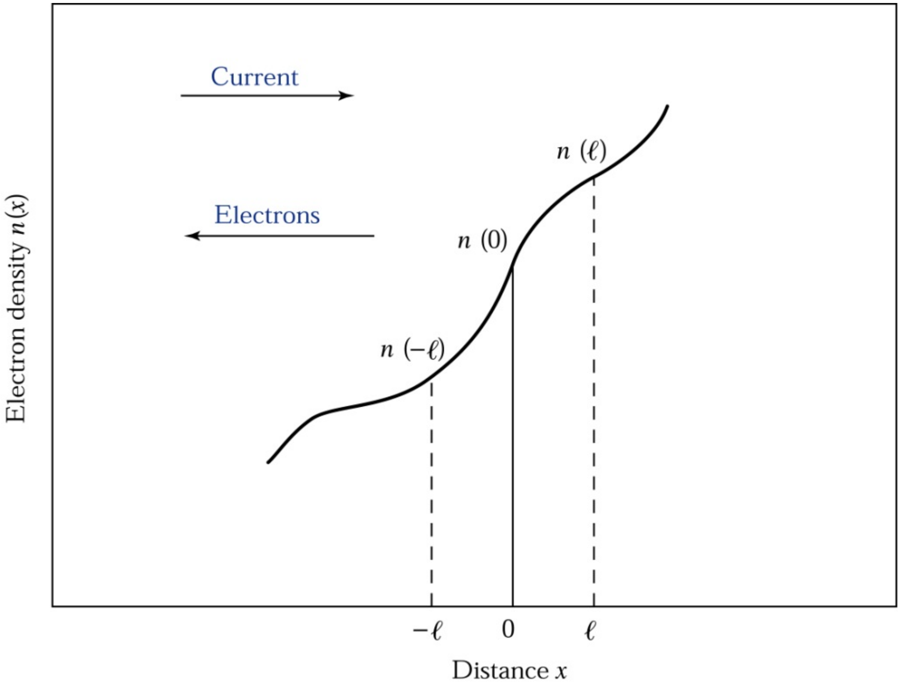


Figure 2.8. Basic setup to measure carrier concentration using the Hall effect.



$$F1 = \frac{(1/2)n(-l)l}{\tau_c} = \frac{1}{2}n(-l)\bullet v_{th}$$

一半向右

$$F2 = \frac{1}{2}n(l)\bullet v_{th}$$

一半向左

$$F = F1 - F2 = \frac{1}{2}v_{th}[n(-l) - n(l)] =$$

$$\frac{1}{2}v_{th}\{[n(0) - l\tfrac{dn}{dx}] - [n(0) + l\tfrac{dn}{dx}]\} =$$

$$-v_{th}l\frac{dn}{dx} = -D_n\frac{dn}{dx}$$

Figure 2.9. Electron concentration versus distance; *l* is the mean free path. The directions of electron and current flows are indicated by arrows.

$$J_n = -qF = qD_n \frac{dn}{dx} \quad (27)$$

F: electron flow
Dn:diffusivity
=Vth*L

L=Vth*τc

μ=qτ/m

$$D_n = \left[\frac{kT}{q}\right]\mu_n \quad (30)$$

Einstein relation

$$J_n = q\mu_n n\varepsilon + qD_n \frac{dn}{dx} \quad (31)$$

drift

diffusion

Current density
eq.

$$J_p = q\mu_p p\varepsilon - qD_p \frac{dp}{dx} \quad (32)$$

$$Jcond. = J_n + J_p \quad (33)$$

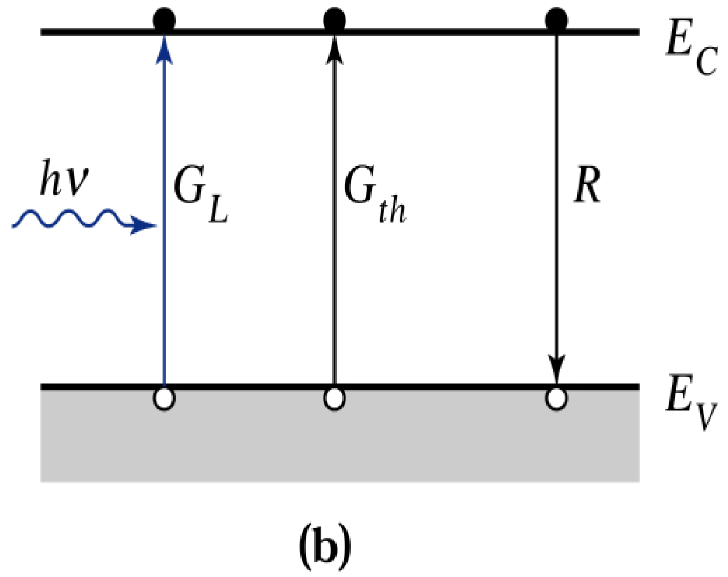


Figure 2.10. Direct generation and recombination of electron-hole pairs: (*a*) at thermal equilibrium and (*b*) under illumination.

§Direct Recombination

Δ熱平衡

$$G_L = R - G_{th} \equiv U$$ (40) Net recombination rate

Light thermal

- For low Inj.(Δp, $p_{no} << n_{no}$) 若 direct-recom.

n-type

$$U \cong \beta n_{n0} \Delta p = \frac{p_n - p_{n0}}{\frac{1}{\beta n_{n0}}}$$ (42)

β:比例常數

Excess hole

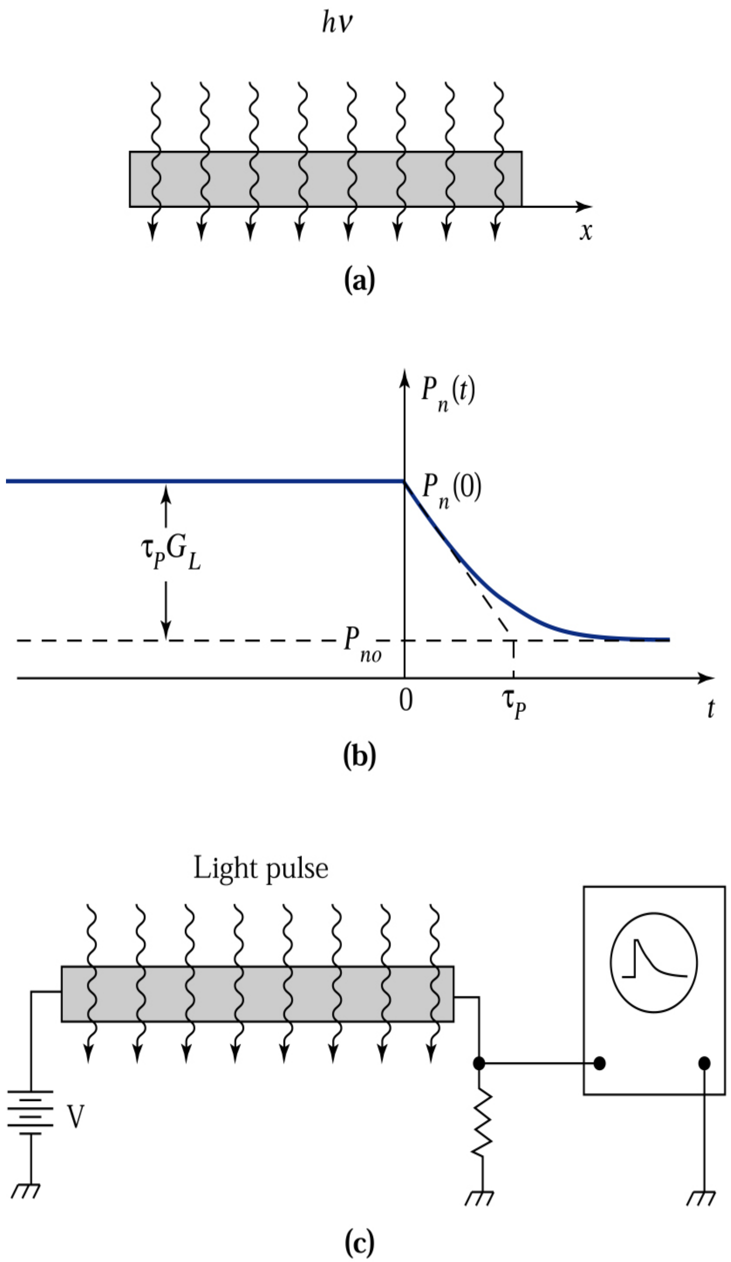
$$U = \frac{p_n - p_{n0}}{\tau_p}$$ (43)

(僅與excess minority carrier有關)

$$\tau_p \equiv \frac{1}{\beta n_{n0}}$$ (44) Lifetime (of minority carrier)

- Direct recombination : 即 band to band (for 三五族)

Figure 2.11.
Decay of photoexcited carriers.
a) *n*-type sample under constant illumination. (*b*) Decay of minority carriers (holes) with time.
(*c*) Schematic setup to **measure minority carrier lifetime.**



$$p_n(t) = p_{no} + \tau_p G_L \exp(-t/\tau_p)$$

Example 7, p.59, quasi-Fermi levels induced by light

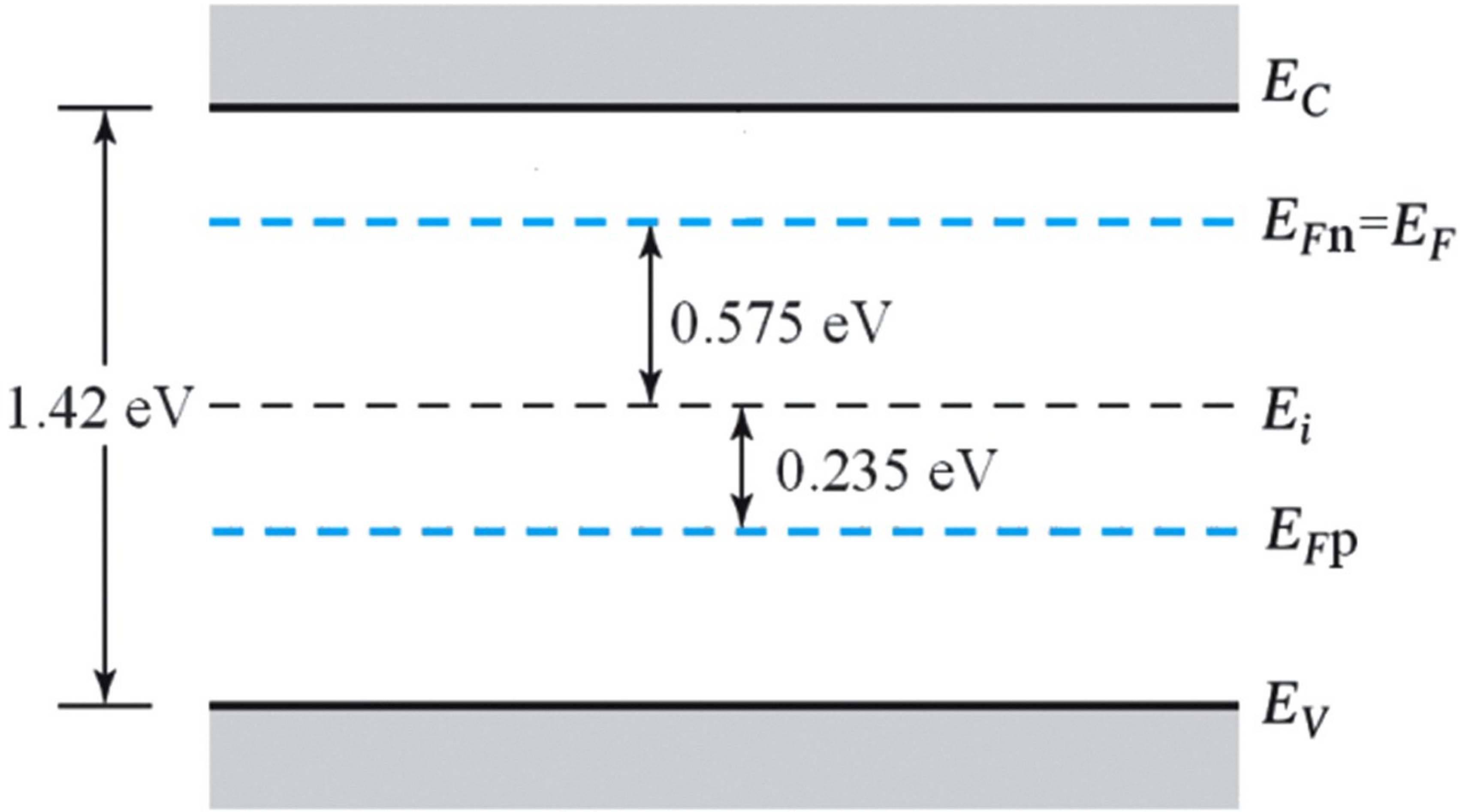


Figure 2.12

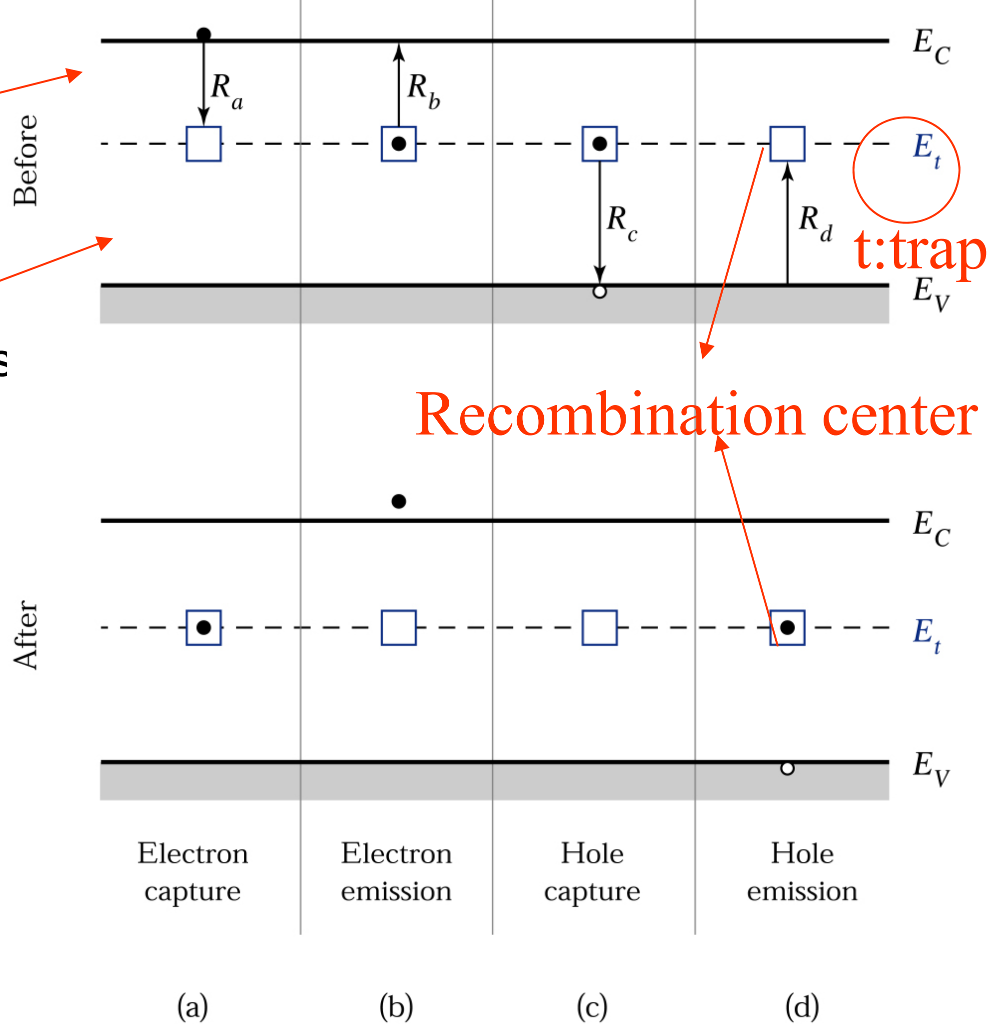
§Indirect Recom. 經由 recombination center 再複合.

- For indirect bandgap (Si)

Donor like

Figure 2.13.
Indirect generation-recombination proces
at thermal equilibrium.

Acceptor like



t:trap

Recombination center

Nt:concentration
of center ↓

$$U \approx V_{th}\sigma_0 N_t \frac{p_n - p_{no}}{1+[\frac{2ni}{n_{no}+p_{no}}]\cosh[\frac{E_t - E_i}{kT}]} = \frac{p_n - p_{n0}}{\tau_r} \qquad (50)$$

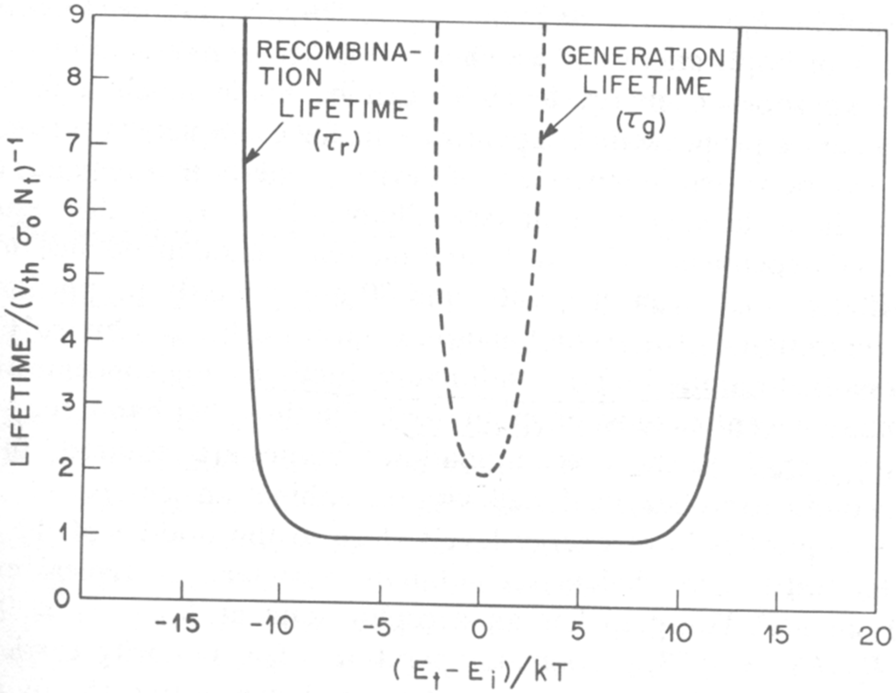


Fig. 15 Recombination lifetime and generation lifetime versus energy level o recombination center.

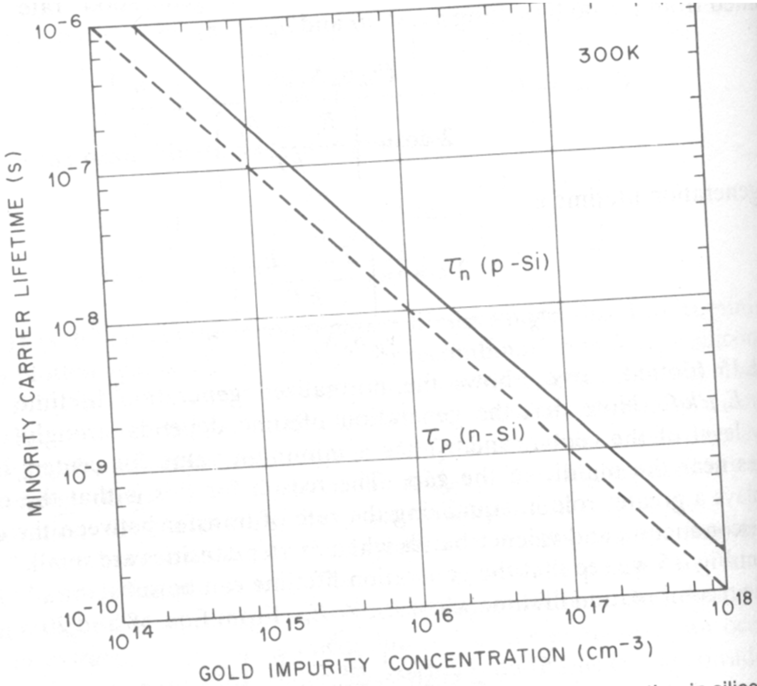
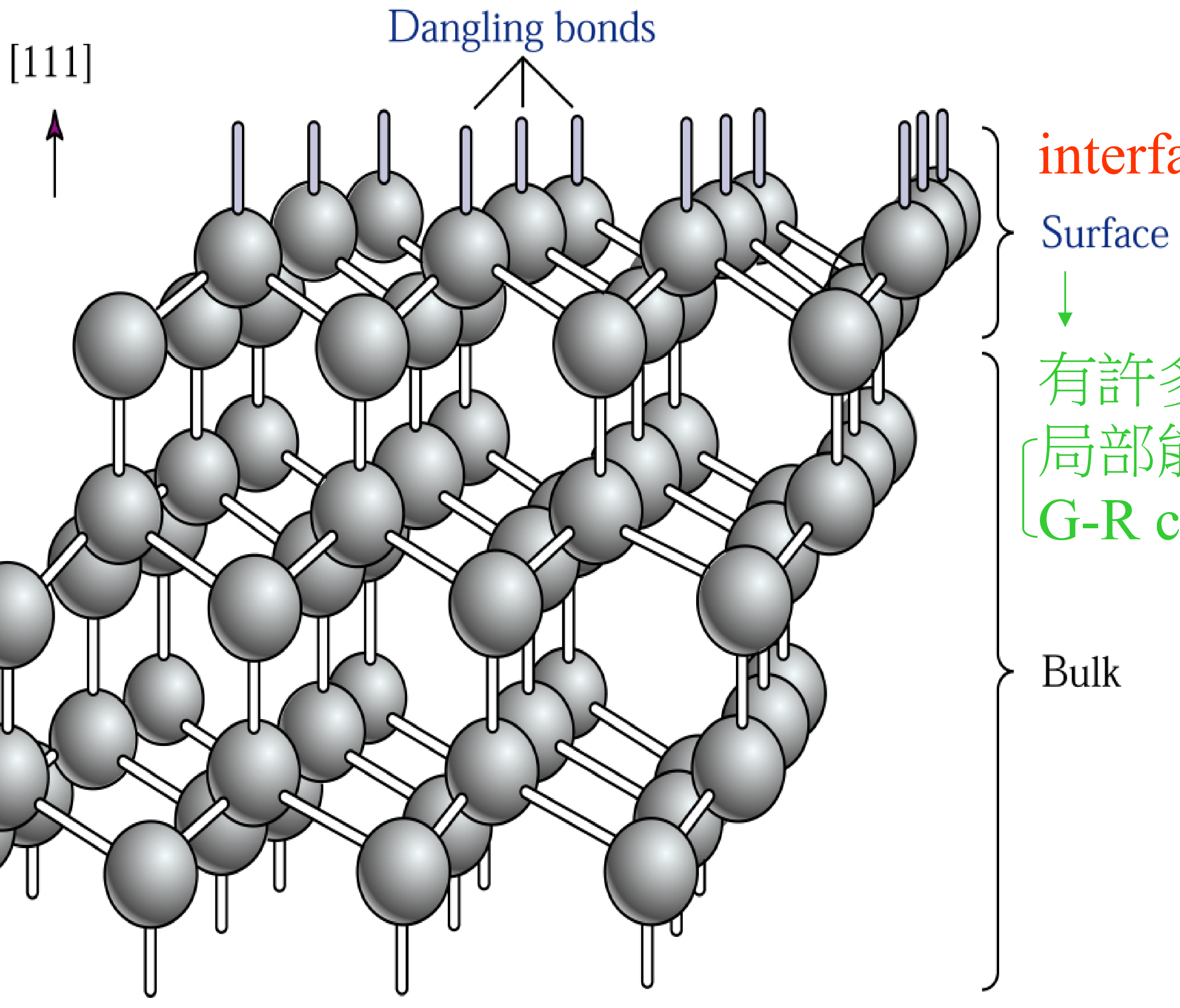


Fig. 16 Recombination lifetime versus gold impurity concentration in silicon.[8]

★ $Sr >> Sg$

(80) (0.1)

$(\tau_g > \tau_r)$



interface

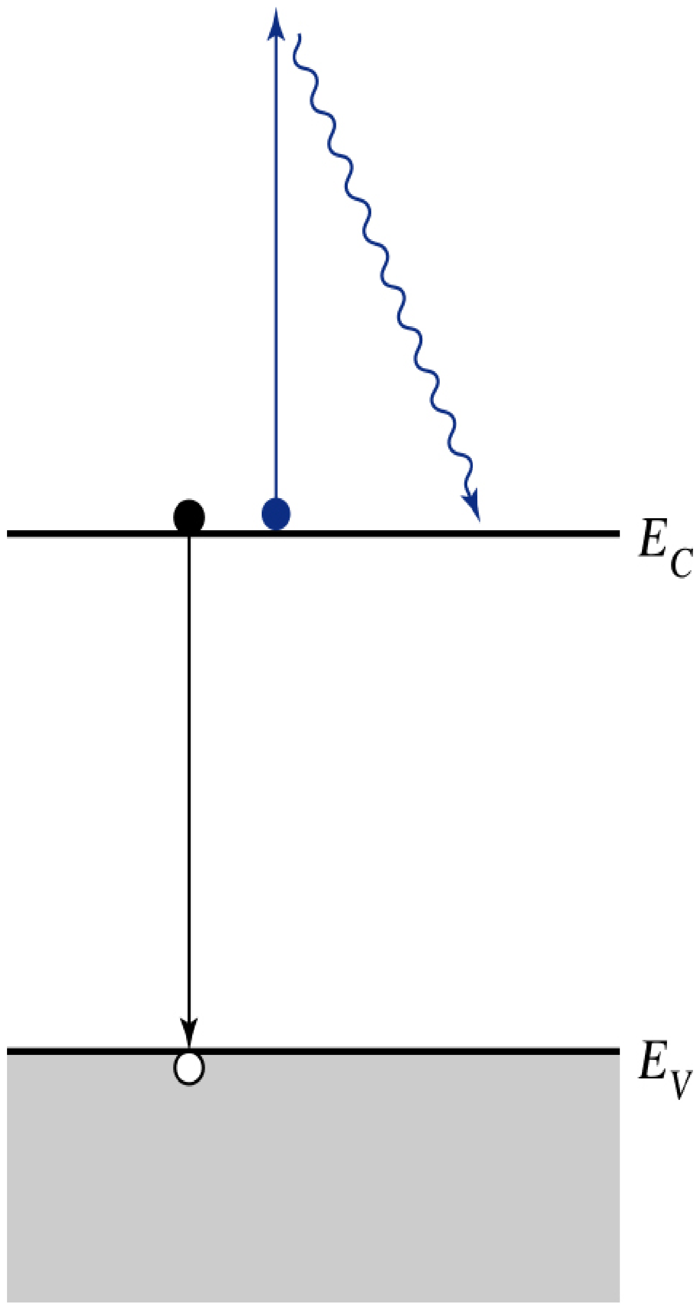
↓

有許多,

局部能階

G-R center

Figure 2.14 Schematic diagram of bonds at a clean semiconductor surface. The bonds are anisotropic and differ from those in the bulk.[5]



Auger recombination.

Def: the transfer of energy and momentum released by e-h recomb to a third e/h

When carrier concentration is very high, Auger recomb is important.

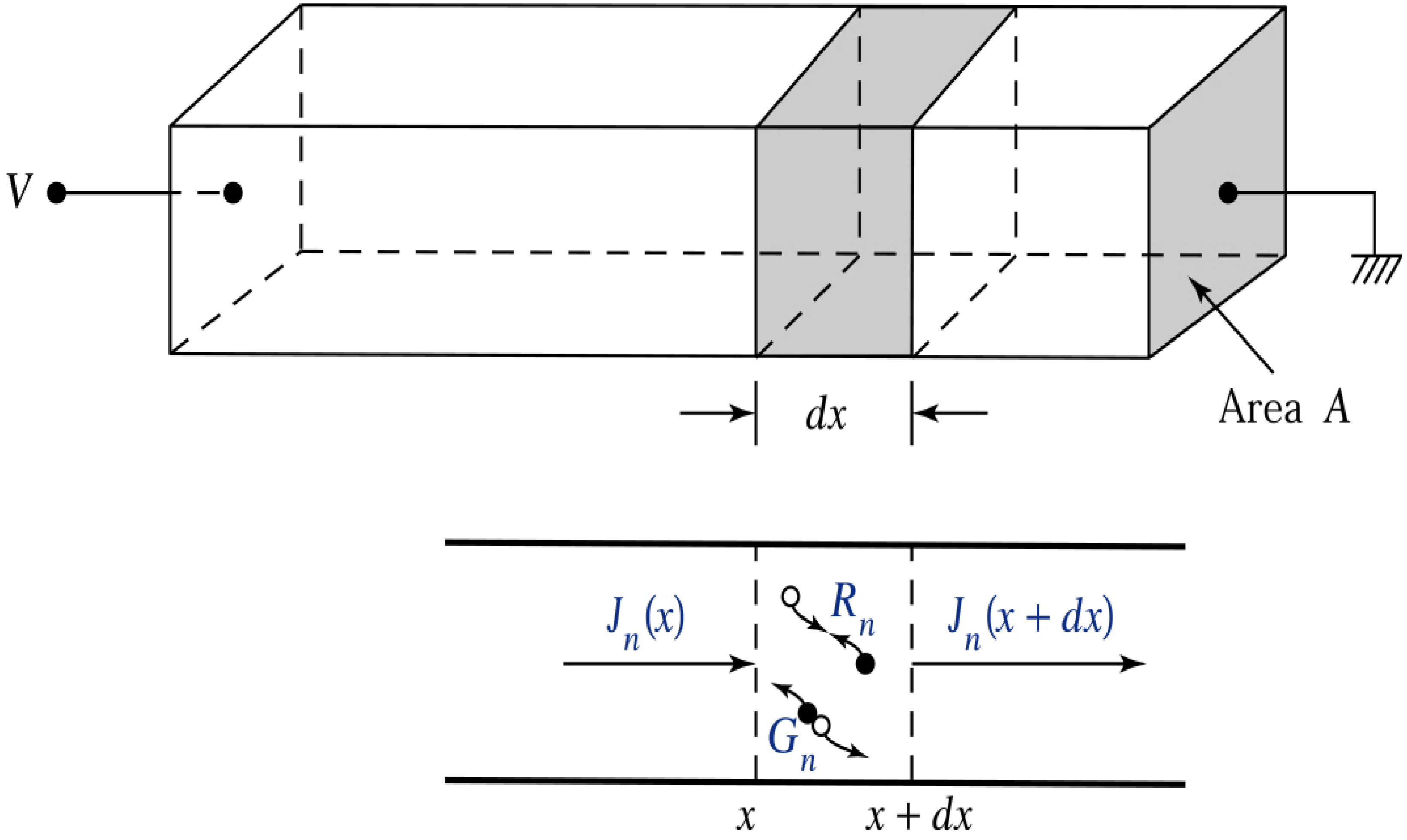


Figure 2.15. Current flow and generation-recombination processes in an infinitesimal slice of thickness *dx*.

drift + diff. + Recom. 均同時發生 ⟹

★ Continuity Eq. (一維)

由 $$\frac{\partial n}{\partial t} = \frac{\partial J_n}{q\partial x} + (G_n - R_n) \qquad (56)$$

又 $$J_n = \underbrace{q\mu_n n\varepsilon}_{\text{drift}} + \underbrace{qD_n\frac{dn}{dx}}_{\text{diff}}$$

(1)
$$\begin{cases} \dfrac{\partial n_p}{\partial t} = \left[n_p\mu_n\dfrac{\partial \varepsilon}{\partial x} + \mu_n\varepsilon\dfrac{\partial n_p}{\partial x}\right] + D_n\dfrac{\partial^2 n_p}{\partial x^2} + G_n - \dfrac{n_p - n_{po}}{\tau_n} & (58) \\[2ex] \dfrac{\partial p_n}{\partial t} = -p_n\mu_p\dfrac{\partial \varepsilon}{\partial x} - \mu_p\varepsilon\dfrac{\partial p_n}{\partial x} + D_p\dfrac{\partial^2 p_n}{\partial x^2} + G_p - \dfrac{p_n - p_{no}}{\tau_p} & (59) \end{cases}$$

*少數carrier之注入

加上 Poisson's eq.

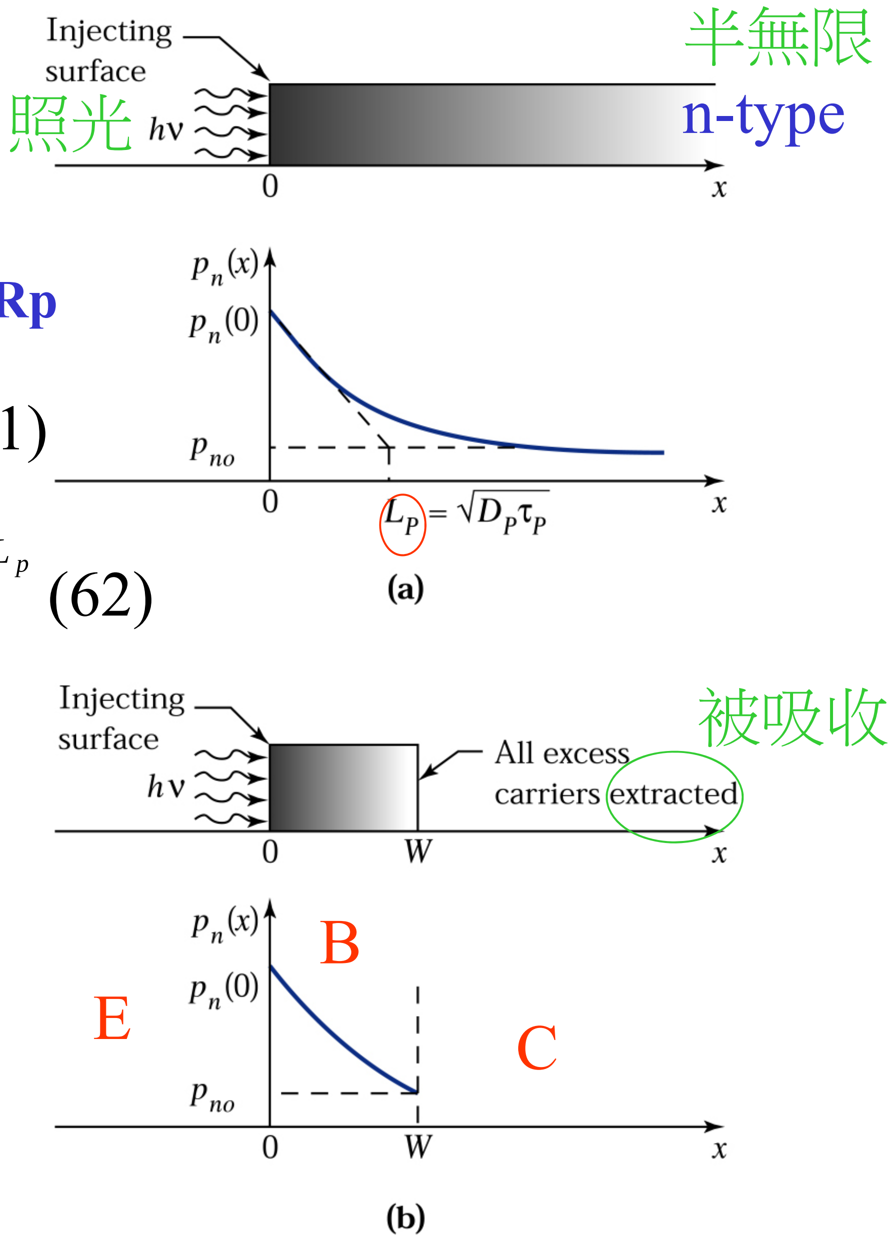
(2) $$\frac{d\varepsilon}{dx} = \frac{\rho_s}{\varepsilon_s} \qquad (60)$$

(3) 及Boundary Condition

⟹ 可解出inj. minority carrier distribution

*G<<R→ G可忽略

例: **Figure 2.16.**
Steady-state carrier injection from one side. (*a*) Semiinfinite sample. (*b*) Sample with thickness *W*.



電流之梯度

=Recomb. rate, Rp

$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{no}}{\tau_p} \quad (61)$$

$$p_n(x) = p_{no} + [\, p_n(0) - p_{no}\,] e^{-x/L_p} \quad (62)$$

如B.C. $p_n(0)=\text{const}$, $p_n(\infty)= p_{no}$

$$L_p = \sqrt{D_p \tau_p}$$

$$*\tau_p \uparrow \Rightarrow \left\{ \begin{array}{l} Lp \uparrow \\ Rp \downarrow \end{array} \right.$$

$\Rightarrow$ 一致

$$\text{B.C.}\quad \begin{cases} P_n(0) \\ P_n(W)=p_{no} \end{cases}$$

$$p_n(x) = p_{no} + [p_n(0) - p_{no}]\left[\frac{\sinh\left[\frac{W-x}{L_p}\right]}{\sinh(W/L_p)}\right] \tag{63}$$

又 $\varepsilon=0$

$$J_p = -qD_p \left.\frac{\partial p_n}{\partial x}\right|_{\textcircled{W}} = q[p_n(0) - p_{n0}]\frac{D_p}{L_p}\frac{1}{\sinh(W/L_p)} \tag{64}$$

*BJT解J時會用到. (E inj.到B,穿越B,到C之J)

$S_{lr} \to \infty$

$P_n(0) \to P_{n0}$

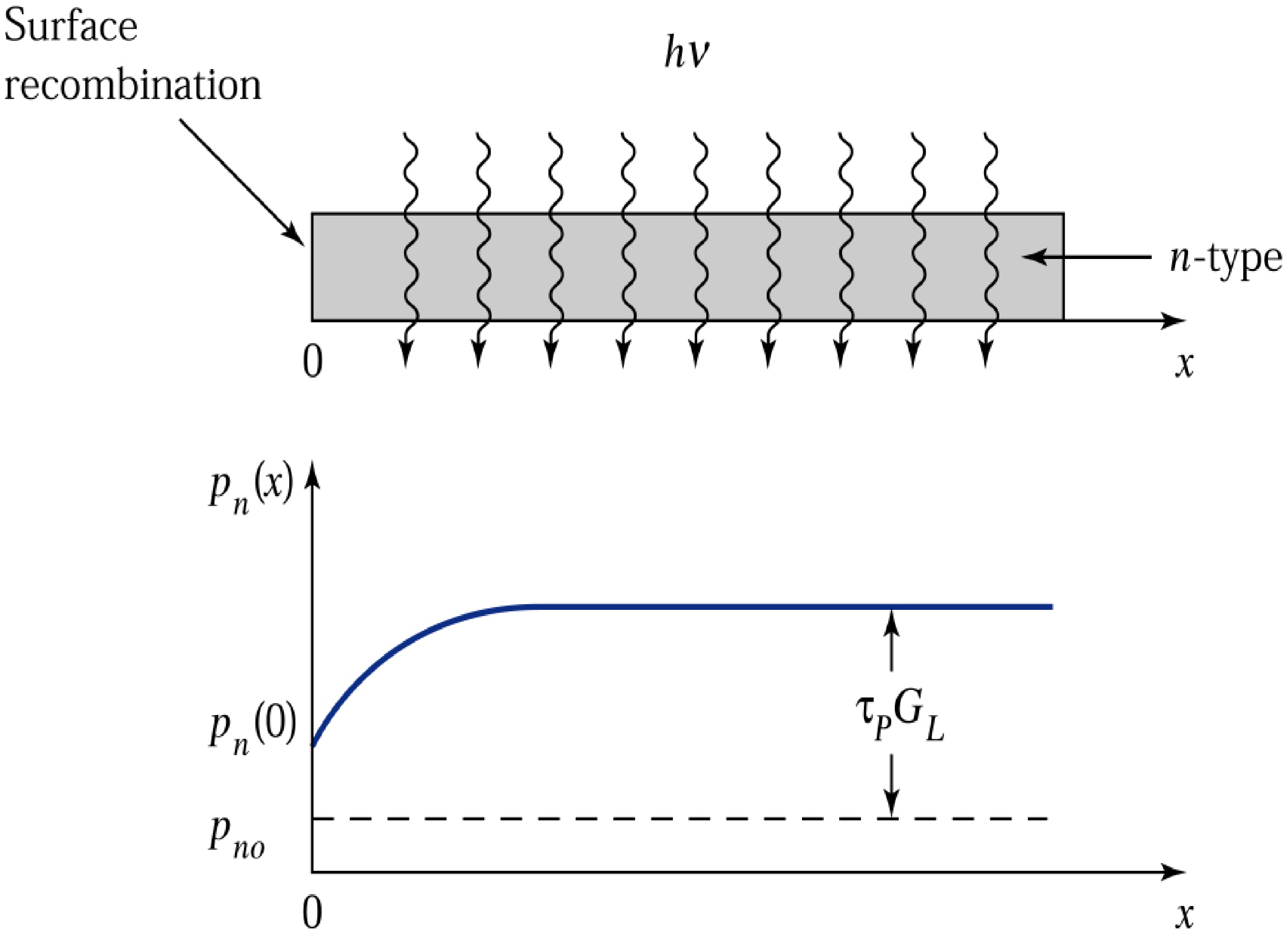


Figure 2.17. Surface recombination at *x* = 0. The minority carrier distribution near the surface is affected by the surface recombination velocity.[6]

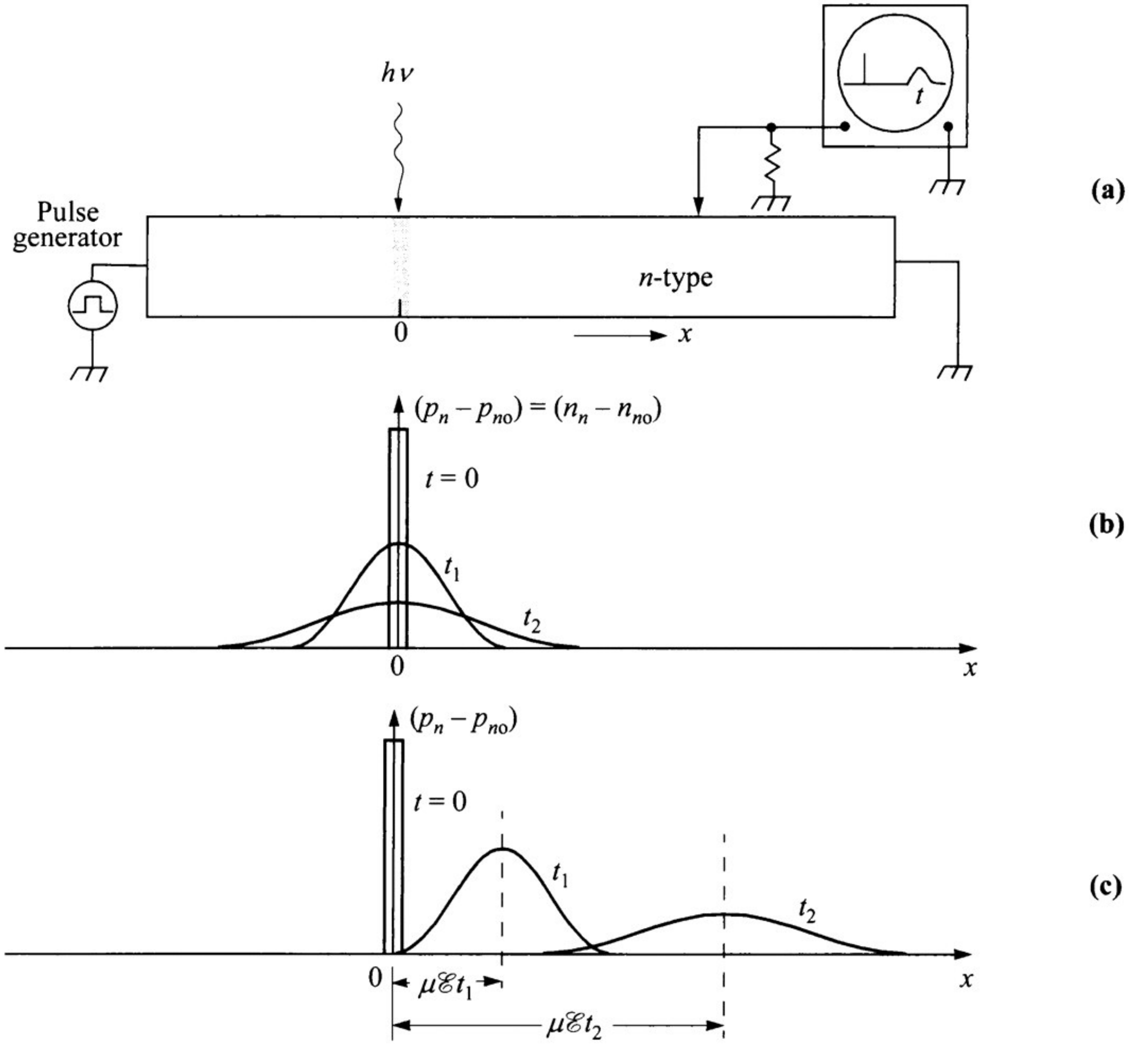
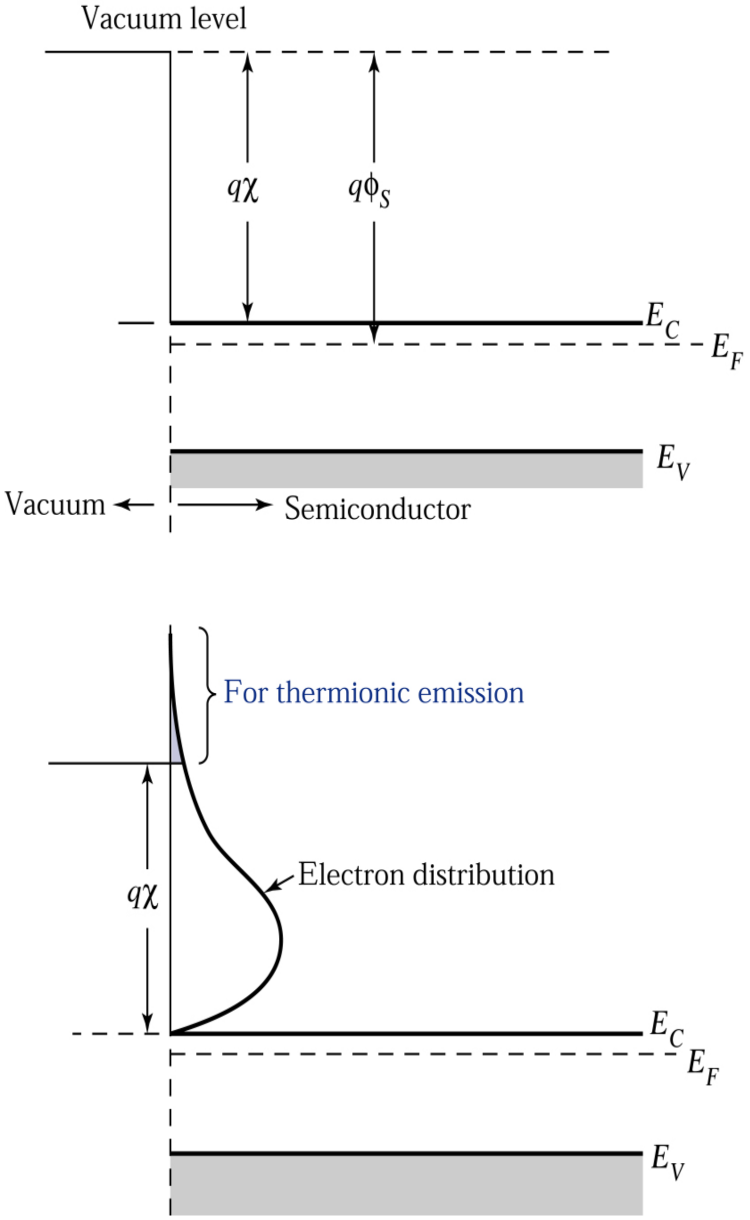


Figure 2.18

Figure 2.18.
The Hayes-Shockley experiment. (*a*) Experimental setup. (*b*) Carrier distributions **without** an applied field.
(*c*) Carrier distributions **with** an applied field.[7]

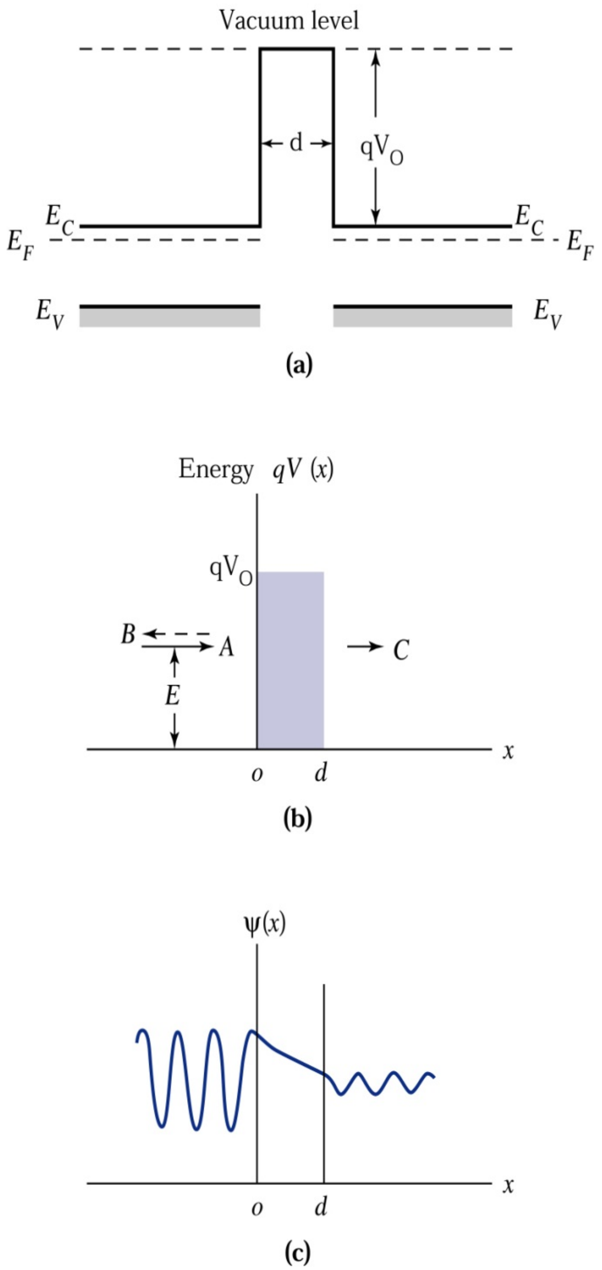
Figure 2.19.
(*a*) The band diagram of an isolated *n*-type semiconductor.
(*b*) The thermionic emission process.



$$n_{th} = \int_{q\chi}^{\infty} n(E)dE = Nc\exp[-\frac{q(\chi+Vn)}{\kappa T}\ ]$$

$$N=Nc\exp[-(Ec-Ef)/kt]$$

Figure 2.20.
(*a*) The band diagram of two isolated semiconductors with a distance *d*. (*b*) One-dimensional potential barrier. (*c*) Schematic representation of the wave function across the potential barrier.



$$\left[\frac{C}{A}\right]^2 = \exp[-2d\sqrt{2m_n(qV_0 - E)/\hbar^2}$$

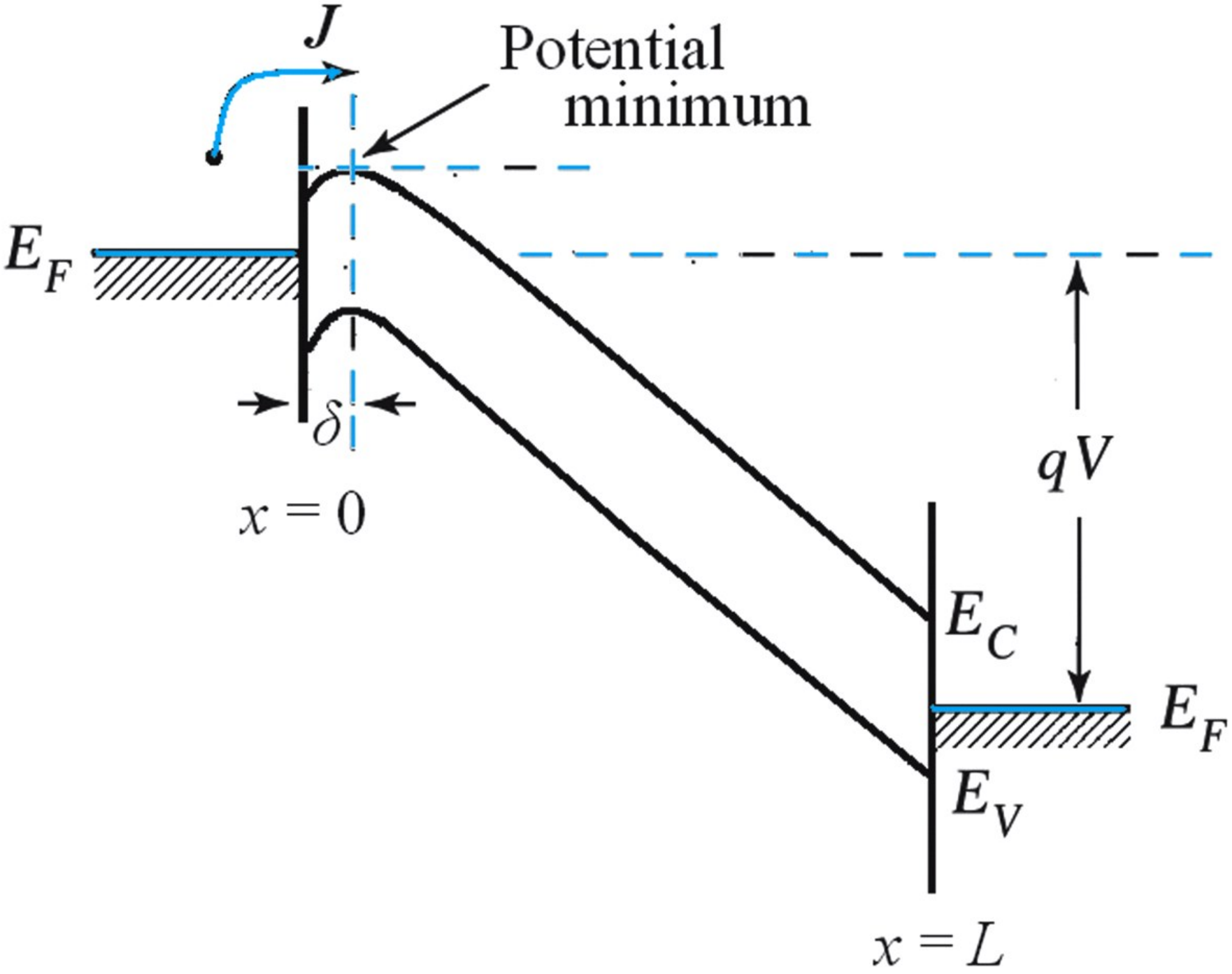


Figure 2.21a
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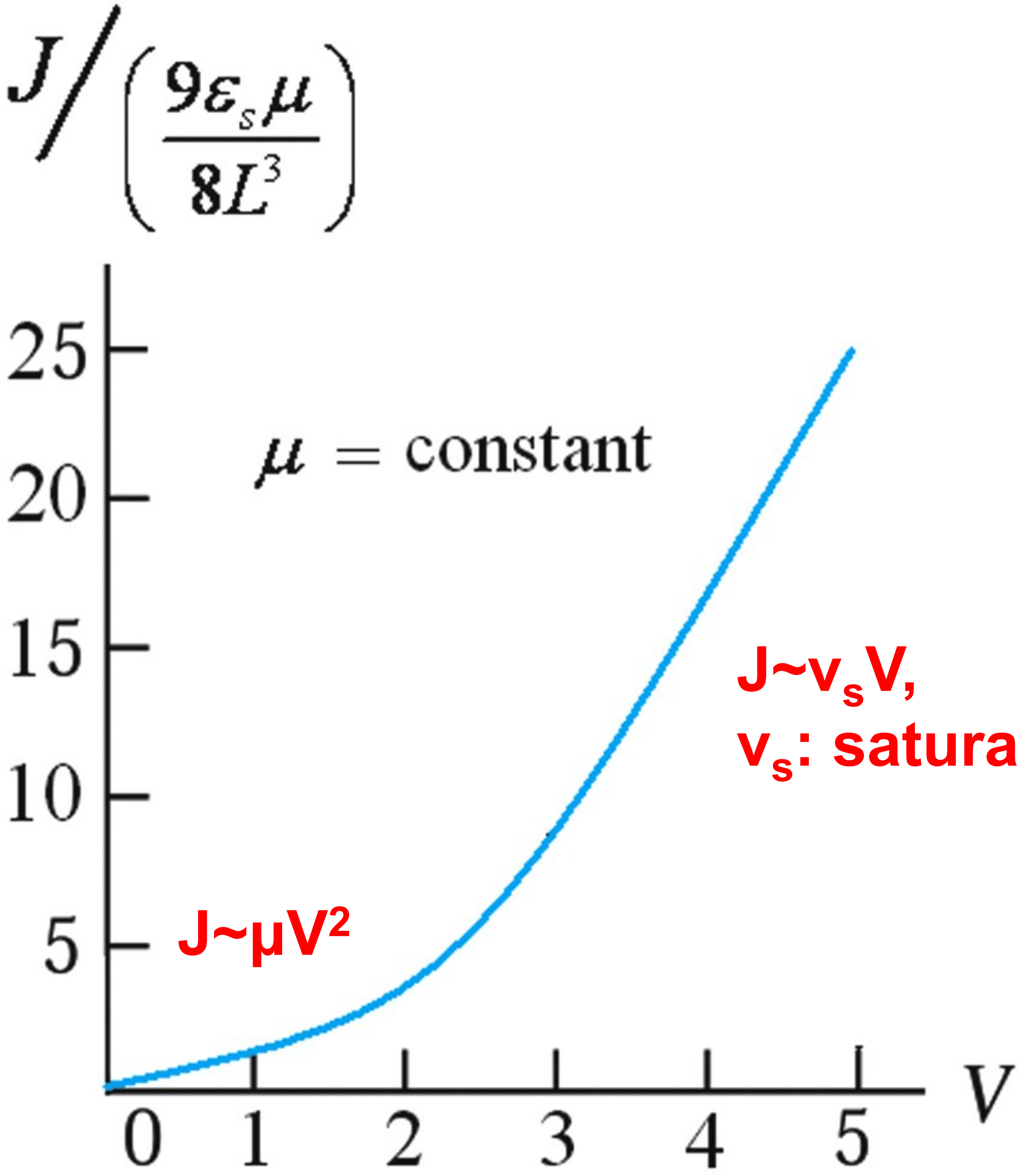
Space charge effects, p.71, 72



$J \sim \mu V^2$

$J \sim v_s V$,
v_s: saturation velocity

Figure 2.21b

$V_{drift} = \mu\ \varepsilon$

●低電場, v正比ε, 即μ定值

$\varepsilon \uparrow$ ➡ $\mu \downarrow$

●高電場, v_{drift}=定值,
飽和了

★當 $v_{drift} \longrightarrow v_{th}$時,
碰撞時間τ_c ↓
($\mu_n \equiv q\tau_c/m_n$)

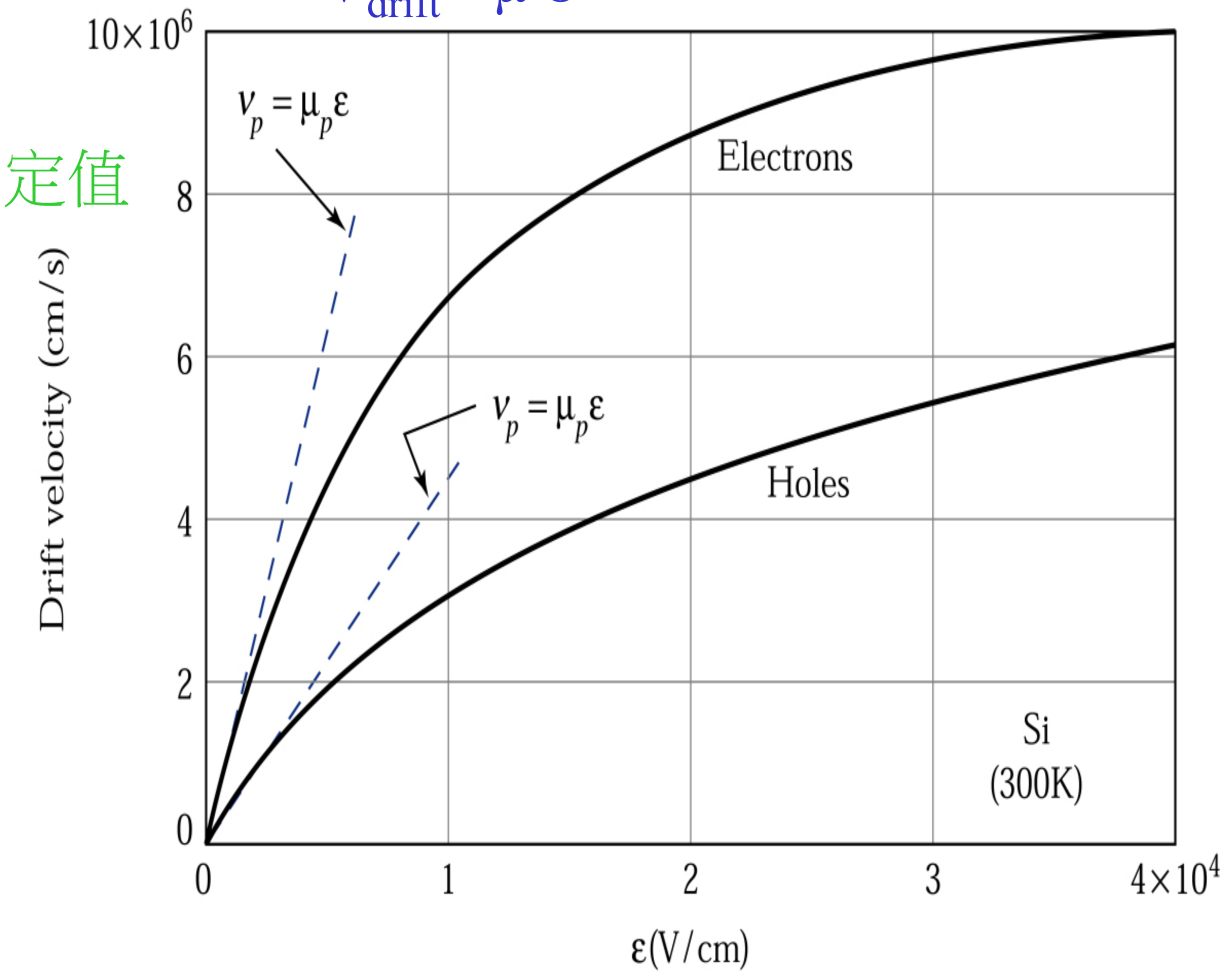


Figure 2.22. Drift velocity versus electric field in Si.[8]

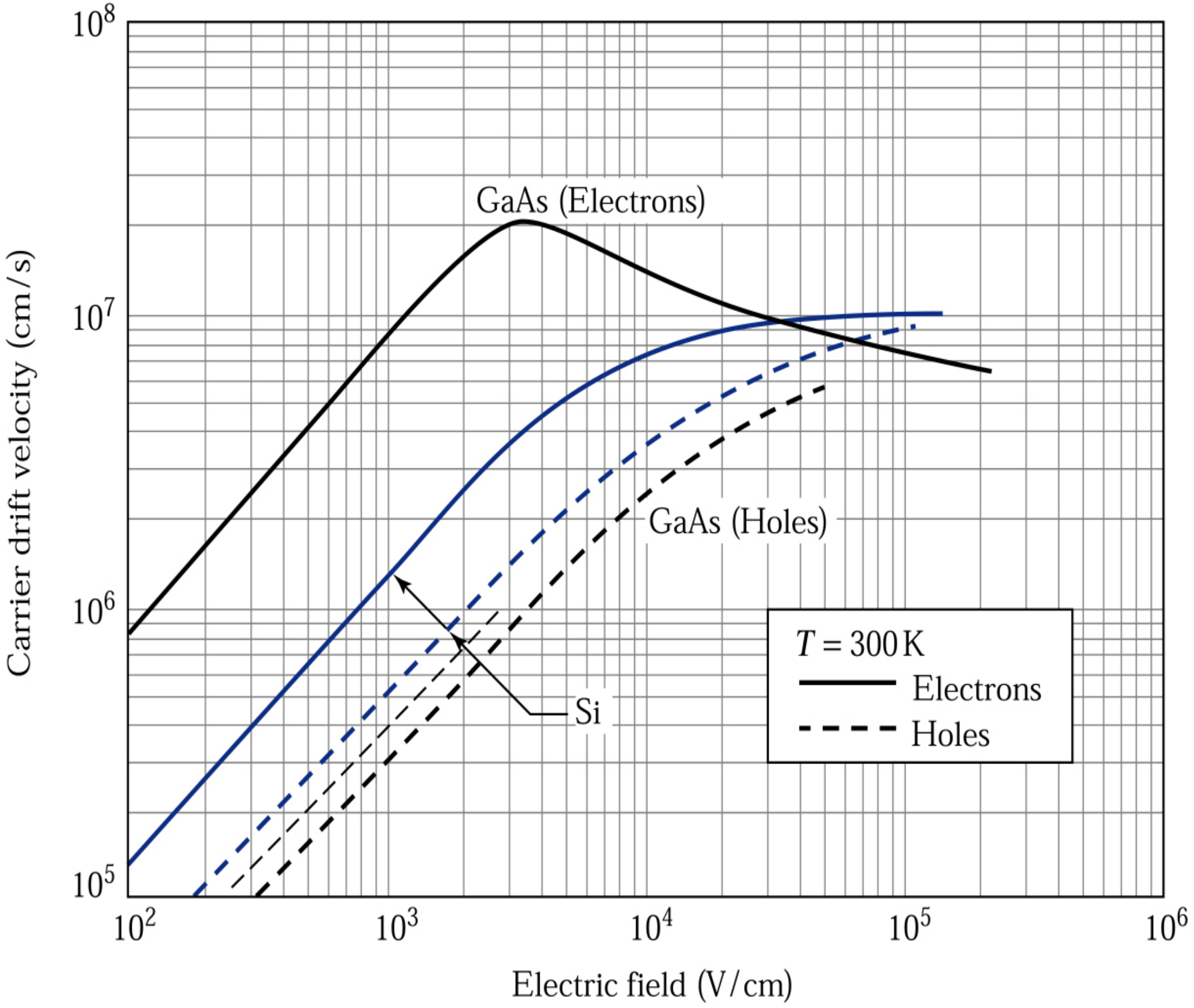


Figure 2.23. Drift velocity versus electric field in Si and GaAs. Note that for ***n*-type GaAs, there is a region of negative differential mobility.**[8,9]

$\varepsilon_b < \varepsilon$

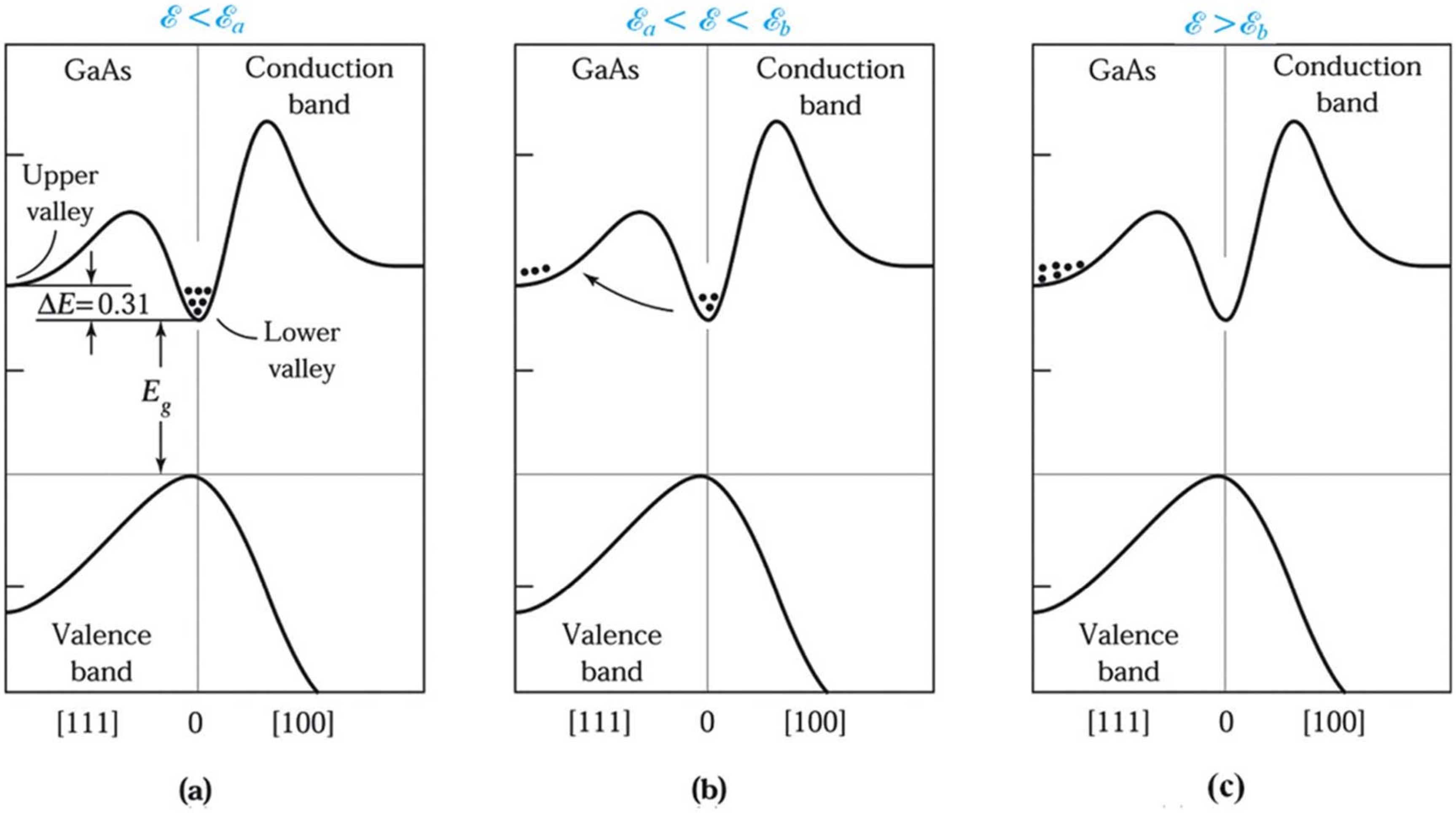


Figure 2.24

Figure 2.24. Electron distributions under various conditions of electric fields for a two-valley semiconductor.

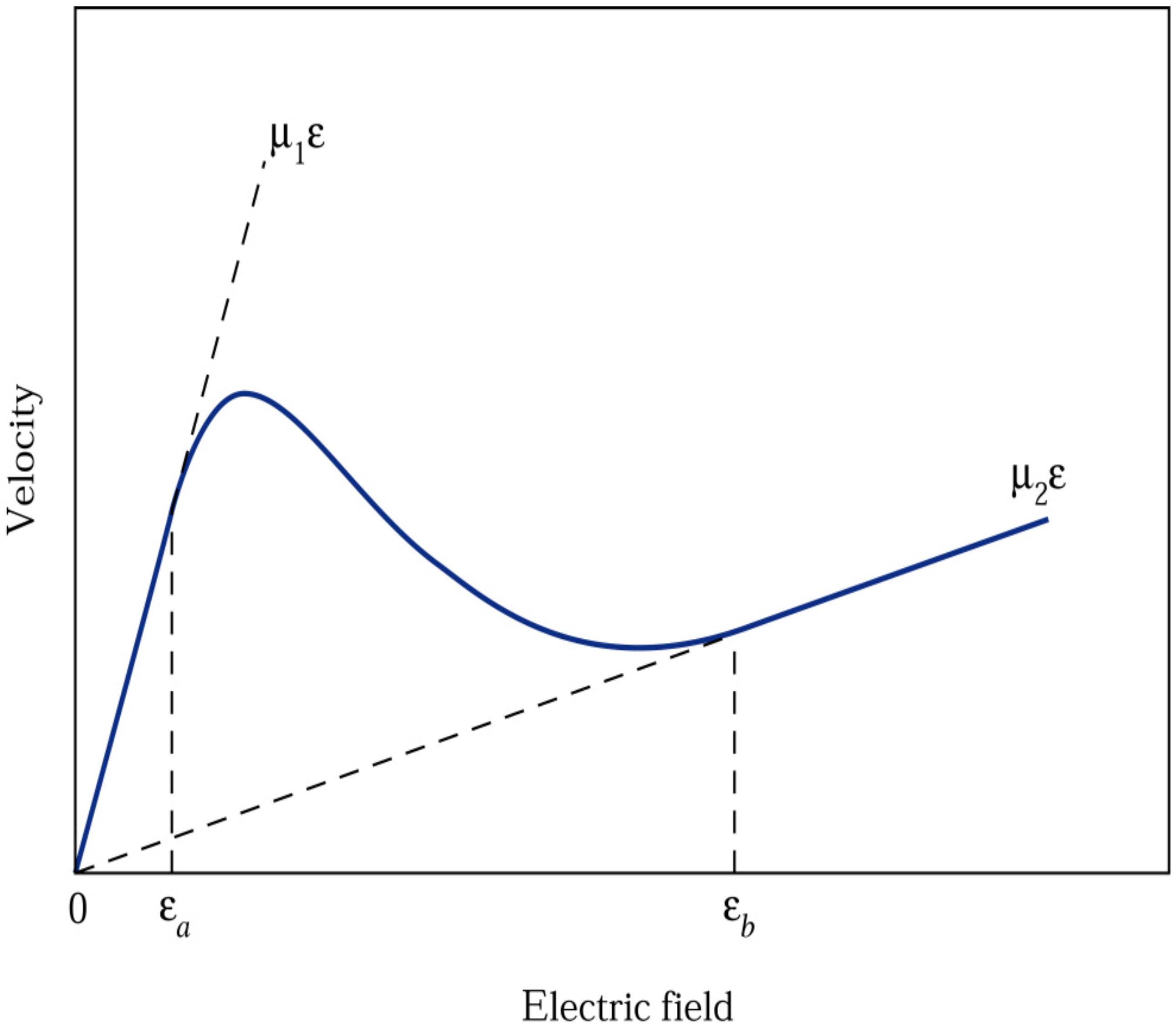


Figure 2.25. One possible velocity-field characteristic of a two-valley semiconductor.

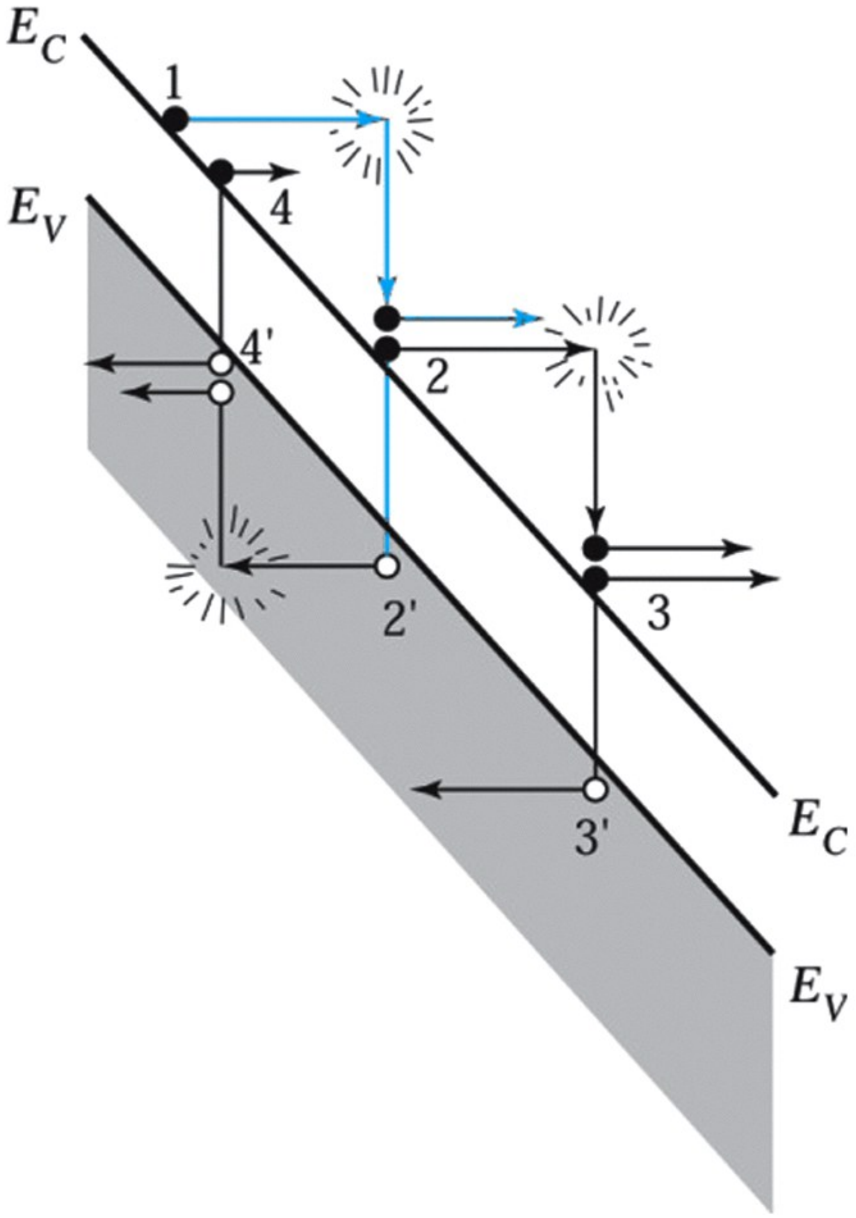
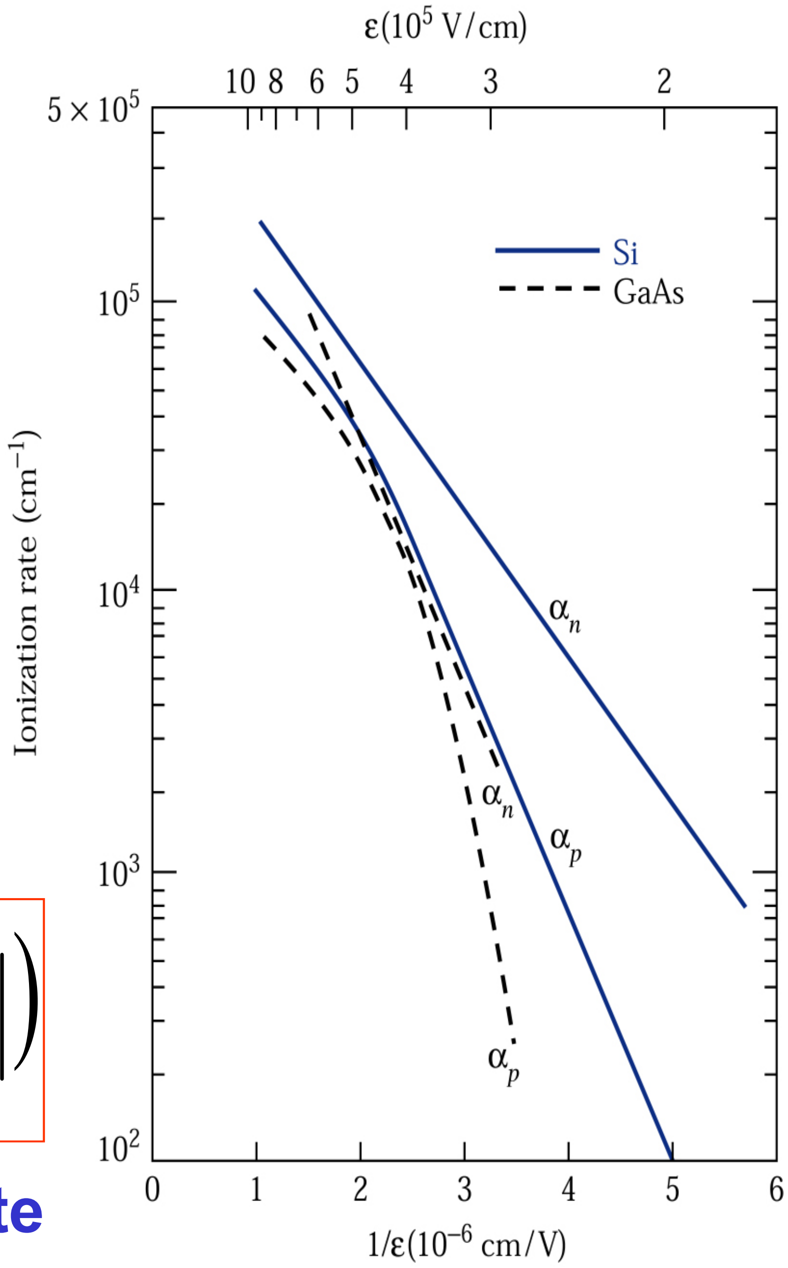


Figure 2.26

Figure 2.26.
Energy band diagram for the avalanche process.

Figure 2.27.
Measured ionization rates versus reciprocal field for Si and GaAs.[9]



α: ionization rate

$$\boxed{G_A = \frac{1}{q}\left(\alpha_n \left|J_n\right| + \alpha_p \left|J_p\right|\right)}$$

G_A=e-h pair generation rate